→ <u>Wakes</u> /

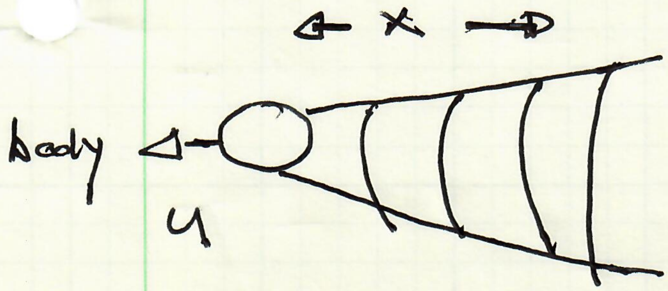
- Physics Ideas → flow created by response flow to separation
- Link:

  Drag → wake flow
- Width: - Laminar
  - Turbulent
- Scalings
- Deficit and punchline.

- Discontinuity stability

  KH simple

Wakes

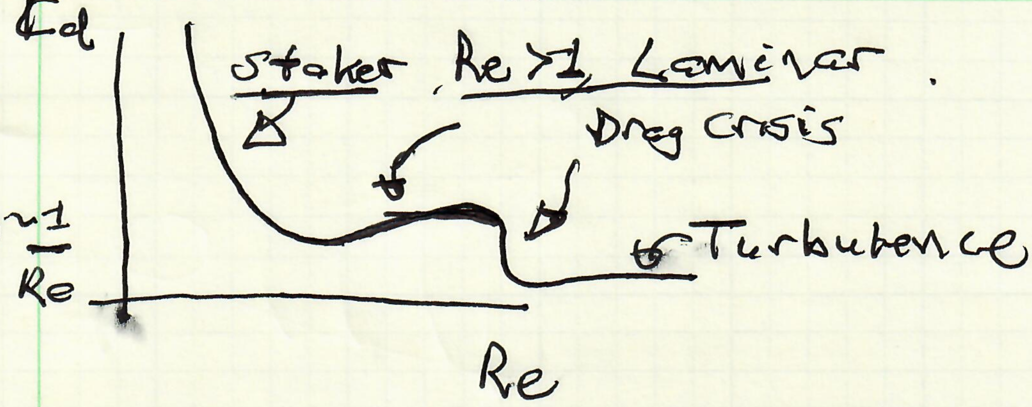


Wake:

- region behind moving body of departure from potential flow. Wake is rotational.
- Wake is consequence of body experiencing drag; (or flow dragging on body)
- region of wake is limited, in angular extent.
- Message of wake: A little viscosity forces a global adjustment in flow structure.

Why? Wakes? Fd vs Re curve, again

Wake flow



$$Re = \frac{UR}{\nu}$$

>1

$$F_d \sim C_D \rho A U^2$$

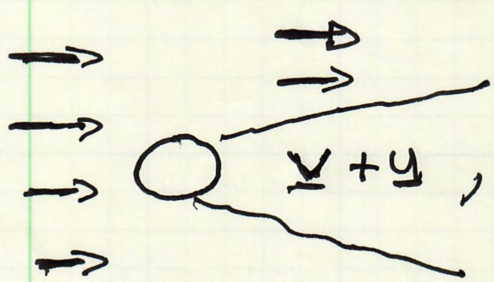
C_D ↓ Drag coefficient

i.e. flow not turbulent, but inertia is relevant.

Furthur:

- distances behind body $x \gg R$, are region of wake

- if body speed U, then if frame where body stationary,

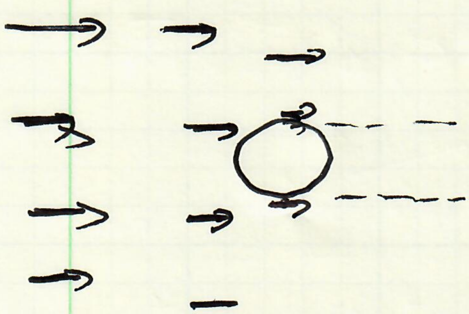


$\underline{v} + \underline{u}$, with $v_x < 0$
(slower in wake)

{ $\underline{v}$ noticeably different from zero in limited region.

(*) How limited? → As laminar, ⊥ signal propagation is diffusion, only.

