PHYSICS 2D FINAL EXAM (9 problems) FALL QUARTER 2020
PROF. HIRSCH

Formulas:

Time dilation; Length contraction: $\Delta t = \gamma \Delta t' \equiv \gamma\, \Delta t_p$; $L = L_p / \gamma$; $c = 3 \times 10^8 m/s$

Lorentz transformation: $x' = \gamma(x - vt)$; $y' = y$; $z' = z$; $t' = \gamma(t - vx/c^2)$; inverse: $v \rightarrow -v$

Velocity transformation: $u_x' = \dfrac{u_x - v}{1 - u_x v / c^2}$; $u_y' = \dfrac{u_y}{\gamma(1 - u_x v / c^2)}$; inverse: $v \rightarrow -v$

Spacetime interval: $(\Delta s)^2 = (c\Delta t)^2 - [\Delta x^2 + \Delta y^2 + \Delta z^2]$

Relativistic Doppler shift: $f_{obs} = f_{source}\sqrt{1 + v/c}\ /\ \sqrt{1 - v/c}$

Momentum: $\vec{\mathrm{p}} = \gamma m \vec{u}$; Energy: $E = \gamma mc^2$; Kinetic energy: $K = (\gamma - 1)mc^2$

Rest energy: $E_0 = mc^2$; $E = \sqrt{p^2c^2 + m^2c^4}$

Electron: $m_e = 0.511\ MeV/c^2$ Proton: $m_p = 938.26\ MeV/c^2$ Neutron: $m_n = 939.55\ MeV/c^2$

Atomic mass unit: $1\ u = 931.5\ MeV/c^2$; electron volt: $1\text{eV} = 1.6 \times 10^{-19}$ J

Stefan's law: $e_{tot} = \sigma T^4$, e_{tot} = power/unit area ; $\sigma = 5.67 \times 10^{-8} W/m^2K^4$

$e_{tot} = cU/4$, U = energy density = $\int_0^\infty u(\lambda,T)d\lambda$; Wien's law: $\lambda_m T = \dfrac{hc}{4.96 k_B}$

Boltzmann distribution: $P(E) = Ce^{-E/(k_BT)}$

Planck's law: $u_\lambda(\lambda,T) = N_\lambda(\lambda) \times \overline{E}(\lambda,T) = \dfrac{8\pi}{\lambda^4} \times \dfrac{hc/\lambda}{e^{hc/\lambda k_B T} - 1}$; $N(f) = \dfrac{8\pi f^2}{c^3}$

Photons: $E = hf = pc$; $f = c/\lambda$; $hc = 12{,}400\ eV A$; $k_B = (1/11{,}600) eV/K$

Photoelectric effect: $eV_s = K_{\max} = hf - \phi$, $\phi \equiv$ work function;

Compton scattering: $\lambda' - \lambda = \dfrac{h}{m_e c}(1 - \cos\theta)$; $\dfrac{h}{m_e c} = 0.0243 A$

Coulomb force: $F = \dfrac{kq_1q_2}{r^2}$; Coulomb energy: $U = \dfrac{kq_1q_2}{r}$; Coulomb potential: $V = \dfrac{kq}{r}$

Force in electric and magnetic fields (Lorentz force): $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

Rutherford scattering: $\Delta n = C \dfrac{Z^2}{K_\alpha^2} \dfrac{1}{\sin^4(\phi/2)}$ $ke^2 = 14.4\ eV A$

Hydrogen spectrum: $\dfrac{1}{\lambda_{mn}} = R(\dfrac{1}{m^2} - \dfrac{1}{n^2})$; $R = 1.097 \times 10^7\ m^{-1} = \dfrac{1}{911.3 A}$; $\hbar c = 1973\ eV\ A$

Bohr atom: $E_n = -\dfrac{ke^2 Z}{2r_n} = -E_0 \dfrac{Z^2}{n^2}$; $E_0 = \dfrac{ke^2}{2a_0} = \dfrac{m_e(ke^2)}{2\hbar^2} = 13.6 eV$; $K = \dfrac{m_e v^2}{2}$; $U = -\dfrac{ke^2 Z}{r}$

