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TOWARDS A SCALING THEORY OF DRAG REDUCTION

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Flexible polymers in dilute solution enhance the viscosity in slow flows. But in strong, rapidly varying, shear fields, they behave *elastically*. A turbulent cascade (from large to small scales) should thus be deeply modified when the elastic stresses become comparable to the Reynold's stress. A (tentative) scaling picture for these effects has been proposed by M. Tabor and the present author[1]): it involves one unknown exponent n relating polymer deformation (λ) and spatial scales (r) in the cascade. We now show that, depending on the control parameters (turbulent power; polymer concentration and molecular weight) the cascade may proceed according to two "scenarios". In the first scenario[1]) the smallest Kolmogorov eddy occurs when the chains are only partly stretched. In the second scenario, the smallest eddies display nearly full chain extension: polymer degradation is expected to be much more serious in the latter case.

We also transpose these ideas to wall turbulence, in the first scenario. At a distance y from the wall, the smallest eddy scale available $r^{**}(y)$ is a *decreasing* function of y (very different from the classical Lumley picture, where it is an increasing function of y). The overall result is again an increase of the buffer layer, provided that the polymer concentration exceeds a – very low but finite – threshold.

We point out finally that the elastic effects discussed here could be present in many other systems: one amusing (although impractical) example is a binary mixture near a consolute point: the concentration fluctuations should be very similar, in this effects, to a polymer solution with coil size ξ (the correlation length), at the overlap concentration c^*.

1. Introduction

Flexible polymers in dilute solution can reduce turbulent losses very significantly[1]). The main (tentative) interpretation of this effect is due to Lumley[2]). He emphasized that remarkable visco-elastic effects can occur only when certain hydrodynamic frequencies become higher than the relaxation rate of one coil $1/T_z$ (the "time criterion"). He then proposed a crucial assumption: namely that, in regions of turbulent flow, the solution behaves as a fluid of *strongly enhanced viscosity* – presumably via regions of elongational flow. On the other hand, Lumley noticed that – for turbulent flow near a wall – the viscosity in the laminar sublayer near the wall should remain *low*: this last observation does agree with the viscometric data on dilute, linear polymers in good solvents, which show shear thinning[3]). Starting from the above assumptions, and performing a careful

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matching of velocities and stresses beyond the laminar layer, Lumley was able to argue that the overall losses in pipe flow should be reduced.

This explanation has been rather generally accepted. However, it is now open to some question: in recent experiments with polymer injection at the center of a pipe, one finds drag reduction in conditions where wall effects are not involved[4,5]).

This observation prompted Tabor and the present author to try a completely different approach[6]): namely to discuss first the properties of homogeneous, isotropic, 3 dimensional turbulence *without any wall effect*, in the presence of polymer additives. This "cascade theory" is described in section 2. The central idea is that polymer effects at small scales (high frequencies) are not described by a viscosity, but by an *elastic modulus*. The general notion of elastic behavior at high frequencies is classical for molten, entangled chains[3]). We claim that it is also important for dilute polymers. In our approach the viscosity effects are mostly trivial, and we do not even discriminate between solvent viscosity and solution viscosity.

In section 3 we return to wall turbulence, and try to set up a modified version of the Lumley approach, where, at each distance y from the wall, we have a cascade, but it is truncated elastically. This gives a law for the minimum eddy size r^{**} versus distance y which is qualitatively different from Lumley's viscous effect. But the net result is still an enhancement of the intermediate "buffer layer". We expect drag reduction from this, although we have not carried out the detailed analog of Lumley's matching.

In section 4 we list some more general systems which can show drag reduction on turbulent flow. Some of the systems are dominantly elastic while others are probably dominantly viscous.

Our whole discussion is very qualitative. But, even at this modest level, it leads to a surprisingly rich classification of possible cascades and flows. For instance, in bulk turbulence, we have three control parameters: a) the dissipation per unit mass ε; b) the polymer chain length, or equivalently the number of monomers per chain N; c) the monomer concentration c (or the number of coils/cm^3 $c_p = c/N$).

This 3 dimensional parameter space can be split into regions where different "scenarios" for the cascade should occur. The identification of these scenarios is a natural aim for future experimental research.

2. The cascade theory

2.1. *The time criterion*

Our starting point is the classical view of Kolmogorov[7]) for homogeneous, isotropic, 3 dimensional turbulence.

