On diffusive and ballistic transport of intermittent coherent structures in the scrape-off layer



In this paper we review some of the theoretical and experimental aspects of coherent mesoscale filaments extending radially along magnetic field lines in the scrape-off layer, often called "blobs". We discuss the importance of turbulence and intermittency in plasma transport and describe the characteristics of the blob structure which reflect its intermittency, including the non-Gaussian statistics seen in profiles of intermittent events. We present different models of transport in order to develop a robust model which combines diffusive and intermittent convective transport in the edge by deriving the power-law exponents for the diffusive and convective processes. Using this, some options for this model are included; a derivation of a proposed transport mechanism is presented. Comparisons to experimental data are discussed.

# **Contents**





Figure 1: Upon completion of her first year as a Ph.D. student, the author reflects on her experience in graduate school.

### <span id="page-2-0"></span>1 Introduction

#### <span id="page-2-1"></span>1.1 Turbulence in magnetically confined plasma

Perhaps the greatest complexity on the road to achieving confinement in magnetic fusion devices comes from the inescapable phenomenon of turbulence in plasma [\[1\]](#page-14-0). Despite over half a century of intense theoretical and experimental study, these turbulent processes are still poorly understood and remain one of the great challenges in developing and managing a device capable of achieving nuclear fusion. Developing this understanding is of critical importance because the effects of turbulence can be deleterious to the effectiveness of the device. Though turbulence is ubiquitous throughout both the core and edge region, the effects of turbulence on both confinement and transport in the plasma edge are widely studied because of the overarching effects of edge turbulence on overall plasma performance. Control of turbulence in the edge has been shown to improve overall confinement in the entire device and affect heat and particle flux to the first wall, which prevents wall erosion (improving material performance and device longevity, and decreasing plasma contamination from wall sources). Heat and particle fluxes are transported in the edge region through the separatrix to the scrape-off layer (SOL) via anomalous processes that exhibit both diffusive and intermittent characteristics.

<span id="page-2-3"></span>

Figure 2: Sketch of a "blob" (in 2D) or a "filament" (in 3D) on the outer midplane of a tokamak. Note that the structure is localized in the plane perpendicular to the magnetic field and is extended along the parallel field lines. The blob will propagate radially. Taken from [\[1\]](#page-14-0).

### <span id="page-2-2"></span>1.2 Intermittency in the edge

Intermittency introduced by turbulence in the edge region of confinement devices has been observed in linear devices, stellarators, and tokamaks and are characterized by "bursts" in signals in edge diagnostics [\[2\]](#page-14-1). These intermittent events are fluctuations from the typical Gaussian distribution (analyzed using conditional averaging over several confinement devices) that feature large higher order moments; namely, the skewness and kurtosis are large due to these intermittent fluctuations.

In the scrape off layer, density and particle flux are intermittent features in space and time. These intermittent, turbulent transport events in the far SOL account for rapid cross-field transport and can carry plasma towards material surfaces at velocities exceeding  $100 \text{ ms}^{-1}$  [\[3\]](#page-14-2). This has been observed both

<span id="page-3-2"></span>

Figure 3: Density profile in the scrape-off layer during a major transport event. Line (a) represents the unperturbed SOL density. Lines (b) and (c) are the subsequent density scans in time. Line (d) is the density scan of the structure itself, with the characteristic form similar to a shock wave. Taken from  $[4]$ .

experimentally and in non-linear turbulence codes and violates the previously-held assumption that crossfield transport in the SOL was due to some random diffusive process (à la mean field theory) that could be characterized with some ad-hoc value. Because of this huge discrepancy between the theoretical model and the actual data, it is important to investigate and characterize these intermittent events to understand SOL transport.

### <span id="page-3-0"></span>1.3 The Blob

The quintessential characteristic figure of intermittency in edge transport is the plasma "blob": an intermittent, filamentary, coherent structure extending radially along field lines that moves rapidly through the plasma via ballistic propagation outwards toward the wall, shown in figure [2.](#page-2-3) The blob itself is characterized by a coherent area of density on the order of the separatrix density (which is much higher than the ambient SOL density; see figure [3\)](#page-3-2) that moves rapidly through the edge region. The density profile of the blob propagates as a shock, with a steep front followed by a long density tail, which is shown in figure [5.](#page-5-1) Particle transport is dominated by these rare-but-significant transport events; the magnitude and timescale of these fluctuations is characteristic of the intermittent nature of the blob.

