PHYSICS 200A : CLASSICAL MECHANICS
PROBLEM SET #1

[1] Minimize the functional

$$F\big[y(x)\big] = \int_0^u dx \left(\tfrac{1}{2}y'^2 + ayy' + \tfrac{1}{2}y^2 + y\right)$$

when the values of y are not specified at the endpoints. Here, u and a are constants.

[2] Find the extrema of the functional

$$F\big[y(x), z(x)\big] = \int_0^{\pi/2} dx \left(y'^2 + z'^2 + 2yz\right)$$

subject to the boundary conditions

$$y(0) = z(0) = 0 \qquad , \qquad y(\pi/2) = z(\pi/2) = 1 \quad .$$

[3] Find the extrema of the functional

$$F\big[y(x)\big] = \int_0^1 dx \left(y'^2 + x^2\right)$$

subject to the boundary conditions

$$y(0) = 0 \qquad , \qquad y(1) = 1 \qquad , \qquad \int_0^1 dx\, y^2 = 2 \quad .$$

[4] Derive the equations of motion for the Lagrangian

$$L = e^{\gamma t}\Big[\tfrac{1}{2}m\dot{q}^2 - \tfrac{1}{2}kq^2\Big] \quad ,$$

where $\gamma > 0$. Compare with known systems. Rewrite the Lagrangian in terms of the new variable $Q \equiv q\,\exp(\gamma t/2)$, and from this obtain a constant of the motion.
[José and Saletan problem 3.24]

[5] A particle of mass m is embedded, a distance b from the center, in a uniformly dense cylinder of mass M. (The mass of the cylinder plus the inclusion is thus $M + m$.) The cylinder rolls without slipping along a plane inclined at an angle α with respect to the horizontal, under the influence of gravity. The axis of the cylinder remains horizontal throughout the motion.

(a) Choose an appropriate generalized coordinate and find the Lagrangian.

(b) Find the equations of motion.

(c) Under what conditions does a stable equilibrium exist?

(d) Find the frequency of small oscillations about the equilibrium.

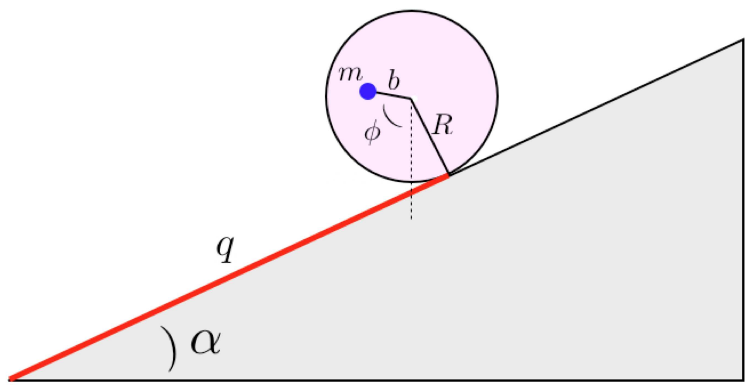

Figure 1: A cylinder of radius R with an inclusion rolls along an inclined plane.