At each spatial scale (r) there is a characteristic fluctuating velocity $U(r)$, related to (r) by the condition

$$\frac{U^3(r)}{r} = \varepsilon = \text{constant}. \tag{2.1}$$

We must compare the characteristic frequencies $U(r)/r$ to the Zimm relaxation rate of one coil^{8})

$$\frac{1}{T_z} \cong \frac{kT}{\eta_0 R^3}, \tag{2.2}$$

where k is the Boltzmann constant, T the temperature, η_0 the solvent viscosity, and R the gyration radius of the coil at rest. We focus our attention on linear, flexible, neutral polymers in good solvents, where the Flory law holds^{9,10})

$$R \cong N^{3/5}a, \tag{2.3}$$

a being a monomer size, and N the number of monomers per coil.

At large scales r, the hydrodynamic frequency U/r is smaller than $1/T_z$. But, if we go down in scale, we may reach a value $r = r^*$ where the two frequencies become equal. Thus

$$U(r^*)T_z = r^*. \tag{2.4}$$

Solving the coupled equations (2.1), (2.4) we arrive at

$$r^* = \left(\varepsilon T_z^3\right)^{1/2} \qquad (\sim N^{2.7}\varepsilon^{1/2}), \tag{2.5}$$

$$U^* = \left(\varepsilon T_z\right)^{1/2} \qquad (\sim N^{0.9}\varepsilon^{1/2}). \tag{2.6}$$

Note that r^* (and U^*) depend on molecular weight, but not on concentration.

Another parameter of interest for our discussion will be the Reynolds number computed *at the scale* r^*, namely

$$\mathrm{Re}^* = \frac{U^* r^*}{\nu} = \frac{\varepsilon T_z^2}{\nu} \quad (\sim N^{3.6}\varepsilon), \tag{2.7}$$

where $\nu = \eta_0/\rho$ is the kinematic viscosity, ρ being the fluid density.

The condition (2.4) defining r^* is the natural expression of Lumley's time criterion^{2}). Most interesting viscoelastic effects will occur only at frequencies higher than $1/T_z$, or equivalently at scales $r < r^*$.

2.2. *The passive range*

If our solute macromolecules are very dilute, their reaction on the flow pattern is weak. Thus we expect that there exists a certain interval $r^* > r > r^{**}$ where eddies of size r are still described by the Kolmogorov cascade, but where the polymer begins to undergo strong distortions.

2.2.1. *Information from laminar flows*

Let us concentrate first on *elongational flows*. Two regimes have been probed in some detail.

(i) *Constant shear rate* $\dot{\gamma}$ (fig. 1). Here one expects that the coils are essentially unperturbed when $\dot{\gamma} < 1/T_z$ and that they are strongly elongated when $\dot{\gamma} > 1/T_z$[11,12,10]). This sharp coil–stretch transition has been observed in an important series of experiments by Keller and coworkers[13,14]). We might think at first sight that this transition should show up in turbulent flows and bring in some important nonlinear effects. We shall argue, however, that this is not correct: for the situations of interest, where $\dot{\gamma}$ (as seen by the molecule) is rapidly varying in time, the coil stretch–transition disappears completely.

(ii) *Variable shear rates*. Two examples are shown on figs. 2a, b: in fig. 2a we have a duct with a periodic modulation of the cross section. In fig. 2b we consider the converging flow towards a very thin (~ 500 Å) capillary. The main conclusion, obtained first from detailed calculations by Daoudi[15]) for a coil under periodic modulations, is the following: whenever the modulation frequency is higher than the Zimm relaxation rate, the coil *follows passively the deformations of the local volume element*. The dimensionless elongation λ of the coil is entirely fixed by the flow.

Of course there are still some local modulations in the coil shape: more specifically, if we call ω the modulation frequency, we can define subunits of p monomers such that

$$\frac{1}{T_z(p)} = \omega, \qquad T_z(p) = \frac{\eta_0 a^3 p^{1.8}}{kT}, \tag{2.8}$$

where $T_z(p)$ is the Zimm type of the subunit. Inside each subunit we still have some relaxation, but at larger scales the coil deforms affinely.

These considerations have been transposed long ago[15]) to the converging flow of fig. 2b. Here the shear rate at a distance r from the entrance point is of order

$$\dot{\gamma}(r) = \text{const.}\, r^{-3}. \tag{2.9}$$

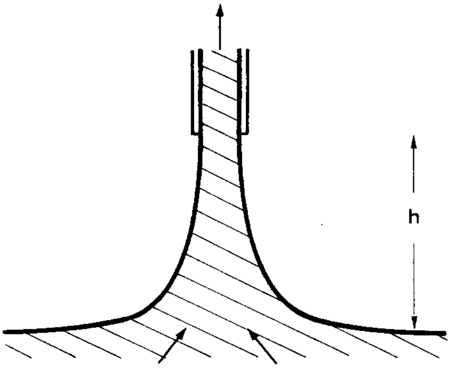

Fig. 1. The tubeless siphon: a dilute solution of long, flexible polymers can be sucked up over large intervals h (~ 20 cm). This shows the dramatic effects of the polymer induced stresses in longitudinal shear flows.