#### <span id="page-3-1"></span>1.3.1 Skewness

Statistically, the formation of blobs is characterized by the radial location at which the third moment of the fluctuations (the skewness, which measures transfer of energy) equals zero. During the formation process, small perturbations in density will grow and disconnect through turbulent processes in the plasma, and form an equal number of blobs (areas of enhanced density) and voids (areas vacated by the redistribution of density). As seen in experiments [\[2\]](#page-14-1) measuring the skewness of the fluctuations through the edge, as in figure [4,](#page-4-2) charge polarization in the structure caused by curvature effects will cause the blobs and holes will move outwards and inwards, respectively. The blobs will move outwards, falling down the density gradient, and the holes will move inwards, going up the density gradient. Here, the skewness vanishes

<span id="page-4-2"></span>

Figure 4: Profiles for low (gray) and high (black) densities in L-mode conditions. The sprinkle-coated area is the separatrix, where the skewness is zero. Taken from [\[2\]](#page-14-1).

close to the separatrix with negative bursts inside the LCFS and positive deviations in the SOL, which is coherent with the characteristic feature of intermittency that higher order moments are large.

#### <span id="page-4-0"></span>1.3.2 Universality and self-similarity

Intermittent transport events have been shown to exhibit universality and self-similarity (per Kolmogorov, as these events are spawned by turbulent phenomena), and many statistical properties of blobs share these traits. Data from simulations measuring statistical properties of turbulence shows strongly non-Gaussian PDFs in turbulent fluctuations, with a portion of rare-but-significant ballistic transport events that propagate radially [\[1\]](#page-14-0). When compared to experimental data, profiles (e.g. power spectra, fluctuation statistics) are similar, across many devices, geometries, and confinement modes [\[5\]](#page-14-4). This indicates that there is a universality in the behavior of these structures, as they can be characterized using similar analytic techniques across many operating regimes. The concept of universality in the blob is important to note because if the PDF is insensitive to turbulent fluctuations, device configuration, and other parameters, then the mechanism for the ballistic transport is the same for all cases, meaning that a phenomenon like avalanching, which is also universal, is a useful tool for characterizing these events. This means we can search for appropriate distribution functions that describe the profiles and spectra and look for a fundamental model that can describe the statistical properties of the fluctuations.

#### <span id="page-4-1"></span>1.3.3 Don't call it what it's not

Many of the statistical characteristics of blobs are similar to mechanisms involved in systems exhibiting self-organized criticality (SOC). Comparisons have been made between blob propagation and the avalanche effects found in SOC structures [\[1\]](#page-14-0) because of similar ballistic transport mechanisms. However, since blobs are treated as coherent and individually propagating structures, these models do not necessarily

<span id="page-5-1"></span>

Figure 5: Time evolution of transport of a blob in the ambient SOL (red). The colormap represents the density; the coordinates represent the poloidal and radial directions. Notice the characteristic trailing wake at the tail and strong density front as time progresses. Taken from [\[4\]](#page-14-3).

apply. Although there are similarities, the blob is a distinct nonlinear entity that is not directly related to avalanche effects. Despite this necessary distinction, however, it is rare that a system is entirely described by SOC (and a full, robust definition of what SOC actually even doesn't seem to exist) [\[6\]](#page-14-5), so while blobs might not explicitly be considered avalanches, it is still interesting to note the similarities between the phenomena (particularly noting the principle of joint reflection symmetry, where blob observers (blobservers) will note that voids will always move up a density gradient and blobs will always move down the density gradient, regardless of orientation, as in figure [6\)](#page-5-2).

### <span id="page-5-0"></span>2 Non-linear model of edge transport

We have established the importance of turbulence and intermittency in the SOL and the existence of intermittent structures, and now move to a discussion of edge transport and how these blobs evolve in time

<span id="page-5-2"></span>

Figure 6: Density gradient in a system following the principle of joint reflection symmetry. Blobs (or bumps) will move down and to the left on the gradient, while voids will always move up and to the right. Taken from [\[7\]](#page-14-6).

and space. Both diffusive and non-local flux can coexist in the same paradigm, which is discussed here.

