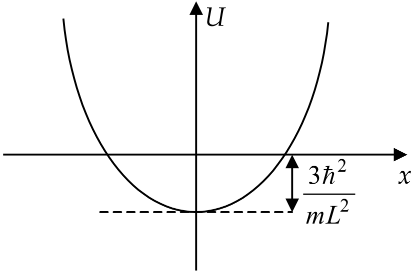
6-5 (a) Solving the Schrödinger equation for U with $E = 0$ gives

$$U = \left(\frac{\hbar^2}{2m}\right)\frac{\left(\frac{d^2\psi}{dx^2}\right)}{\psi}.$$

If $\psi = Ae^{-x^2/L^2}$ then $\frac{d^2\psi}{dx^2} = \left(4Ax^3 - 6AxL^2\right)\left(\frac{1}{L^4}\right)e^{-x^2/L^2}$, $U = \left(\frac{\hbar^2}{2mL^2}\right)\left(\frac{4x^2}{L^2} - 6\right)$.

(b) $U(x)$ is a parabola centered at $x = 0$ with $U(0) = \frac{-3\hbar^2}{mL^2} < 0$:



6-14 (a) Still, $\frac{n\lambda}{2} = L$ so $p = \frac{h}{\lambda} = \frac{nh}{2L}$

$$K = \left[c^2p^2 + \left(mc^2\right)^2\right]^{1/2} - \left(mc^2\right) = E - mc^2$$

$$E_n = \left[\left(\frac{nhc}{2L}\right)^2 + \left(mc^2\right)^2\right]^{1/2},$$

$$K_n = \left[\left(\frac{nhc}{2L}\right)^2 + \left(mc^2\right)^2\right]^{1/2} - mc^2$$

(b) Taking $L = 10^{-12}\ \text{m}$, $m = 9.11\times10^{-31}\ \text{kg}$, and $n = 1$ we find $K_1 = 4.69\times10^{-14}\ \text{J}$. The nonrelativistic result is

$$E_1 = \frac{h^2}{8mL^2} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{8\left(9.11\times10^{-31}\ \text{kg}\right)\left(10^{-24}\ \text{m}^2\right)} = 6.03\times10^{-14}\ \text{J}$$

Comparing this with K_1, we see that this value is too big by 29%.