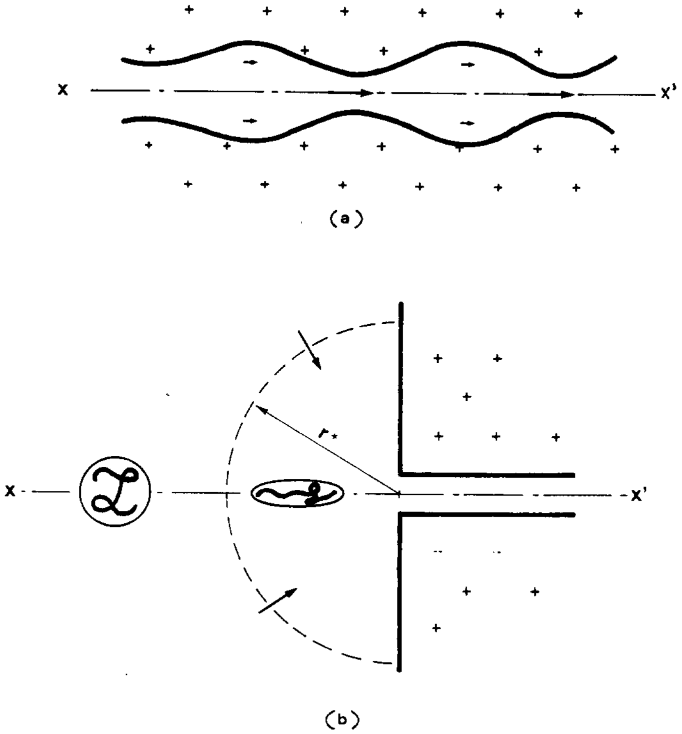

Fig. 2. Two (approximate) examples of longitudinal shear flows. a) Tube with periodic constriction; b) entry of a capillary. In both cases the molecules which lie exactly on the axis of symmetry (xx') experience a purely longitudinal shear.

Viscoelastic effects occur for $r < r^*$, where:

$$\dot{\gamma}(r^*) = 1/T_z. \tag{2.10}$$

At distances $r > r^*$ the coils are not deformed. At distances $r < r^*$ they deform affinely, and their elongation is

$$\lambda = \left(\frac{r^*}{r}\right)^2 \quad (3\mathrm{d}). \tag{2.11a}$$

A similar discussion can be given for a 2 dimensional flow, where the fluid converges towards a slit. Here the result is

$$\lambda = \frac{r^*}{r} \quad (2\mathrm{d}). \tag{2.11b}$$

Thus for a simple longitudinal flow, there is always a simple power law relating the striction parameter r^*/r and the polymer elongation.

2.2.2. *Transposition to the Kolmogorov cascade*[6])

We now make a bold assumption: namely that for flows which are admixtures of longitudional shear and simple shear, and which are turbulent, there remains a power law between polymer elongation and spatial scale

$$\lambda(r) = \left(\frac{r^*}{r}\right)^n, \tag{2.12}$$

where n is an unknown exponent. The extreme case quoted in eq. (2.11) suggests that $n < 2$. In practical discussions we shall attempt to use $n = 1$ and $n = 2$ as possible values.

It may be worthwhile to return here to the definition of molecular elongation. For one particular coil, the dimensionless elongation $\lambda_{(1)}$ can be constructed from the radius of gyration $\tilde{R}$ in the distorted state

$$\lambda^2_{(1)} = \frac{\tilde{R}^2}{R^2}, \tag{2.13}$$

where R is the radius at rest [eq. (2.3)]. For an ensemble of coils in a turbulent flow, we should select the coils which belong, in real space, to eddies of size r (and which do not belong to any smaller eddy). Then the average of $\lambda_{(1)}$ over this population is what we call λ. One immediate question concerns the distribution of $\lambda_{(1)}$ values within the population. In the present, naive, approach, we assume that this distribution is reasonably *narrow*, so that, for instance, the average of the squares is given by

$$\langle\lambda^2_{(1)}\rangle = k_2\lambda^2 \tag{2.14}$$

with a constant k_2 which is independent of r, and of order unity. It may well be that this assumption of narrow distributions is not satisfactory, and that independent exponents $n_1 n_2 \ldots$ would be required to describe the successive moments of the elongation. But, at our present level of ignorance, we shall omit this complication.