#### <span id="page-6-0"></span>2.1 Diffusion

In the edge region, transport is dominated by ballistic processes [\[1\]](#page-14-0). Since transport is largely ballistic, the conventional picture of diffusion does not adequately describe the transport in the edge. However, diffusion does still exist, and there are some diffusive processes in the SOL which are to be considered in addition to these ballistic, non-localized events. The generic, one-dimensional, scalar model for diffusion in plasma takes the form

$$
\frac{\partial n}{\partial t} = \frac{\partial}{\partial x} \Big[ D(x) \frac{\partial n}{\partial x} \Big] + S(x, t), \tag{1}
$$

where *D* is the diffusion coefficient and *S* is a source. This model is closely related with a local Brownian random walk model, with a Gaussian distribution and appropriate time and spatial correlation scales [\[8\]](#page-14-7). Since the diffusion coefficient can vary with x and t (and n, in nonlinear models), this equation works under many circumstances, but experiences many limitations, particularly in the intermittent and turbulent SOL. As such, a more robust description of transport is necessary.

#### <span id="page-6-1"></span>2.2 Ballistic propogation

As previously discussed, the dominant transport mechanism in the SOL is ballistic. The derivation presented in [\[9\]](#page-14-8) is a widely-used physical model for radial ballistic transport in the edge and is closely followed here. Since blob transport accounts for most of the particle transport in the SOL, we model SOL transport via the ballistic propagation of blobs.

First, we assume the SOL plasma temperature, T, is constant and the plasma resistivity electrostatic potential is small and constant, meaning the equation for electric current is

<span id="page-6-2"></span>
$$
\nabla \vec{j}_{\perp} + \nabla \vec{j}_{\parallel} = 0,\tag{2}
$$

with  $\nabla \vec{j}_{\perp} = c(\vec{B} \times \nabla P)/B^2$ ). We integrate Eq. [\(2\)](#page-6-2) along the field line, with boundary condition  $j_{\parallel}|_{target} =$  $en<sub>t</sub>C<sub>s</sub>(e\phi/T)$ , leading to

$$
\frac{e\varphi}{T} = \frac{\rho_i}{2n_t B} \int d\ell \, \nabla \ln B \cdot (\vec{B} \times \nabla n),\tag{3}
$$

where  $n_t$  is the plasma density near the targets,  $C_s = \sqrt{T/M}$  is the plasma sound speed, *M* is the ion mass, *e* is the elementary charge,  $\rho_i$  is the ion gyro-radius, and  $\ell$  is the coordinate along the magnetic field line.

<span id="page-6-3"></span>Now, to characterize an actual structure, we consider a blob of density  $n_b$  and parallel length  $\ell_b$  situated at the midplane in radial and poloidal coordinates *x* and *y*, respectively. We neglect magnetic shear and take  $\nabla$ ln*B* =  $\vec{e}_x/R$ , where *R* is the tokamak major radius, and arrive to

$$
\frac{e\varphi}{T} = \frac{\ell_b \rho_i}{2Rn_t} \frac{\partial n_b}{\partial y}.
$$
\n(4)

We now use Eq. [\(4\)](#page-6-3) to find the  $\vec{E} \times \vec{B}$  drift velocity, yielding

$$
\frac{\partial n_b}{\partial t} + C_s \frac{\rho_i^2}{2} \frac{\ell_b}{R} \left\{ \frac{\partial}{\partial x} \left[ n_b \frac{\partial}{\partial y} \left( \frac{1}{n_t} \frac{\partial n_b}{\partial y} \right) \right] - \frac{\partial}{\partial y} \left[ n_b \frac{\partial}{\partial x} \left( \frac{1}{n_t} \frac{\partial n_b}{\partial y} \right) \right] \right\} = 0. \tag{5}
$$

In the case where  $n_t = \xi n_b$ , where  $\xi$  is constant, we arrive to a separable solution of the form

$$
n_b(t, x, y) = n_b^{(x)}(t, x)n_b^{(y)}(x),
$$
\n(6)

with  $n_h^{(y)}$  $b<sup>(y)</sup>(y) \propto exp(-(y/\delta)^2)$ , where  $\delta$  is the poloidal scale length of the blob, arrives to the ballistic equation for blob density along x,

$$
(\partial_t + V_b \partial_x) n_b^{(x)} = 0,\t\t(7)
$$

where the velocity of blob propagation in the radial direction is

$$
V_b = C_s \left(\frac{\rho_i}{\delta}\right)^2 \frac{\ell_b}{R} \frac{n_b}{n_t}.\tag{8}
$$

