Problem Set 1

Due October 21 (20 pts.)

1) Estimates:

a) Derive the shear viscosity and thermal diffusivity of a hard sphere gas by heuristic methods.

b) Consider a heavy particle of mass M radius d_2 in a gas of light particles of mass m, radius d_1 ($d_1 < d_2$, $m \ll M$).

   i) What is the mobility of the heavy particle?

   ii) What is the deflection length for the heavy particle?

   iii) When will heavy particle energy equilibrate with that of a light particle?

c) $K = l_{mfp}/L$, where L is a macroscale (system size) defines the Knudsen number. For $K > 1$, estimate the effective thermal conductivity and shear frictional force/area on its container wall.

2) Calculate the heat conduction coefficient of a dilute, hard sphere gas using the Krook-model collision operator. Use the method of moment hierarchy truncation.

3) (a) The motion of an electron belonging to a molecule in a rarefied gas may, in some cases, be replaced by that of a harmonic oscillator: it is determined by

$$\frac{dx}{dt} = \frac{p}{m}, \qquad \frac{dp}{dt} = -m\omega_0^2 x - eE(t),$$

where x and p denote the radius vector and the momentum of the electron within the molecule respectively, m the mass, $-e$ the electric charge, ω_0 the characteristic angular frequency, and $E(t)$ an external electric field. Show that the average $\bar{f}(x,p,t)$ of the electron distribution function $f(x,p,t)$, taken over all possible values of the time and of the position at which collisions occur, obeys the equation

$$\frac{\partial \bar{f}}{\partial t} + \frac{p}{m}\cdot\frac{\partial \bar{f}}{\partial x} + \{-m\omega_0^2 x - eE(t)\}\cdot\frac{\partial \bar{f}}{\partial p} = -\frac{\bar{f} - f_0}{\tau}$$

by assuming that collisions between molecules occur with a mean free flight time τ, and that the electron distribution function immediately after a collision reduces to a given distribution function $f_0(x, p, t)$.

(b) Derive the low frequency electric current in the limit of small τ. Note $E = E(t)$ here.

(c) What is the effective Ohm's Law for this system? Discuss.

(d) For the cases of large τ (weak collisionality) and $\omega_o^2 \to 0$, propose and implement a mean field approach to computing how f evolves.

4) Now, use the Boltzmann Equation to calculate systematically:

(a) the diffusion coefficient for a heavy particle in a gas of light particles

(b) the diffusion coefficient for a light particle in a gas of heavy particles.

Hint: See "Physical Kinetics", Sections 11, 12.