To summarize: we expect that any coil, located in eddies of size $r < r^*$, will follow passively the surrounding volume element, and will deform accordingly. We postulate a scaling law describing this effect [eq. (2.12)] in terms of a single exponent n.

This simple behavior, with affine deformation, and without significant reactions of the coils on the flow, will hold in a finite interval of spatial scales $r^* > r > r^{**}$. We call this interval the *passive range*.

2.3. *The first scenario*: *semi-stretched chains*

2.3.1. *Stresses in a partly stretched state*

Let us now consider the reaction of the polymers on the flow: when our coils are stretched by a factor λ, a certain elastic energy is stored in each of them. We shall, for the moment, assume that the coils are significantly stretched ($\lambda \gg 1$) but that they are still far from full extension: the deformed size $\tilde{R}$ is still much smaller than the contour length Na: or returning to eqs. (2.3–13):

$$1 \ll \lambda \ll N^{2/5}. \tag{2.15}$$

Since $N \sim 10^4$–10^5 in typical experiments, we may go up to $\lambda \sim 100$.

What is the elastic energy of a coil in this regime? In the harmonic approximation, it would be proportional to $(\lambda - 1)^2$ (or equivalently to λ^2, since $\lambda \gg 1$). However, for coils in good solvents, the harmonic approximation is not very good: the shape, and the monomer repulsions, change with λ. This has been analyzed by Pincus^{16}).

The final result is an anharmonic energy

$$F_1 \cong kT\lambda^{5/2} \quad (1 \ll \lambda \ll N^{2/5}) \tag{2.16a}$$

or a free energy per unit volume

$$F_{\mathrm{el}} = \frac{c}{N}kT\lambda^{5/2} = G\lambda^{5/2}, \tag{2.16b}$$

where G has the dimensions of one elastic modulus, and is linear in concentration. Equivalently the restoring force on one spring is

$$f_1 \cong \frac{kT}{R}\lambda^{3/2} \tag{2.17a}$$

and the stress due to c/N springs/cm^3 is

$$\tau \cong \frac{c}{N} f_1 \lambda R \cong F_{\mathrm{el}}. \tag{2.17b}$$

2.3.2. *The elastic limit* r_1^{**}

Whenever τ is much smaller than the Reynolds stresses ρU^2, the reaction of the polymer on the flow is negligible. If we go towards smaller and smaller scales, λ and τ increase, while the local Reynolds stress $\rho U^2(r)$ decreases. Thus, at a certain scale r_1^{**}, the two stresses become equal.

$$G[\lambda(r_1^{**})]^{5/2} = \rho U^2(r_1^{**}). \tag{2.18}$$

Using the Kolmogorov formula (2.1) this leads to

$$\frac{r_1^{**}}{r^*} = X^v, \quad v = \left(\frac{5n}{2} + \frac{2}{3}\right)^{-1}, \tag{2.19}$$

where we have introduced a dimensionless parameter

$$X \equiv \frac{G}{\rho U^{*2}}(\sim cN^{-2.8}\varepsilon^{-1}). \tag{2.20}$$

X is a natural measure of concentration effects. Consider a typical case with $U^* = 1$ m/s, $\rho = 1$ g/cm^3, $ca^3 = 10^{-4}$, $a = 2$ Å $T = 300$ K, $N = 10^4$. Then $G = 60$ erg/cm^3 and $X = 6 \times 10^{-3}$. Depending on our choice of n, the exponent v might be in range $\frac{1}{3}$- $\frac{1}{6}$.

2.3.3. *Comparison with the Kolmogorov limit*

Can we effectively go down to the elastic limit r_1^{**}, retaining the inertial cascade all the time? The Kolmogorov scheme is always truncated by viscous dissipation at a scale r_κ defined by[7]):

$$\frac{r^*}{r_\kappa} = (\mathrm{Re}^*)^{3/4}. \tag{2.21}$$

The elastic limit is observable only if $r_1^{**} > r_\kappa$. Comparing (2.19) and (2.21) we find that this condition is equivalent to

$$X > (\mathrm{Re}^*)^{-3/4v}. \tag{2.22}$$

The condition (2.22) has no counterpart in the Lumley scheme, where drag reduction was expected to occur at arbitrarily low polymer concentrations. Here, we do find a minimum X, or equivalently, a concentration threshold c_m, below which the polymer should have no visible effect. Using eqs. (2.20) for X and (2.7) for Re^*, we can find how the threshold concentration c_m depends on N and ε:

$$c_\mathrm{m} \sim N^{2.8-2.7/v} \varepsilon^{1-3/4v}. \tag{2.23}$$

Since $1/v$ is expected to be in the range 3–6, c_m should be a strongly decreasing function of N. It may well be that for long chains, c_m is extremely small and practically invisible. But systematic experiments at variable N might detect the threshold c_m.