#### <span id="page-7-0"></span>2.3 Anomalous diffusion

<span id="page-7-1"></span>Ideally, we are looking for a description which reflects the underlying Gaussian nature of the "classical" diffusion and the intermittent nature of the fluctuations. We will examine anomalous (or strange) diffusion; that is, where the moments of radial displacement in the system scale as

$$
\langle [x - \langle x \rangle]^n \rangle \sim t^{n\mathbf{V}},\tag{9}
$$

where  $\nu$  is the diffusion exponent (that does not have to be an integer) which depends on the transport process. In diffusive scaling,  $v = 1/2$ , and in ballistic scaling,  $v = 1$ , as shown in Section [2.1](#page-6-0) and Section [2.2,](#page-6-1) respectively. Generally, this equation is used in conjunction with the mean squared displacement of the distribution, or

$$
\langle [x - \langle x \rangle]^2 \rangle = Bt^{\mu}, \tag{10}
$$

<span id="page-7-2"></span>where *B* is some scaling parameter and  $\mu$  is the scaling exponent (similar to Eq. [\(9\)](#page-7-1),  $\mu = 1$  for diffusion and  $\mu = 2$  in ballistic transport). Both Eq. [\(9\)](#page-7-1) and Eq. [\(10\)](#page-7-2) correspond to different continuous time random walk (CTRW) models, depending on the range of the exponential parameters. This model can describe anomalous or strange transport, where the mean squared displacement will behave asymptotically with a power law [\[10\]](#page-14-9).

### <span id="page-8-0"></span>3 Fractional transport model

#### <span id="page-8-1"></span>3.1 Model of diffusive and ballistic transport

As we have established, there are both diffusive and ballistic elements to transport in the SOL. This corresponds with a typical Gaussian distribution (the diffusive process) with a rare-but-significant element (the ballistic process). Together, they form a unique distribution characteristic of many intermittent processes: a Gaussian-like distribution with a thick tail as the distribution extends to infinity. The idea of a "fat-tailed" transport model has been widely studied in many fields, from physics to biology to economics to many more. Here, we discuss two such examples as to prelude the development of a model which combines both diffusive and ballistic transport. Such models are useful because they can describe stochastic processes without employing spatio-temporal scales [\[11\]](#page-14-10).

#### <span id="page-8-2"></span>3.1.1 Collisional electron distribution

One such analysis, performed by the Soviet Union's gold medalist in Algebra Gymnastics circa 1988 in [\[12\]](#page-14-11), shows the self-similar solution of an exact solution of the electron kinetic equation with a powerlaw (fat tail) distribution. In this approach, our star athlete solves the collisional kinetic electron equation in several ranges across the distribution function. Along the electron distribution function, lower energy electrons are diffusive due to collisions and results in heat convection, while high energy electrons along the distribution tail propagate ballistically.

This model is effective in that it achieves the appropriate transport mechanism, but it is limiting because of the clunky algebra borne of the necessity to discretize the PDF based on energy. It is not a streamlined or robust approach, so it will not be employed in this analysis, but such a model *could*, in theory, be applied to anomalous blob transport for anyone interested in such a highly specific brand of masochism.

#### <span id="page-8-3"></span>3.1.2 Hydrogen trapping

Similarly, the broadband distributions of energy in hydrogen trapping in plasma-facing components found in tokamaks also follows a similar probability distribution which we can analyze in our search for a robust transport model [\[13\]](#page-14-12). These models have a robust consideration of the distribution of hydrogen trapping sites that takes both the standard hydrogenic diffusion and the possibility of Lévy flights. Such processes are described with a power-law distribution in both waiting time and step size,

$$
\frac{\partial^{\alpha}[H]}{\partial t^{\alpha}} = D_{eff} \frac{\partial^{2/(3-\beta)}[H]}{\partial t^{2/(3-\beta)}},\tag{11}
$$

where hydrogen transport will be affected by Lévy flights when  $0 < \alpha < 1$  and  $1 < \beta < 2$  [\[14\]](#page-14-13). Otherwise, when  $\alpha = 1$  and  $\beta = 2$ , the hydrogen concentration will follow the "classic" picture of diffusion.

