PHYSICS 2D FINAL EXAM FALL QUARTER 2020
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Formulas:

Time dilation; Length contraction: $\Delta t = \gamma \Delta t' \equiv \gamma\, \Delta t_p$; $L = L_p / \gamma$; $c = 3 \times 10^8 m/s$

Lorentz transformation: $x' = \gamma(x - vt)$; $y' = y$; $z' = z$; $t' = \gamma(t - vx/c^2)$; inverse: $v \to -v$

Velocity transformation: $u_x' = \dfrac{u_x - v}{1 - u_x v / c^2}$; $u_y' = \dfrac{u_y}{\gamma(1 - u_x v / c^2)}$; inverse: $v \to -v$

Spacetime interval: $(\Delta s)^2 = (c\Delta t)^2 - [\Delta x^2 + \Delta y^2 + \Delta z^2]$

Relativistic Doppler shift: $f_{obs} = f_{source} \sqrt{1 + v/c} \;/\; \sqrt{1 - v/c}$

Momentum: $\vec{p} = \gamma\, m \vec{u}$; Energy: $E = \gamma\, mc^2$; Kinetic energy: $K = (\gamma - 1) mc^2$

Rest energy: $E_0 = mc^2$; $E = \sqrt{p^2 c^2 + m^2 c^4}$

Electron: $m_e = 0.511\, MeV/c^2$ Proton: $m_p = 938.26\, MeV/c^2$ Neutron: $m_n = 939.55\, MeV/c^2$

Atomic mass unit: $1\, u = 931.5\, MeV/c^2$; electron volt: $1 \text{eV} = 1.6 \times 10^{-19}$ J

Stefan's law: $e_{tot} = \sigma T^4$, e_{tot} = power/unit area ; $\sigma = 5.67 \times 10^{-8} W/m^2 K^4$

$e_{tot} = cU/4$, U = energy density = $\int_0^\infty u(\lambda, T) d\lambda$; Wien's law: $\lambda_m T = \dfrac{hc}{4.96 k_B}$

Boltzmann distribution: $P(E) = Ce^{-E/(k_B T)}$

Planck's law: $u_\lambda(\lambda, T) = N_\lambda(\lambda) \times \overline{E}(\lambda, T) = \dfrac{8\pi}{\lambda^4} \times \dfrac{hc/\lambda}{e^{hc/\lambda k_B T} - 1}$; $N(f) = \dfrac{8\pi f^2}{c^3}$

Photons: $E = hf = pc$; $f = c/\lambda$; $hc = 12{,}400\ eV\,A$; $k_B = (1/11{,}600) eV/K$

Photoelectric effect: $eV_s = K_{max} = hf - \phi$, $\phi \equiv$ work function;

Compton scattering: $\lambda' - \lambda = \dfrac{h}{m_e c}(1 - \cos\theta)$; $\dfrac{h}{m_e c} = 0.0243 A$

Coulomb force: $F = \dfrac{kq_1 q_2}{r^2}$; Coulomb energy: $U = \dfrac{kq_1 q_2}{r}$; Coulomb potential: $V = \dfrac{kq}{r}$

Force in electric and magnetic fields (Lorentz force): $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

Rutherford scattering: $\Delta n = C \dfrac{Z^2}{K_\alpha^2} \dfrac{1}{\sin^4(\phi/2)}$ $ke^2 = 14.4\ eV\,A$

Hydrogen spectrum: $\dfrac{1}{\lambda_{mn}} = R(\dfrac{1}{m^2} - \dfrac{1}{n^2})$; $R = 1.097 \times 10^7\ m^{-1} = \dfrac{1}{911.3 A}$; $\hbar c = 1973\ eV\ A$

Bohr atom: $E_n = -\dfrac{ke^2 Z}{2r_n} = -E_0 \dfrac{Z^2}{n^2}$; $E_0 = \dfrac{ke^2}{2a_0} = \dfrac{m_e (ke^2)}{2\hbar^2} = 13.6 eV$; $K = \dfrac{m_e v^2}{2}$; $U = -\dfrac{ke^2 Z}{r}$

$hf = E_i - E_f$; $r_n = r_0 n^2$; $r_0 = \dfrac{a_0}{Z}$; $a_0 = \dfrac{\hbar^2}{m_e ke^2} = 0.529 A$; $L = m_e vr = n\hbar$ angular momentum

de Broglie: $\lambda = \dfrac{h}{p}$; $f = \dfrac{E}{h}$; $\omega = 2\pi f$; $k = \dfrac{2\pi}{\lambda}$; $E = \hbar\omega$; $p = \hbar k$; $E = \dfrac{p^2}{2m}$

Wave packets: $y(x,t) = \sum_j a_j \cos(k_j x - \omega_j t)$, or $y(x,t) = \int dk\, a(k)\, e^{i(kx - \omega(k)t)}$; $\Delta k \Delta x \sim 1$; $\Delta\omega \Delta t \sim 1$

group and phase velocity : $v_g = \frac{d\omega}{dk}$; $v_p = \frac{\omega}{k}$; Heisenberg: $\Delta x \Delta p \sim \hbar$; $\Delta t \Delta E \sim \hbar$

Probability: $P(x)dx = |\Psi(x)|^2\, dx$; $P(a \le x \le b) = \int_a^b dx P(x)$ $\hbar c = 1973\ eVA$