2.3.4. *Ultimate fate of the turbulent energy*

At the scales $r = r^{**}$, the liquid should behave like a strongly distorted rubber, carrying elastic waves (longitudinal and transverse) with comparable kinetic and elastic energies.

At the scales $r < r^{**}$ inertial nonlinearities are not, by themselves able to generate smaller structures. But the *elasticity* is also nonlinear, and may have the ingredients required to produce shock waves in the "rubber". This process may imply a further thinning of scales – down to the natural width of a shock front. This scheme is completely conjectural: we may deal with a sea of rarefaction waves and shock waves, with very peculiar couplings between them: Thus, we do not know the ultimate fate of the turbulent energy.

On the whole, it is tempting to assume that the formation of new eddies is strongly restricted for $r < r^{**}$. Using the description of eqs. (2, 18–23), this would then lead to a truncation in the cascade at $r = r_1^{**}$ (the index 1 stands for the "first scenario" with partly elongated chains, which was the only one discussed above).

The result is

$$r_1^{**} = r^* X^v, \tag{2.24}$$

$$r_1^{**} \sim N^{2.7-2.8v} \varepsilon^{1/2-v} c^v. \tag{2.25}$$

We expect at last a qualitative change, and possibly a truncation, of the cascade at $r = r_1^{**}$. Note that r_1^{**} should increase rapidly with N.

Recent experiments on pipe turbulence or with planar mixing layers[17]), using strophometry or laser Doppler anemometry, do suggest that polymer additives can suppress certain small scales. But detailed proposals, such as (2.25), for the truncation, remain to be checked.

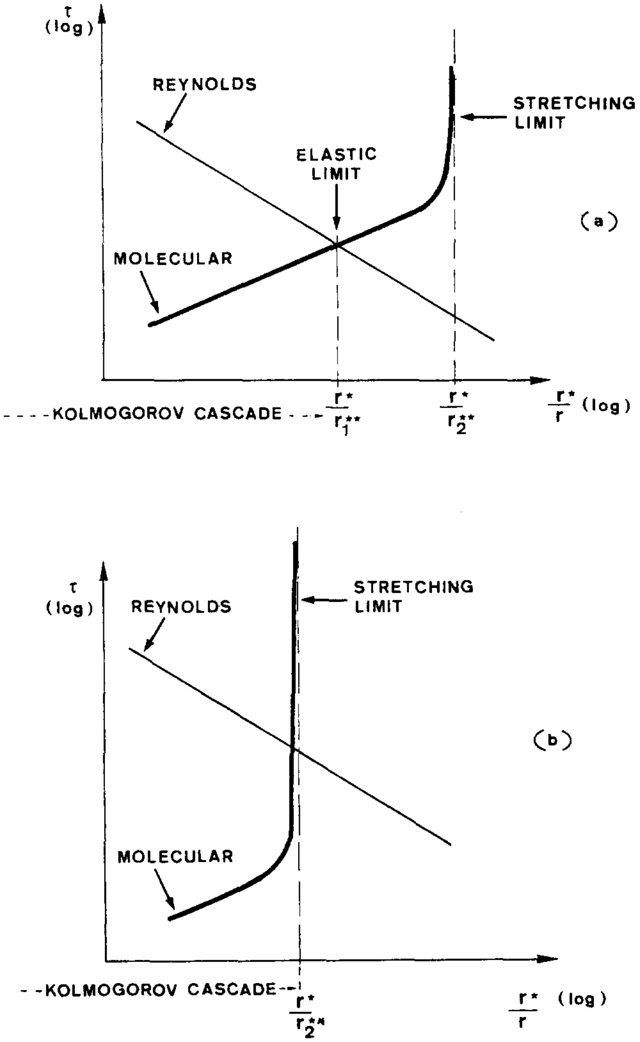

Fig. 3. Reynolds stresses and molecular stresses in a Kolmogorov cascade with a dilute polymer solution. a) In the first scenario, the Kolmogorov cascade is truncated at a certain scale r_1^{**}: at this scale, the chains are only partly stretched; b) in the second scenario, the trunction occurs (at a scale r_2^{**}) where the chains are totally stretched.