This is applicable to the discussion of intermittency in the edge in that both diffusion and ballistic propagation can be handled in the same equation to define similar properties in the system. Diffusion and ballistic propagation are not affecting the system on the same scale (as in the edge/SOL) but they result in the same mechanism: transport. As such, we can characterize the "combination" of diffusion and ballistic propagation by introducing a fractional dimension. Where diffusive properties are governed by the

 $\alpha = 2$  derivative and ballistic properties are governed by the  $\alpha = 1$  derivative, the existence of both properties causing transport in the same direction means that we can creatively massage our integer derivative equation into the space of fractional derivatives.

#### <span id="page-9-0"></span>3.2 Application to blobs

Our approach will incorporate elements from both Section [3.1.1](#page-8-2) and Section [3.1.2,](#page-8-3) and modify the de-scription of non-local transport presented in and [\[10\]](#page-14-9) and [\[15\]](#page-14-14), paired with similar assumptions and approximations used in [\[16\]](#page-14-15). Our discussion will be limited to one dimension, which is appropriate for the axisymmetric radial geometry of a tokamak.

<span id="page-9-3"></span>We assume a seperable PDF of density distribution

$$
\xi(x - x', x', t - t', t) = p(x - x', x'; t)\psi(x'; t - t'),
$$
\n(12)

where *p* and  $\psi$  are statistically independent, with *p* for the size of the step in the motion of the blob and  $\psi$  for the waiting time. We are looking for a "master equation" [\[10\]](#page-14-9), which will describe the evolution of the density of the blob. The resulting equation must be of the form of a gain-loss balance,

$$
\partial_t n(x,t) = \int_0^t dt' \int d^d x' [-K(x,x'-x;t',t-t')n(x,t') + K(x',x-x';t',t-t')n(x',t')],\tag{13}
$$

where  $K(x', x - x'; t', t - t')$  is the transition PDF of finding at *x* at a time *t* the blob with was in *x'* at time  $t'$ . This introduces memory into our system. The transition PDF  $K$  can be related to the functions  $p$ and  $\psi$  through a considerable amount of stimulating algebra painstakingly detailed in both [\[10\]](#page-14-9) and [\[15\]](#page-14-14). Ultimately, the result is a master equation of the form

<span id="page-9-1"></span>
$$
\partial_t n(x,t) = -\int_0^t dt' \phi(t-t';x) n(x,t') + \int dx' p(x-x';x',t) \int_0^t dt' \phi(t-t';x') n(x',t'), \tag{14}
$$

where  $\phi$  is an auxiliary distribution resulting from a convolution of the probability density and the waiting time PDF and the transition probability is

$$
K(x', x - x'; t', t - t') = p(x - x', x'; t) \psi(x'; t - t').
$$
\n(15)

We can now start to apply Eq.  $(14)$  to the blob. We will restrict the coordinate x to values between 0 and 1 through normalization of the coordinates via introduction of a boundary at  $X = 0$  and  $X = 2a$  (with *a* as the minor radius of the torus in the tokamak) such that  $x = X/2a$ . We will define this system such that if any blob hits the boundary  $(x = 0 \text{ or } x = 1)$  the blob is lost to the machine walls and Dave Hill gets to write a check for a new wall panel. Applying these boundary conditions and introducing a source term,  $f(x,t)$ , which characterizes the excitement of the blob from fluctuations due to instability [\[16\]](#page-14-15), we obtain

<span id="page-9-2"></span>
$$
\partial_t n(x,t) = -\int_0^t dt' \phi(t-t';x) n(x,t') + \int_0^1 dx' p(x-x';x',t) \int_0^t dt' \phi(t-t';x') n(x',t') + f(x,t). \tag{16}
$$

The introduction of the  $f(x,t)$  term also enables the ability to find a steady state solution: because of the introduced constraint that blobs coming in contact with the wall effectively removes the blob from the system, we can still see new blobs being excited through the introduction of this term.