(*) How does wake form?



— No slip boundary condition 'slows down' fluid flowing past body

— discontinuity → KHI results on surface

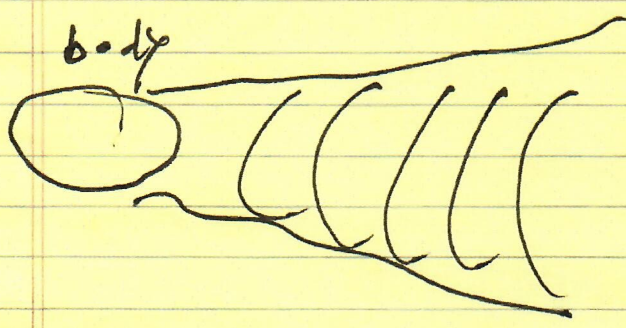
∴ but — viscosity smoothes out discontinuity.

n.b. if turbulent, then turbulent mixing $(\tilde{v} \cdot \tilde{v})$ smooths discontinuity faster than viscous mixing.

B.) Wakes – Simple physics cf.: Prandtl – Tietjens, Falkovich, Lander

Wake is:

– region of departure from potential flow behind object moving thru water and experiencing drag



– wake is ~~in~~ inexorably coupled to drag

– Message of wakes:

A little ~~r~~ ~~force~~ forces a global adjustment of flow structure *

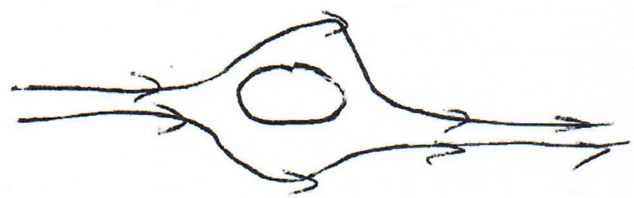
– drag – thinking in frame where object at rest, drag results from loss of flow momentum to object.

i.e. ←O ⇒ ⇉ O

* → **wake** is region of flow where loss of momentum is evident.

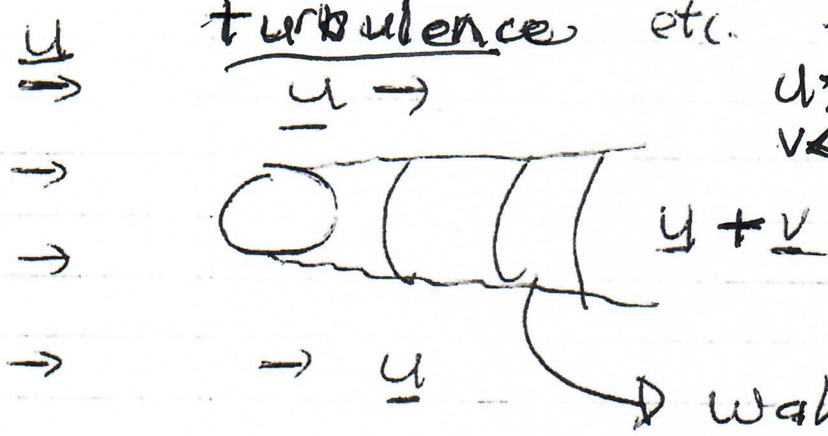
i.e.,

– if potential flow (no drag)



symmetry upstream, downstream in ⊥ displacement of fluid element

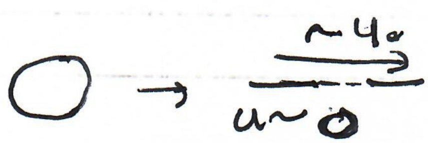
– with no-slip b.c., viscosity, turbulence etc.



$u > 0$
$v < 0$

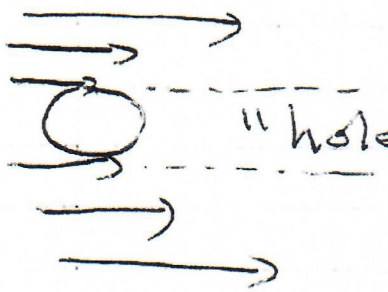
$\underline{v}$ opposite $\underline{u}$ i.e. $\begin{cases} u > 0 \\ v < 0 \end{cases}$

results from evolution of discontinuity.



unstable → KH.