$hf = E_i - E_f$; $r_n = r_0 n^2$; $r_0 = \dfrac{a_0}{Z}$; $a_0 = \dfrac{\hbar^2}{m_e ke^2} = 0.529 A$; $L = m_e vr = n\hbar$ angular momentum

de Broglie: $\lambda = \dfrac{h}{p}$; $f = \dfrac{E}{h}$; $\omega = 2\pi f$; $k = \dfrac{2\pi}{\lambda}$; $E = \hbar\omega$; $p = \hbar k$; $E = \dfrac{p^2}{2m}$

Wave packets: $y(x,t) = \sum_j a_j \cos(k_j x - \omega_j t)$, or $y(x,t) = \int dk\, a(k)\, e^{i(kx - \omega(k)t)}$; $\Delta k \Delta x \sim 1$; $\Delta\omega\Delta t \sim 1$

group and phase velocity : $v_g = \dfrac{d\omega}{dk}$; $v_p = \dfrac{\omega}{k}$; Heisenberg: $\Delta x \Delta p \sim \hbar$; $\Delta t \Delta E \sim \hbar$

Probability: $P(x)dx = |\Psi(x)|^2\, dx$; $P(a \le x \le b) = \int_a^b dx P(x)$ $\hbar c = 1973\ eVA$

Schrodinger equation: $-\dfrac{\hbar^2}{2\mathrm{m}}\dfrac{\partial^2 \Psi}{\partial x^2} + \mathrm{U(x)}\Psi(\mathrm{x,t}) = \mathrm{i}\hbar\dfrac{\partial \Psi}{\partial t}$; $\Psi(\mathrm{x,t}) = \psi(\mathrm{x})\mathrm{e}^{-\mathrm{i}\frac{\mathrm{E}}{\hbar}t}$

Time – independent Schrodinger equation: $-\dfrac{\hbar^2}{2\mathrm{m}}\dfrac{\partial^2 \psi}{\partial x^2} + \mathrm{U(x)}\psi(\mathrm{x}) = \mathrm{E}\psi(\mathrm{x})$; $\int_{-\infty}^{\infty} dx\ \psi^* \psi = 1$

∞ square well: $\psi_{\mathrm{n}}(x) = \sqrt{\dfrac{2}{L}}\sin(\dfrac{n\pi x}{L})$; $E_n = \dfrac{\pi^2\hbar^2 n^2}{2mL^2}$; $\dfrac{\hbar^2}{2m_e} = 3.81 eVA^2$ (electron)

Harmonic oscillator: $\Psi_{\mathrm{n}}(x) = H_n(x)e^{-\frac{m\omega}{2\hbar}x^2}$; $E_n = (n+\dfrac{1}{2})\hbar\omega$; $E = \dfrac{p^2}{2m} + \dfrac{1}{2}m\omega^2 x^2 = \dfrac{1}{2}m\omega^2 A^2$

Expectation value of$[Q]$: $< Q >= \int \psi^*(x)[Q]\psi(x)\ dx$; Momentum operator: $p = \dfrac{\hbar}{i}\dfrac{\partial}{\partial x}$

Eigenvalues and eigenfunctions: $[\mathrm{Q}]\ \Psi = q\ \Psi$ (q is a constant) ; uncertainty: $\Delta Q = \sqrt{< Q^2 > - < Q >^2}$

Step potential: reflection coef : $R = \dfrac{(k_1 - k_2)^2}{(k_1 + k_2)^2}$, $T = 1 - R$; $k = \sqrt{\dfrac{2m}{\hbar^2}(E - U)}$

Tunneling: $\psi(x) \sim \mathrm{e}^{-\alpha x}$; $T = e^{-2\alpha\Delta x}$; $T = e^{-2\int_{x_1}^{x_2}\alpha(x)dx}$; $\alpha(x) = \sqrt{\dfrac{2m[U(x) - E]}{\hbar^2}}$

Schrodinger equation in 3D: $-\dfrac{\hbar^2}{2\mathrm{m}}\nabla^2\Psi + \mathrm{U}(\vec{\mathrm{r}})\Psi(\vec{\mathrm{r}},\mathrm{t}) = \mathrm{i}\hbar\dfrac{\partial \Psi}{\partial t}$; $\Psi(\vec{\mathrm{r}},\mathrm{t}) = \psi(\vec{\mathrm{r}})\mathrm{e}^{-\mathrm{i}\frac{\mathrm{E}}{\hbar}t}$