We can restrict the CTRW model of blobs to a steady state, where  $\partial_t n(x,t) = 0$ . Then, the source term in the RHS of Eq.  $(16)$  must be time independent, such that

$$
f(x) = n(x) \int_0^t dt' \phi(t - t'; x) - \int_0^1 dx' p(x - x'; x', t) n(x') \int_0^t dt' \phi(t - t'; x'). \tag{17}
$$

This means that we cannot chose *p* and  $\psi$  arbitrarily due to the selection of the joint PDF in Eq. [\(12\)](#page-9-3). We will choose to satisfy our constraint by making the following adjustments:

$$
p(x - x'; x', t) \rightarrow p(x - x'; x')
$$
  
\n
$$
\phi(t - t'; x') = \frac{1}{\tau_D} \delta(t - t')
$$
  
\n
$$
\psi(t - t'; x') = \frac{1}{\tau_D} \exp\left(-\frac{t - t'}{\tau_D}\right),
$$
\n(18)

<span id="page-10-1"></span><span id="page-10-0"></span>where  $\tau_D = 1/(\bar{\chi}_{\parallel} v)$  is the timescale for a localized group of particles to diffuse (with collisionality) over a distance of the correlation length of the fluctuations. This means our master equation in Eq. [\(16\)](#page-9-2) will reduce to

$$
\partial_t n(x,t) = \frac{1}{\tau_D} \int_0^1 dx' p(x - x'; x') n(x', t) - \frac{1}{\tau_D} n(x,t) + f(x). \tag{19}
$$

By satisfying our induced constraint using Eq. [\(18\)](#page-10-0), we have created a Markovianization of Eq. [\(16\)](#page-9-2). However, this model is still valid to describe Lévy flights because it is still nonlocal in space.

Since we are aiming to find a solution which features the coexistence of both local and non-local transport, we are looking for some threshold to characterize the transition between the two regimes. Since the characteristic nature of the blob is its dramatic density difference to its surroundings, the introduction of a critical density gradient allows for a distinction between the regimes; that is, when diffusive transport is dominant, the density gradient will be small, and when ballistic transport dominates, there will be a sharp density gradient between the surrounding areas and the blob itself.

Such a density gradient can be defined as

$$
\nabla n(x',t) \equiv \frac{\partial n(x,t)}{\partial x}\bigg|_{x=x'},\tag{20}
$$

which makes the jump PDF time-dependent:

$$
p(x - x'; x', t) = p[x - x'; \nabla n(x', t)].
$$
\n(21)

This introduces additional freedom (and complexity) to the analysis because it makes Eq. [\(19\)](#page-10-1) nonlinear. Additionally, there is now a dependence on locality rather than being homogeneous. This means Eq. [\(19\)](#page-10-1) now reads

$$
\partial_t n(n,t) = \frac{1}{\tau_D} \int_0^1 dx' p[x - x'; \nabla n(x',t)] n(x',t) - \frac{1}{\tau_D} n(x,t) + f(x). \tag{22}
$$

Now, we must look for an equation for *p*. As discussed previously, our threshold for crossing between local and non-local transport will be a positive critical gradient, such that  $(\nabla n)_c > 0$ . Such a qualification is useful for this purposes because we are only considering blobs (and not holes left by the blobs). When the local density gradient is smaller than this critical value, we expect "classical", diffusive transport with a Gaussian distribution. When the local density gradient is larger than this critical value, we expect ballistic or superdiffusive transfer with a Lévy flight, with a Cauchy distribution. In the diffusive case,  $\alpha = 2$ , and in the ballistic case,  $\alpha = 1$ , as expected from our previous discussions.

Following the characterization of Lévy distributions in [\[10\]](#page-14-9), we obtain a PDF of the form

$$
P_L(x, \alpha, \sigma) = \frac{1}{2\pi} \int dk \cdot \exp(-ikx - \sigma^\alpha |k|^\alpha) = \int dk \cdot \exp(-ikx) \hat{n}_\alpha(\sigma k), \tag{23}
$$

where  $\alpha$  dictates which transport regime (local or non-local) and  $\sigma$  is some scaling parameter. For  $\alpha = 2$ , we know  $P_L$  will be Gaussian and for  $\alpha = 1$ ,  $P_L$  will be a Cauchy PDF. We introduce one final parameter to manage the threshold at which we will change regimes,

$$
\zeta(x',t) = \Theta[|\nabla n(x',t)| - (\nabla n)_c],\tag{24}
$$

where  $\Theta(x)$  is the Heaviside function. This leads us to a final equation for our step size PDF,

$$
p_{ss}[x-x';\nabla n(n',t) = \zeta(x',t)P_L(x-x,1,\sigma_1)] + [1-\zeta(x',t)]P_L(x-x',2,\sigma_2). \tag{25}
$$