2.4. *The second scenario*: *strongly stretched chains*

Let us return to the passive regime, and assume that we can reach very small scales r, so that the polymer under flow may become fully elongated. Returning to (2.15), we see that this corresponds to $\lambda = \lambda_{\max} \cong N^{2/5}$. Inserting this value into the scaling law (2.12), we arrive at a certain characteristic scale

$$r_2^{**} = R^* N^{-2/(5n)}. \tag{2.26}$$

We call r_2^{**} the stretching limit. The meaning of r_2^{**} is made more apparent by the construction of fig. 3 which is a log–log plot of the stresses as a function of the striction ratio r^*/r. When λ gets very close to $\lambda_{\max}$, the stresses tend to diverge (detailed laws for this have been constructed on simple molecular models): this fixes r_2^{**}.

In this second scenario the behavior at scales r smaller than r^{**} is probably rather different: viscous forces are very important when we have rod particules immersed in a fluid; it may well be that viscosity takes over at $r = r_2^{**}$.

3. Flow near a wall

3.1. *A reminder on pure fluids*[7])

Wall turbulence is characterised by a velocity U_τ such that the wall stress is equal to ρU_τ^2. The average velocity profile for a pure fluid is represented in fig. 4:

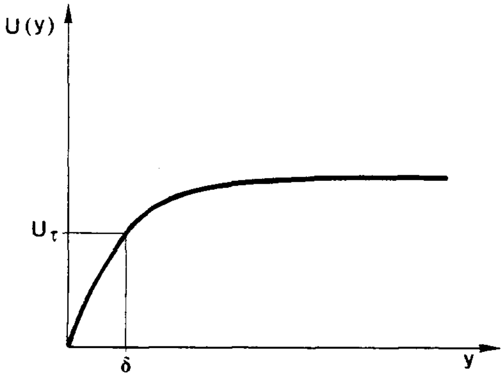

Fig. 4. Average velocity profile in the wall turbulence of a pure fluid (qualitative). y is the distance from the wall, and $\delta = \nu/U_\tau$ is the thickness of the laminar sublayer. At $y \gg \delta$ the profile is logarithmic.

it is linear very close to the wall, in a laminar sublayer of size

$$\delta = \frac{\nu}{U_\tau} \tag{3.1}$$

then, at long distances from the wall, it is logarithmic. At any distance y ($\gg \delta$) from the wall we find eddies with waves vectors k lying between two limits: the largest size (or the smallest wave vector $k_{\min}^{-1}$) is given by the distance y itself; the smallest size (or the largest wave vector $k_{\max}^{-1}$) is given by a Kolmogorov limit r_κ similar to eq. (2.21). The main difference is that now the dissipation ε is a function of the distance to the wall.

$$\varepsilon \cong U_\tau^3/y. \tag{3.2}$$

A formula, equivalent to (2.21, for $r_\kappa(y)$ is

$$r_\kappa = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4} = \delta^{3/4} y^{1/4}. \tag{3.3}$$

Thus the eddies expected around the observation point (y) have wave vectors in the range

$$\frac{1}{y} < k < \frac{1}{\delta^{3/4} y^{1/4}}. \tag{3.4}$$

We call the lower limit the geometrical limit, and the upper limit the viscous limit. The range of turbulent eddies is shown on fig. 5.

3.2. *The Lumley model for polymer solutions*[2])

Lumley kept the viscosity in the laminar sublayer at its Newtonian value, but he assumed an increased viscosity η_t in the turbulent regions. This implied that, at any distance y from the wall, the Kolmogorov limit $r_\kappa(y)$ was shifted upwards; in the log/log plot of fig. 5, the new viscous limit is represented by a dotted line, parallel to the original limit (slope -4 according to eq. (3.3)). The higher the concentration, the higher the shift of the viscous limit.

The net result is a shrinkage of the turbulent domain: beyond the laminar layer, we now find a buffer layer, of size increasing with the polymer concentration. It is then natural to expect that the turbulent losses be *reduced*; Lumley gave a detailed argument to show this[2]). Note that the whole effect occurs at arbitrarily low c: as soon as we add some polymer, the viscous limit shifts.