3D square well: $\Psi(\mathrm{x,y,z}) = \Psi_1(x)\Psi_2(y)\Psi_3(z)$; $\mathrm{E} = \dfrac{\pi^2\hbar^2}{2m}(\dfrac{n_1^2}{L_1^2} + \dfrac{n_2^2}{L_2^2} + \dfrac{n_3^2}{L_3^2})$

Spherically symmetric potential: $\Psi_{\mathrm{n},\ell,\mathrm{m}_\ell}(r,\theta,\phi) = R_{n\ell}(r)Y_\ell^{m_\ell}(\theta,\phi)$; $Y_\ell^{m_\ell}(\theta,\phi) = P_\ell^{m_\ell}(\theta)e^{im_\ell\phi}$

Angular momentum: $\vec{\mathrm{L}} = \vec{r} \times \vec{p}$; $[L_z] = \dfrac{\hbar}{i}\dfrac{\partial}{\partial\phi}$; $[L^2]Y_\ell^{m_\ell} = \ell(\ell+1)\hbar^2 Y_\ell^{m_\ell}$; $[\mathrm{L_z}]Y_\ell^{m_\ell} = m_\ell\hbar Y_\ell^{m_\ell}$

Radial probability density: $P(r) = r^2 |R_{n\ell}(r)|^2$; Energy: $\mathrm{E_n} = -(ke^2/2a_0)(Z^2/n^2)$

Ground state of hydrogen-like ions: $\Psi_{1,0,0} = \dfrac{1}{\pi^{1/2}}(\dfrac{Z}{a_0})^{3/2}e^{-Zr/a_0}$; $\int_0^\infty dr\ \mathrm{r}^n e^{-\lambda r} = \dfrac{n!}{\lambda^{n+1}}$

Orbital magnetic moment: $\vec{\mu} = \dfrac{-e}{2m_e}\vec{L}$; $\mu_z = -\mu_B m_l$; $\mu_{\mathrm{B}} = \dfrac{e\hbar}{2m_e} = 5.79 \times 10^{-5} eV/T$

Spin 1/2: $s = \dfrac{1}{2}$, $|S| = \sqrt{s(s+1)}\hbar$; $S_z = m_s\hbar$; $m_s = \pm 1/2$; $\vec{\mu}_s = \dfrac{-e}{2m_e}g\vec{S}$

Orbital + spin mag moment: $\vec{\mu} = \dfrac{-e}{2m_e}(\vec{L} + g\vec{S})$; Energy in mag. field: $U = -\vec{\mu}\cdot\vec{B}$

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$\vec{J} = \vec{L} + \vec{S}$; $|\vec{J}| = \sqrt{j(j+1)}\hbar$; $|\ell - s| \leq j \leq \ell + s$

Two particles : $\Psi(\vec{r}_1, \vec{r}_2) = +/- \Psi(\vec{r}_2, \vec{r}_1)$; symmetric/antisymmetric

Problem 1 (10 points)
An electron has kinetic energy 511,000 eV.
(a) Find its speed. Give your answer as v/c.
(b) Find its momentum. Give your answer in units eV/c.
(c) According to a clock in the reference frame of this electron, how long does it take for this electron to travel from the earth to the sun? It takes light 500 seconds to travel from the sun to the earth. Give your answer in seconds. Justify your answer.

Problem 2 (10 points)
Assume the temperature at the surface of the sun is 5800K, and at the surface of the earth is 300K. Assume the earth is a black body and there is nothing between the sun and the earth that blocks radiation coming from the sun on the earth, and that you can ignore any radiation from other sources (e.g. stars) incident on the earth.
(a) At which wavelength does the earth emit maximum radiation per unit wavelength? Give your answer in A.
(b) At which wavelength does the earth absorb maximum radiation per unit wavelength? Give your answer in A.
(c) How much total power is incident on the earth from the sun? Give your answer in W. The radius of the earth is 6378 km. Calculate and justify your answer, don't look it up.