This leads us to our final model,

$$
\partial_t n(n,t) = \frac{1}{\tau_D} \int_0^1 dx' p_{ss}[x - x'; \nabla n(x',t)] n(x',t) - \frac{1}{\tau_D} n(x,t) + f(x), \tag{26}
$$

which features two regimes: local and non-local transport, distinguished by some critical density gradient.

#### <span id="page-11-0"></span>3.3 Applicability to experiment

Intermittent features in the edge have been identified in both simulations and experiments. Blobs have been identified in many tokamaks using gas-puffing techniques (as in ASDEX; see figure [7\)](#page-12-0) and through statistical analysis of signal fluctuations. As discussed in Section [1.2,](#page-2-2) intermittent events are characterized by large higher order moments. As seen in figure [8,](#page-13-1) skewness and kurtosis measurements on Alcator C-Mod are large outside of the LCFS, in the SOL. Gas puff imaging signal fluctuations are consistent with the "shock"-like burst wave and slow decay shapes characteristic of blobs [\[5\]](#page-14-4). Waiting times between burst

<span id="page-12-0"></span>

Figure 7: Illuminated coherent filamentary structures seen from the gaspuffing position in ASDEX. Taken from [\[1\]](#page-14-0).

events are exponentially distributed and show that the appearance of blobs in the far SOL are uncorrelated, and peak burst times are distributed with a large exponential tail, which is characteristic of intermittency.

The experimental data is consistent with the predictions and assumptions we have made to this point: the blobs are coherent structures and move independently and ballistically, the blobs form and propagate through the far SOL and account for the majority of cross-field transport, and feature large high-order moments and non-Gaussian statistics. It is reasonable to assume our theoretical models will hold when intermittent events are rare.

An important parameter discussed in [\[5\]](#page-14-4) in the characterization of intermittent structures is the degree of burst overlap,

$$
\frac{\Phi_{rms}}{\langle \Phi \rangle} = \left(\frac{\tau_w}{\tau_d}\right)^{1/2},\tag{27}
$$

where  $\Phi$  is the superposition of uncorrelated pulses,  $\tau_w$  is the average burst waiting time, and  $\tau_d$  is the duration of the pulse (skewness and kurtosis are also measured using these parameters - increasing with the ratio  $\tau_w/\tau_d$ ). An expansion of this model would be necessary if fluctuations happen at high frequency, as overlap between bursts would decrease the  $\tau_w/\tau_d$  and decrease the magnitude of the skewness and kurtosis. For strong burst overlap, this would predict a normal distribution (which is consistent with the central limit theorem, and is independent of the "shock"-like behavior of the fluctuation). Further studies could investigate the possibilities of fractional diffusion in such a model and the validity of a normal distribution of rapid-fire large-scale events.

<span id="page-13-1"></span>

(a) Radial variation of the skewness of GPI signal fluctuations.

(b) Radial variation of the kurtosis of GPI signal fluctuations.

Figure 8: Skewness and kurtosis of measured signals in the SOL of Alcator C-Mod. The gray shading on both figures is the LCFS. Notice the large increase in both quantities outside of the LCFS, which is characteristic of the intermittency of blob transport. Taken from  $[5]$ .

## <span id="page-13-0"></span>4 Conclusions

Intermittency in the scrape-off layer introduces immense complexity to the transport of both particles and heat. In particular, the coherent structures of perturbed density known as blobs (formed by turbulence in plasma) reflect the strong influence of ballistic, anomalous transport on the plasma dynamics in the SOL. The synthesis of diffusion with this ballistic transport results in a diffusion equation that describes the anomalous transport introduced by the intermittent blob behavior that is characterized by Gaussian and Cauchy distributions. Such a model seems to agree well with experimental results.

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