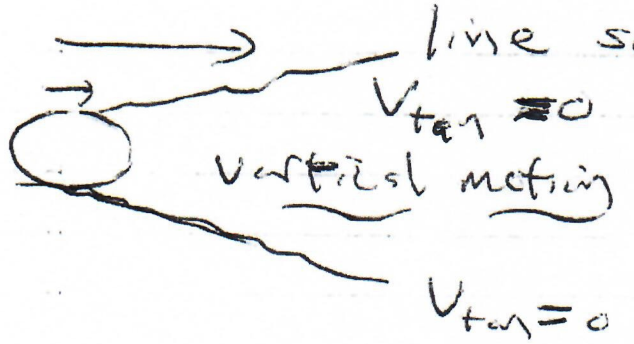
* — origin of wake is no-slip b.c. + $\left\{ \begin{array}{l} \text{viscosity} \\ \text{turbulence} \end{array} \right.$, after separation



but flow is unstable!

{ how high in Re can one go?

oo



$\underline{\omega} = \nabla \times \underline{V} \neq 0$

* — boundary of wake traced by fluid particles:

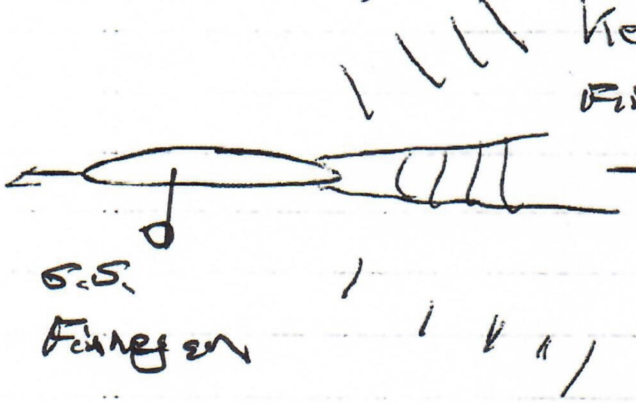
→ passing close to body,

→ scattered by diffusion (and turbulent mixing)

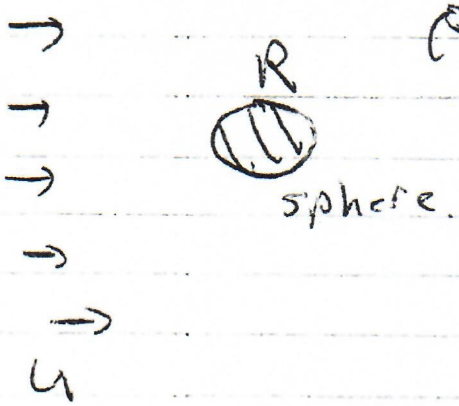
⇒ expansion.

Note:

* – in general, wake multi-component



– here consider spherical cow of wake problems



so $F_d \sim \rho u^2 R^2 f(Re)$

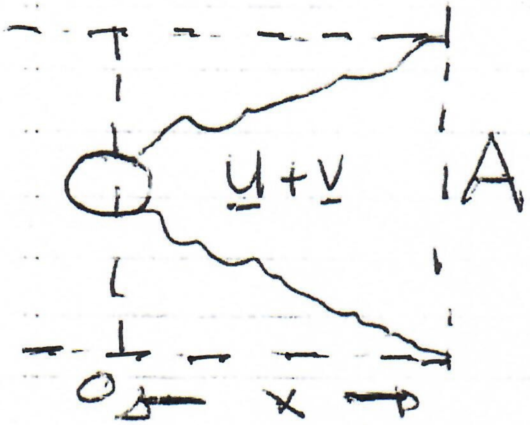
→ no surface effects.

→ How calculate wake structure?

Force of Drag $\equiv$ { Net Rate of Momentum Loss from Flow }

Simply put

i.e.



downstream distance

Rate Momentum Loss ≡

$$-A\,P_{total}(x) + A\,P_{Total}(0) = F_d$$

$$P_{Tot}(0) \cong P + \rho U^2/2$$
$$P_0 + p' + \rho U^2/2$$
total head. Bernoulli applies

$$P_{tot}(x) \cong P + \rho(U+V)^2/2$$
$$P_0 + p' + \rho(U+V)^2/2$$

$$A \sim \pi\, w(x)^2$$

$w \equiv$ width of wake at x downstream

conical symmetry.