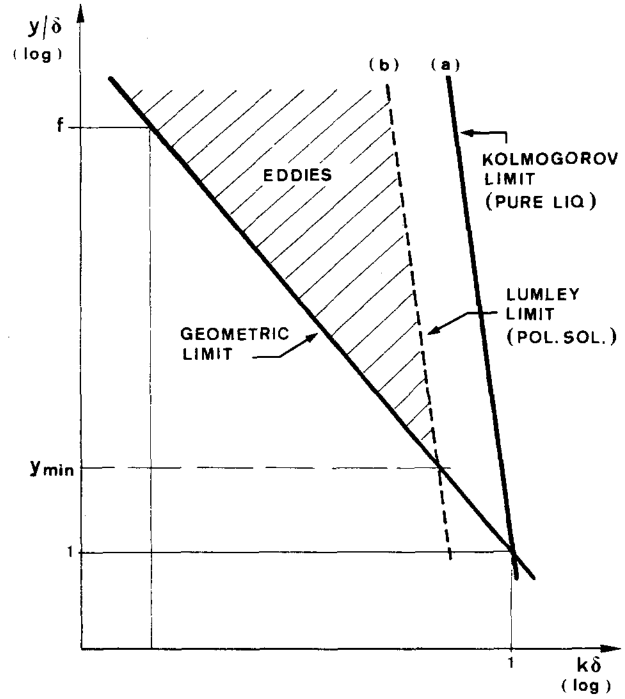

Fig. 5. The distribution of eddy wave vectors k at various distances y from the wall. a) Pure fluid: the smallest eddies are defined by the viscous limit of Kolmogorov. b) Polymer solution in the Lumley scheme: the limit is shifted to the left but the slope of the limiting line remains the same (-4).

3.3. *A modified version in the first scenario*

Let us assume that the discussion of section (2.3) holds: the chains are partly stretched, but never fully extended, and there is an elastic limit r_1^{**}, given, in terms of the local energy dissipation ε, by eqs. (2.21, 24, 25). Inserting eq. (3.2) for $\varepsilon(y)$, this gives

$$r_1^{**}(y) \sim U_\tau^{3(1/2-v)} y^{v-1/2} N^{2.7-2.8v} c^v. \tag{3.5}$$

Let us further assume that no singular dissipation occurs at scales smaller than the elastic limit. Then we are led to a modified Lumley construction, shown in fig. 6. There is an elastic limit, described by the dotted line, and the slope of this line is reversed in sign. At very low c, the new limiting line intersects the geometrical limit at thickness y smaller than the laminar sublayer δ: in this regime, we expect no macroscopic effect. But, beyond a certain threshold c_o (o stands for: onset),

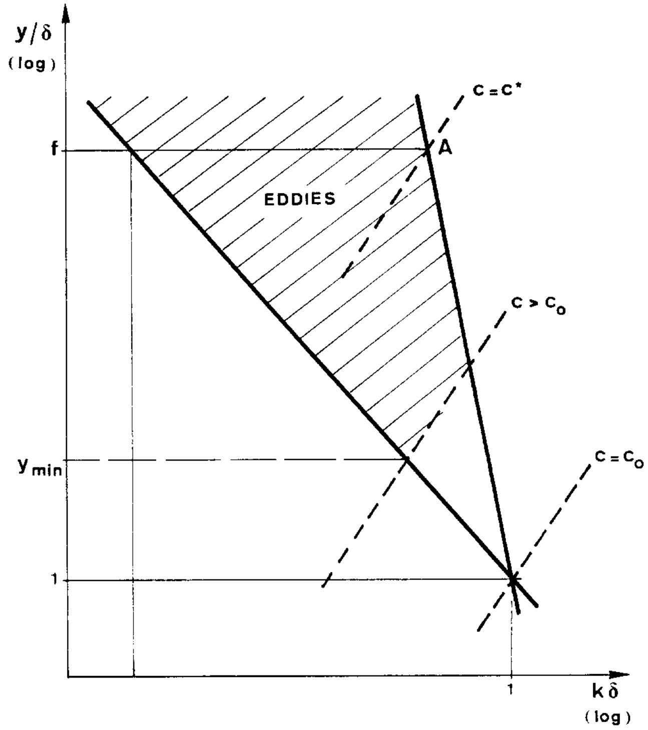

Fig. 6. The plot of fig. 5, transposed to the present model, in the first scenario. The maximum wave vector of turbulent eddies can depend upon an elastic limit: note the difference in slope with fig. 5. At very low polymer concentrations $c < c_0$, this limit is never relevant. Above $c = c_0$, it becomes relevant. When the elastic limit reaches point A the polymer concentration just corresponds to coils in contact ($c = c^*$).

the turbulent domain is actually truncated. The scaling structure of c_0 can be extracted from (2.5), writing $r_1^{**}(\delta) = \delta$. c_0 is the wall analog of the concentration c_m introduced in our discussion of homogeneous turbulence. In many practical cases, c_0 will be very small. But, conceptually, the existence of c_0 reveals a significant difference with the Lumley model.

If we increase c beyond c_0, the elastic limit shifts upward in fig. 6, and we again find a buffer layer: we conjecture that, in this regime, dissipation is reduced.

At a certain higher concentration, the intersection of the elastic limit and the Kolmogorov limit reaches point A. At this moment, the largest eddies cease to satisfy the time criterion. We expect that any addition of polymer beyond this point will be less effective.

