UC San Diego
PHYSICAL SCIENCES
Physics

Problem Set 3

(30 pts.)

1) Consider a simple system with kinetic equation

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \frac{q}{m}E_{ext}(x,t)\frac{\partial f}{\partial v} = c(f).$$

Take f_o initially Maxwellian, formed by a very slow collisional process. $c(f)$ is negligible on dynamical time scales. $E_{ext}(x,t)$ varies slowly in time and space.

a) Compute the **linear** response δf to E_{ext}. From this, derive an expression for the conductivity.

b) Use δf to derive a mean field evolution equation for f_o, on $t < \tau_{coll}$.

c) What physics determines the evolution of f_o?

2) a) Consider a weakly damped linear harmonic oscillator driven by white noise.

i) Derive the fluctuation spectrum at thermal equilibrium.

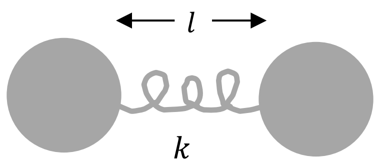
ii) What value of forcing is required to achieve stationarity at temperature T?

b) Now consider a forced nonlinear oscillator

$$\ddot{x} + \gamma x + \omega_0^2 x + \alpha x^3 = \tilde{f}.$$

Again, assume $\tilde{f}$ is white noise. Characterize the equilibrium fluctuation spectrum. Hint: You may find it useful to review Section 29 of "Mechanics", by Landau and Lifshitz.

3) Consider an elastic dumbbell of Stokesian particles in a fluid flow $\underline{v}(\underline{x}, t)$, at temperature T.



a) Derive the Fokker–Planck equation for the length l. (Hint: Consider expansion.)

b) What is the mean square length l?

Assume the dumbbell has spring constant k. The fluid has viscosity ν.

c) Now take the flow as turbulent, so $\underline{v}(\underline{x}, t) = \tilde{v}(\underline{x}, t)$, where $\tilde{v}$ is random. Repeat a) and b), above.

4) Consider a function q which satisfies:

$$\tau \frac{\partial q}{\partial t} = -a(T, T_c)q - bq^3 + \tilde{f}$$

Here $\langle \tilde{f}^2 \rangle = \left|\tilde{f}_0\right|^2 \tau_c \delta(t_1 - t_2)$.

a) What is this system?

b) Derive the Fokker–Planck equation for $P(q, t)$. Solve and discuss the stationary solution for $T > T_c$, $T < T_c$, $T = T_c$.

c) How does $P(q, t)$ evolve if T passes adiabatically thru T_c? Here "adiabatically" means $\tau_c T^{-1} \left(\frac{\partial T}{\partial t}\right) \ll 1$.

d) Discuss the behavior when

$$a = a_0 + \tilde{a}$$

$$\langle \tilde{a}^2 \rangle = \bar{a}^2 \tau_0 \delta(t_1 - t_2).$$

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5) a) Calculate the mobility of a Brownian particle $\mu(\omega)$ using the Kubo formalism.

b) Continue the cumulant expansion of the function F derived in class. Calculate the correction to diffusion. What does it depend on?