

Justify all your answers to all three problems. Write clearly.

Formulas:

Time dilation; Length contraction : $\Delta t = \gamma \Delta t' \equiv \gamma \Delta t_p$; $L = L_p / \gamma$; $c = 3 \times 10^8 \text{ m/s}$

Lorentz transformation :

$$x' = \gamma(x - vt)$$

$$x = \gamma(x' + vt')$$

$$y' = y, \quad z' = z$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$y = y', \quad z = z'$$

$$t' = \gamma(t - vx/c^2)$$

$$t = \gamma(t' + vx'/c^2)$$

Velocity transformation :

$$u_x' = \frac{u_x - v}{1 - u_x v / c^2}$$

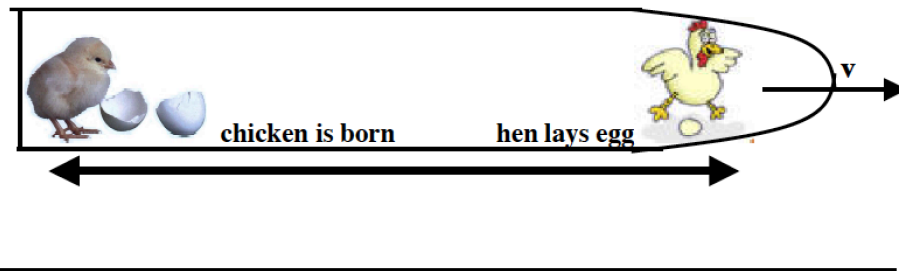
$$u_x = \frac{u_x' + v}{1 + u_x' v / c^2}$$

$$u_y' = \frac{u_y}{\gamma(1 - u_x v / c^2)}$$

$$u_y = \frac{u_y'}{\gamma(1 + u_x' v / c^2)}$$

Relativistic Doppler shift: $f_{obs} = f_{source} \sqrt{(1 + v/c) / (1 - v/c)}$

Problem 1 (10 points)



- ground
- The spaceship shown in the figure is moving at speed v with respect to the ground. According to an observer on the ground, the event in the back of the ship (chicken is born) happened $1\mu\text{s}$ ($=10^{-6}\text{s}$) earlier than the event in the front of the ship (hen lays egg). The length of the spaceship measured by an observer on the ground is 600m.
- (a) How fast is this ship moving if these two events were simultaneous for an observer on the spaceship? Give your answer as v/c . Hint: Use Lorentz transformation to find the answer.
- (b) What is the length of this spaceship as measured by an observer on the spaceship?
- (c) Assume now that $v=0.8c$ rather than the value found in (a), with the same spaceship. Again assume that as seen from the ground chicken event occurs $1\mu\text{s}$ earlier than egg event. Now the events as seen from the spaceship are not simultaneous: how much later was the chicken event than the egg event as seen from the spaceship, in μs ?

Problem 2 (10 points)

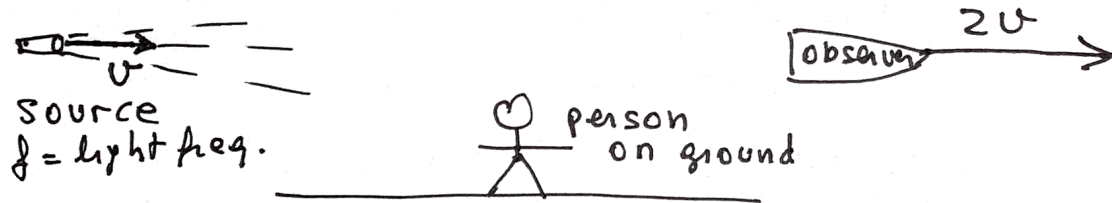
When twins A and B turn 20 years old, twin B departs on a spaceship traveling at speed $0.6c$, twin A stays on Earth. On their respective 21st birthday, both twins A and B lit candles to celebrate.

- How old is twin A when twin B lits up her candle, as measured by clocks in the Earth's reference frame?
- How old is A when the light from the candle lit by B reaches him (as measured in A's reference frame)?
- How old is B when the light from the candle lit by A reaches her (as measured in B's reference frame)?

Hint: ignore any effects that could have resulted from the fact that B was undergoing acceleration for a short period until it reached its traveling speed $0.6c$.

Problem 3 (10 points)

A light source is moving at speed v with respect to the ground. An observer on a spaceship is moving in the same direction as the light source, moving away from the light source at speed $2v$ with respect to the ground, as shown in the picture. Assume $v=0.25c$.



- Find the speed of the spaceship relative to the light source.
- If the frequency of the emitted light is f , what is the frequency measured by the observer on the spaceship, f' ?
- Assume a person on the ground is standing between source and spaceship. Find the frequency f_g that this person measures, in terms of f . Then, assuming this person on the ground emits light with frequency f_g , find the frequency that the spaceship observer would measure for this light, f'' . Show all steps in your calculations. Explain why f'' is larger, smaller or equal to f .