Schrodinger equation: $-\frac{\hbar^2}{2\mathrm{m}}\frac{\partial^2 \Psi}{\partial x^2} + \mathrm{U(x)\Psi(x,t)} = \mathrm{i}\hbar\frac{\partial \Psi}{\partial t}$; $\Psi(\mathrm{x,t}) = \psi(\mathrm{x})e^{-\mathrm{i}\frac{E}{\hbar}t}$

Time – independent Schrodinger equation: $-\frac{\hbar^2}{2\mathrm{m}}\frac{\partial^2 \psi}{\partial x^2} + \mathrm{U(x)}\psi(\mathrm{x}) = \mathrm{E}\psi(\mathrm{x})$; $\int_{-\infty}^{\infty} dx\ \psi^* \psi = 1$

∞ square well: $\psi_n(x) = \sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L})$; $E_n = \frac{\pi^2\hbar^2 n^2}{2mL^2}$; $\frac{\hbar^2}{2m_e} = 3.81 eVA^2$ (electron)

Harmonic oscillator: $\Psi_n(x) = H_n(x)e^{-\frac{m\omega}{2\hbar}x^2}$; $E_n = (n+\frac{1}{2})\hbar\omega$; $E = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2 A^2$

Expectation value of$[Q]: \langle Q \rangle = \int \psi^*(x)[Q]\psi(x)\,dx$; Momentum operator: $p = \frac{\hbar}{i}\frac{\partial}{\partial x}$

Eigenvalues and eigenfunctions: $[\mathrm{Q}]\,\Psi = q\,\Psi$ (q is a constant) ; uncertainty: $\Delta Q = \sqrt{\langle Q^2 \rangle - \langle Q \rangle^2}$

Step potential: reflection coef: $R = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}$, $T = 1 - R$; $k = \sqrt{\frac{2m}{\hbar^2}(E - U)}$

Tunneling: $\psi(x) \sim \mathrm{e}^{-\alpha x}$; $T = e^{-2\alpha\Delta x}$; $T = e^{-2\int_{x_1}^{x_2}\alpha(x)dx}$; $\alpha(x) = \sqrt{\frac{2m[U(x) - E]}{\hbar^2}}$

Schrodinger equation in 3D: $-\frac{\hbar^2}{2\mathrm{m}}\nabla^2\Psi + \mathrm{U}(\vec{\mathrm{r}})\Psi(\vec{\mathrm{r}},\mathrm{t}) = \mathrm{i}\hbar\frac{\partial \Psi}{\partial t}$; $\Psi(\vec{\mathrm{r}},\mathrm{t}) = \psi(\vec{\mathrm{r}})e^{-\mathrm{i}\frac{E}{\hbar}t}$

3D square well: $\Psi(\mathrm{x,y,z}) = \Psi_1(x)\Psi_2(y)\Psi_3(z)$; $\mathrm{E} = \frac{\pi^2\hbar^2}{2m}(\frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2} + \frac{n_3^2}{L_3^2})$

Spherically symmetric potential: $\Psi_{n,\ell,m_\ell}(r,\theta,\phi) = R_{n\ell}(r)Y_\ell^{m_\ell}(\theta,\phi)$; $Y_\ell^{m_\ell}(\theta,\phi) = P_\ell^{m_\ell}(\theta)e^{im_\ell\phi}$

Angular momentum: $\vec{\mathrm{L}} = \vec{r} \times \vec{p}$; $[L_z] = \frac{\hbar}{i}\frac{\partial}{\partial\phi}$; $[L^2]Y_\ell^{m_\ell} = \ell(\ell+1)\hbar^2 Y_\ell^{m_\ell}$; $[\mathrm{L_z}]Y_\ell^{m_\ell} = m_\ell\hbar Y_\ell^{m_\ell}$

Radial probability density: $P(r) = r^2 |R_{n\ell}(r)|^2$; Energy: $\mathrm{E_n} = -(ke^2/2a_0)(Z^2/n^2)$

Ground state of hydrogen-like ions: $\Psi_{1,0,0} = \frac{1}{\pi^{1/2}}(\frac{Z}{a_0})^{3/2}e^{-Zr/a_0}$; $\int_0^\infty dr\ \mathrm{r}^n e^{-\lambda r} = \frac{n!}{\lambda^{n+1}}$

Orbital magnetic moment: $\vec{\mu} = \frac{-e}{2m_e}\vec{L}$; $\mu_z = -\mu_B m_l$; $\mu_B = \frac{e\hbar}{2m_e} = 5.79\times10^{-5} eV/T$

Spin 1/2: $s = \frac{1}{2}$, $|S| = \sqrt{s(s+1)}\hbar$; $S_z = m_s\hbar$; $m_s = \pm 1/2$; $\vec{\mu}_s = \frac{-e}{2m_e}g\vec{S}$

Orbital + spin mag moment: $\vec{\mu} = \frac{-e}{2m_e}(\vec{L} + g\vec{S})$; Energy in mag. field: $U = -\vec{\mu}\cdot\vec{B}$

$\vec{J} = \vec{L} + \vec{S}$; $|\vec{J}| = \sqrt{j(j+1)}\hbar$; $|\ell - s| \le j \le \ell + s$

Two particles : $\Psi(\vec{r}_1, \vec{r}_2) = +/- \,\Psi(\vec{r}_2, \vec{r}_1)$; symmetric/antisymmetric