So

$$F_d \approx w(x)^2\left[-\left(P + \frac{\rho}{2}(U+V)^2\right) + \left(P + \frac{\rho U^2}{2}\right)\right]$$

p' unchanged → (not) straight streamlines

Formally,

$$F_i = \oint \Pi_{ik}\, df_k$$

$$= \oint (p_0 + p')\,\delta_{ik} + \rho (u_i + v_i)(u_k + v_k)\, df_k$$

$$\rho \oint v_k\, df_k = 0$$

$\rightarrow$ for

$$\boxed{F_i = \left(\iint_{x_0} - \iint_{x} \right) (p' + \rho u v_i)\, dy\, dz}$$

— outside → Bernoulli →

— inside, v_x large

• Pressure ~~[illegible]~~ unchanged → start streamlines.

So

$$F_d \sim -w(x)^2 \left[\cancel{p} + \cancel{\frac{\rho U^2}{2}} + \frac{2\rho U V_x}{2} - \cancel{p} - \cancel{\frac{\rho U^2}{2}} \right]$$

$$\sim -\rho U V_x \, w(x)^2$$

n.b. why $p(0) \sim p(x)$?

$$\therefore \quad \boxed{F_d \sim -\rho U V_x \, W(x)^2}$$

$V_x < 0$

$F_d > 0$

$\to$ ok.

Now, need $w(x)$ to get V_x !

→ Observe:

- problem now reduced to one of scale

(within

- wakes are self-similar!

$\Rightarrow \; w \sim x^{\alpha}$, α ?

- wakes can be laminar [redacted]
  or turbulent)

i.) Laminar $\qquad UR/\nu < 1$

now $\cancel{\frac{\partial \underline{V}}{\partial t}} + \underline{V}\cdot\nabla\underline{V} - \nu\nabla^2\underline{V} = -\frac{\nabla p}{\rho}$

st state

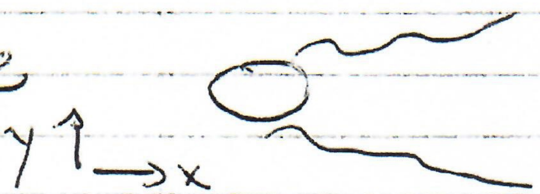
rel to

$$\underline{U}\cdot\nabla\underline{V} + \cancel{\underline{V}\cdot\nabla\underline{V}} - \nu\nabla^2\underline{V} = -\frac{\nabla p}{\rho}$$

oseen

defines laminar vs turbulent

so



$\nabla\cdot\underline{V} = 0$

narrow wake

$$U\partial_x V_y - \nu(\cancel{\partial_x^2} + \partial_y^2)V_y = -\frac{\partial_y p}{\rho}$$

and

$$U\partial_x V_x - \nu\partial_y^2 V_x = -\frac{\partial_x p}{\rho}$$

Scaling

take $\partial_x \sim 1/x$ → downstream distance

$\partial_y \sim 1/w$ → $\perp$ scale

Obv.: $\frac{1}{\sqrt{(\nu/U)x}}\exp\left[\frac{-y^2}{\nu x/U}\right]$ $\qquad w \sim \sqrt{\nu x/U}$

$$\left(\frac{U}{x} - \frac{\nu}{w^2}\right) V_y \sim -\frac{p}{w\rho}$$

$$\left(\frac{U}{x} - \frac{\nu}{w^2}\right) V_x \sim -\frac{p}{x\rho}$$

$$\underline{\nabla} \cdot \underline{V} = 0 \quad \Rightarrow \quad \frac{V_x}{x} \sim \frac{V_y}{w}$$

as p negligible (will show) $\Rightarrow$

$$\frac{U}{x} \sim \frac{\nu}{w^2}$$

$$\Rightarrow \quad \boxed{w \sim (\nu x/U)^{1/2}}$$

→ diffusive spreading of momentum by ν

→ $\sim (\nu t)^{1/2}$ with $t \sim x/U$.

so

$$W \sim \left(\frac{x}{R}\right)^{1/2} \left(\frac{\nu R}{u}\right)^{1/2}$$

$$\boxed{W/R \sim \left(\frac{x}{R}\right)^{1/2} 1/Re^{1/2}}$$

$$\boxed{|V_x| \sim \frac{F_d}{\rho u W^2}}$$

diffusive

→ skin Blasius B.L thickness.