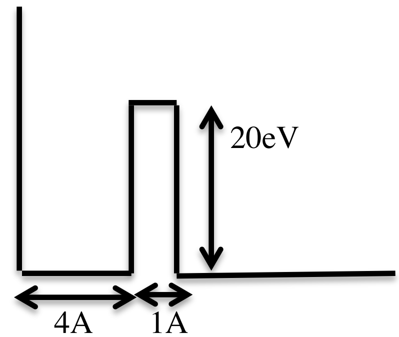
Problem 3 (10 points)
An electron moving in one dimension is described by the wavefunction

$$\psi(x) = Ce^{-|x|/a}$$

(a) Find C in terms of a.
(b) Find the uncertainty in the position of this electron, Δx. Give your answer in terms of a.
(c) What is the probability of finding the electron at distance smaller than a from the origin?
Hint: use that the wavefunction is even to simplify the calculations.

Problem 4 (10 points)
An electron is in the ground state of a one-dimensional harmonic oscillator potential. It can only absorb photons of wavelength 5000A.
(a) Find the energy of this electron, in eV. Justify your answer.
(b) Find the classical amplitude of oscillation for this electron, in Angstrom.
(c) How much more or less likely is it to find this electron at x=0 versus finding it at the classical amplitude position (with x>0)? Give the ratio of probabilities.

Problem 5 (10 points)



An electron is in the ground state of the potential well shown in the figure of width 4A. On the left the wall is infinitely high, on the right it is 20eV high and 1A in width.
(a) Calculate the ground state energy assuming it is an infinite well.
(b) Find a better estimate for the ground state energy taking into account the finite height of the barrier.
(c) The electron travels back and forth inside the well. Estimate the probability that when it hits the right wall it will tunnel through and escape.

Problem 6 (10 points)
There are 7 electrons in a two-dimensional box of side length 5A. The electrons have spin and obey the Pauli exclusion principle. The system is in its ground state.
(a) What is the largest wavelength photon (in A) that this system can absorb?
(b) What is the next largest wavelength photon (in A) that this system can absorb?
(c) What is the ground state energy of this system? Give your answer in eV.

Problem 7 (10 points)
Bohr, Schrodinger and de Broglie meet (through zoom) to discuss the stationary state of an electron in a hydrogen atom. Bohr tells Schrodinger and de Broglie that this electron has magnitude of angular momentum $4\hbar$.
(a) de Broglie says: Monsieur Bohr, in that case, my wavelength for this electron must be x Angstroms.
What is x? Justify your answer.
(b) Schrodinger says: this is impossible, Herr Bohr. I measured the energy of this electron and conclude from it that the maximum possible magnitude of its angular momentum is $y\hbar$.
What is y and what is the value of the energy measured by Schrodinger?
(c) Bohr responds: I agree with you on the value of its energy, Hr. Schrodinger. Would you agree with me that its distance from the nucleus is z? Schrodinger responds: Herr Bohr, that is not so clear. All I can say is that for the largest possible angular momentum for this electron its most likely distance from the nucleus is w.
Give z and w, in Angstrom. Justify clearly how you got the w value, showing all the steps.

Problem 8 (10 points)
An electron in a hydrogen atom has quantum numbers n=3, ℓ=2, m_ℓ=-1. Ignore electron spin.
(a) It makes a transition to a state with n=2 emitting a photon. Give the wavelength (in A) of the photon that is emitted, and list all the possible finite states for the electron (give their quantum numbers) according to selection rules.
(b) Assume a magnetic field of magnitude 50T pointing in the z direction is now turned on. Give all the possible wavelengths of photons emitted when this electron makes a transition to a state with n=2. Take into account selection rules.
(c) Assume this atom is at temperature 300K, the electron is in a state with principal quantum number n=3. (i) What is the probability that it is in a state with m_ℓ=-1 versus in a state with m_ℓ=-2 in the absence of applied magnetic field? (allow for all possible values of ℓ). Give the ratio of probabilities. (ii) Same as (i) in the presence of a 50T magnetic field in the z direction.

Problem 9 (10 points)
For an electron in one dimension described by the wavefunction

$$\psi(x) = \sqrt{\frac{2}{L}}\sin(\frac{\pi x}{L}) \quad \text{for } 0 \le x \le L$$

$$\psi(x) = 0 \text{ elsewhere}$$

and the following operators:
(i) [Q]=[p]
(ii) [Q]=[p^2],
(a) State whether [Q] is a sharp or a fuzzy observable, and if applicable, give its eigenvalue.
(b) Calculate the expectation value <Q> and its uncertainty ΔQ.
Justify all your answers.