It turns out that the polymer concentration associated with point A is a familiar object of solution theory: it is the concentration.

$$c^* = N/R^3 = a^{-3}N^{-3/4} \tag{3.6}$$

at which neighboring coils begin to overlap. Thus, in the interval $c_0 < c < c^*$ we expect a significant drag reduction, increasing steadily with c. At concentrations beyond c^*, the behavior is more complex, because some eddies are operating in a Newtonian regime, and the trivial increase of Newtonian viscosity due to the polymer may become the leading feature.

4. Conclusions and perspectives

4.1. *Elasticity versus viscosity*

The main idea of this paper is that flexible coils, even in the dilute regime, behave elastically at high frequencies: a description of small eddies in terms of a renormalised (real) viscosity is not entirely adequate.

A Kolmogorov cascade remains unaltered by polymer additives only down to a certain limit r^{**} where the polymer stresses balance the Reynolds stresses. In the second scenario, this occurs at full stretching. Chemical degradation is probably severe in this last case.

The fate of the turbulent energy below the limiting scale r^{**} is unclear: in the first scenario, it might result from a delicate balance between elastic shock waves and rarefaction waves. In the second scenario, viscous effects may be immediately dominant.

4.2. *Other elastic systems*

a) Apart from linear polymers, many other systems may show an elastic behavior at high frequencies. One obvious example is *branched* polymers, which may show an improved resistance to degradation. However, the regime of extreme deformation (the analog of the second scenario) is reached at much small elongations λ: we need a special discussion of branched systems, and the details of the branching statistics (e.g. the number of internal loops) will be important.

b) Another family of interesting systems is obtained with *binary fluid mixtures* near a consolute point. Ruiz and Nelson^{18}) have studied situations where, at the starting point, the two fluids are not fully mixed. They pointed out that the resulting concentration gradients induced elastic stresses in the system, and that these stresses *react on the turbulent field*.

c) We may also consider a modified version of the Nelson–Ruiz problem, where the binary mixture is macroscopically homogeneous; there remains an effect due to fluctuations of concentration, with a characteristic size ξ (the correlation length). These fluctuations are remarkably similar to polymer coils: ξ is the analog of the unperturbed polymer size R. There is a characteristic time (first introduced long ago by Ferrell and Kawazaki) which is the exact analog of the Zimm time (eq. (2.2)) with $R \to \xi$. There are elastic tensions (although, to our knowledge, the analog of the nonlinear Pincus formula, eq. (2.16), has not been worked out). The fluctuations span all space: this is the analog of a polymer system at the concentration of first overlap $c^* = N/R^3$. In practice, the main limitation is that, to reach long times T_z (needed to satisfy the time criterion) we need a large ξ, i.e. a good temperature stabilisation, in the one phase region, near the critical point.

d) Similar effects may exist in the 2-phase region, where small droplets are constantly broken (and coalesce) in turbulent flow: here, the elasticity of the droplets can be expressed in terms of one interfacial tension, following the classic papers of Taylor. Very near to the critical point, however, we should return to a microscopic description: the basic formulas are described by Aronowitz and Nelson[19]).

4.3. *The third scenario*

All our discussions have used, as a starting point, the "time criterion" of Lumley: spatial effects have been ignored because the coil size R is usually much smaller than the smallest eddy size. However, after stretching, the situation may be different. A deformed coil of length λR, with $\lambda \gg 1$, may become comparable to the eddy size r: this may lead to a third scenario – an idea first suggested to us by E. Siggia. It is easy to construct the scale r_3^{**} at which we would meet this effect. It is far more difficult to perceive what would happen at even smaller scales. In practice, the third scenario should occur only if we achieve turbulence even at very small scales, with extremely large Reynold's numbers; it will probably be associated with strong degradation.

4.4. *Open problems*

All our discussions are extremely conjectural a) The very existence of a single exponent n characterizing the elongation at different scales (eq. (2.12)) is unproven. (Among other things, we might need a family of exponents to give separate scaling law for the various moments λ^m.) b) The behavior of the cascade beyond the elastic threshold is entirely unclear. c) The intermittency features which are omitted in the Kolmogorov description of the cascade may be much more significant for polymer solutions than for pure liquids.

However, we feel that the proposed scheme, the classification of scenarios, and even the tentative scaling laws which we propose, should be of some use to guide future experiments.

Apart from the elastic systems discussed here, there also exist some interesting questions with rigid rods: they do have an orientational entropy, which leads to some analog of an elastic energy in aligning flows: but they cannot follow the local deformation affinely, as done by the coils. Viscous dissipation is thus much stronger. It may be that the Lumley scheme holds for rods.

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