→ in case you are wondering:

y: $\frac{P}{\rho u} \sim \frac{\nu V_y}{w^2}$ (~~if assume~~) (drop $u\partial_x$ tel)

and

$$\frac{V_x}{x} \sim \frac{V_y}{w}$$

$$\Rightarrow \quad P \sim \rho\nu V_x/x$$

and $P/\rho x \sim \frac{\nu V_x}{x^2} \ll \frac{\nu V_x}{w^2}$ ✓

$\therefore$ drop P. and safely $\nu V_y/w^2$

by analogy with K.T. gases

$$\underline{v}\cdot\nabla\underline{v} \to -\nu_T \nabla^2 \underline{v}$$

$$\nu_T \sim \tilde{w}\, l_{mix}$$

(ii) Turbulent

$$Re \sim UR/\nu \gg 1$$

$$\underline{U}\cdot\nabla\underline{v} + \underline{v}\cdot\nabla\underline{v} - \cancel{\nu\nabla^2\underline{v}} = -\frac{\nabla p}{\rho}$$

ignore

$$\Rightarrow \quad \frac{U}{x} v_x \sim \frac{\tilde{v}_y}{W} v_x$$

wake spreads by advection, not diffusion

$\tilde{v}_y \sim$ turbulent velocity

$$\boxed{W \sim \frac{\tilde{v}_y}{U} x}$$

Take wake turbulence isotropic,

so $\tilde{v}_x \sim \tilde{v}_y$.

[Fair? Test]?

$$W \sim x\tilde{v}_x/U$$

but from drag:

$$\tilde{v}_x \sim F_d/\rho U W^2$$

$\Rightarrow$

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$$W \sim x \frac{F_d}{\rho u^2 W^2} \sim x \left(F_d / \rho u^2 W^2 \right)$$

$$W^3 \sim F_d\, x / \rho u^2$$

$$\Rightarrow \boxed{\begin{aligned} W &\sim \left(F_d / \rho u^2 \right)^{1/3} x^{1/3} \\ &\sim \left(C_D R^2 \right)^{1/3} x^{1/3} \end{aligned}}$$

then, comparing widths:

laminar: $w/R \sim (x/R)^{1/2}\, Re^{-1/2}$

$Re \sim UR/\nu$

turbulent: $\cancel{W} w/R \sim (x/R)^{1/3}\, C_D^{1/3}$

interestingly laminar wake expands with downstream length more rapidly!

Why?

→ turbulence can relax TV behind object (due separation) rapidly, and faster than ν. Thus surrounding flow penetrates the dead water region more rapidly, ∴ less wake expansion.

Also observe: <u>Wake</u> Re drops with x.

⇒

$$Re \sim \frac{W V_y}{\nu} \sim \frac{\cancel{W} V_x}{\nu} \sim \frac{\cancel{W}}{\nu} \frac{F_d}{\rho U W^2}$$

y direction (spr) — wake flow Re

$$Re \sim F_d / \rho U W \nu$$

$$\sim \frac{U^{\cancel{2}} R^2 \cancel{\rho_0} C_D}{\nu \cancel{\rho_0} \cancel{U} (C_D R^2)^{1/3} x^{1/3}}$$

$C_D \sim 1$

$$\sim \left(\frac{UR}{\nu}\right)\left(R/x\right)^{1/3}$$

so

$$\boxed{Re(x) \sim Re_o \left(R/x\right)^{1/3}}$$

and $Re(x) \to 1$ at

$$x_L \sim R \left(Re_o\right)^3$$

distance behind boat where turbulent wake transitions to laminar.

i.e. akin l_d: transition from turbulent mixing to viscous mixing

N.B. [In wake, vortical/rotational region can expand into irrotational region, but never reverse!

i.e. would really violate H-Thm ...

Later discussion

B/a [redacted]

Wakes - Supplement Stuff

→ Revisit turbulent wake using turbulent viscosity, i.e.

$$W \sim \left(\nu x / u \right)^{1/2} \qquad (\nu \to D_T)$$

$$\to \left(D_T x / u \right)^{1/2}$$

i.e. is width of turbulent wake set by turbulent diffn, following Blasius Law

but $D_T \sim W \tilde{V}$ ⇒ turbulent viscosity at mixing length level.

$$\sim W \left(F_d / \rho u W^2 \right)$$

$$\sim F_d / \rho u W \quad \sim \quad \text{const} / W$$

⇒

$$W \sim \left(F_d x / \rho u^2 W \right)^{1/2}$$

$$W^{3/2} \sim \left(F_d / \rho u^2 \right)^{1/2} x^{1/2} \sim \left(C_D R^2 \right)^{1/2} x^{1/2}$$

$$\sim C_D^{1/2} R \, x^{1/2}$$

$$W \sim \left(C_D \right)^{1/3} R^{2/3} x^{1/3}$$

$\Rightarrow$ $$\boxed{W/R \sim C_D^{1/3} (X/R)^{1/3}}$$ agrees ✓.

Now, $D_T \sim \tilde{\nu} W$

$$\sim \frac{(\tilde{\nu} W^2)}{W}$$

$$\sim \frac{\rho U \tilde{\nu} W^2}{\rho U W}$$

$$\sim \frac{Q}{W} \sim Q/R\,(X/R)^{1/3}$$

∴ – Point is that turbulent viscosity, mixing drops downstream, relative to constant viscous mixing.

– follows from $\tilde{\nu} W \sim \overset{\to \text{const.}}{Q} / W$

– explains why turbulent wake spreads more slowly than laminar wake.

→ Some Observations for Wake Flows

→ note, replace A with cut.

$$F_x = -\rho U \int_{\text{wake}} v_x \, dy \, dz$$

Now $Q = \rho \int_{[redacted]\ \text{wake}} \cancel{V_x} \, dy \, dz$

↓

mass flow due wake

⇒ deficit.

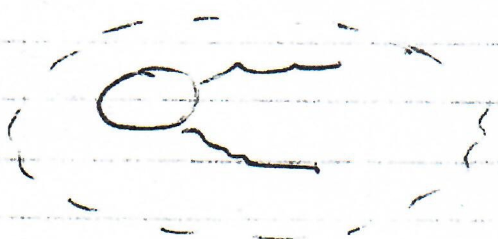
Deficit flux Q

→ difference with without the body

→ fluid flow thru wake area.

→ $\left\{ \begin{array}{l} Q \text{ is } x \text{ independent} \\ \text{i.e. } F_x / U \end{array} \right.$

→ but if encircle body



$\int_{\text{outside wake}} V_x \, dy \, dx \sim \int_{\text{outside wake}} \underline{V} \cdot da$

↓

$V_x \sim 1/r^2$

$\sim$ <u>in dep</u> X

as

$$\rho \int_{\text{tot}} \underline{v} \cdot d\underline{a} = 0$$

i.e. <u>continuity!</u>

Now total $\underline{V}$ → $\left\{ \begin{array}{l} \text{velocity field} \\ \text{departure from } \underline{U} \end{array} \right.$

= vertical wake flow + potential flow.

$\underline{\Rightarrow}$ $\int_{\substack{\text{outside} \\ \text{(pot flow)}}} \underline{V} \cdot d\underline{a} \sim \int_{\substack{\text{inside} \\ \text{wake}}} \underline{V} \cdot d\underline{a}$ → const.

so $V_s \sim 1/r^2$ <u>outside</u>

so, must have $\underline{V}_{\text{pot flow}}$ s/t

$$\int \underline{V} \cdot da = Q/\rho$$ to compensate

then, for area at r:

$$V \pi r^2 \sim Q/\rho$$

$$\Rightarrow V \sim Q/r^2$$

$$\phi \sim Q/r$$

monopole !

global adjustment in potential flow due wake/viscosity (localized)

Message:

A little ν forces a global adjustment in flow structure.

Note is dominant far from body !

- pot flow $\phi \sim 1/r^2$ → dipole
- wake consequence of ν.