Analysis of Fast Ion Intermittent Transport due to Alfvén Eigenmodes on DIII-D

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Experiments in the DIII-D tokamak show that fast ion losses to the wall in experiments with multiple Alfvén eigenmodes (AEs) can have intermittent behavior. Increases in neutral beam injection (NBI) power and AE amplitude correlate with larger and more frequent bursts of losses. Avalanches are likely caused by overlaps of the AEs, as using electron cyclotron heating (ECH) to modify the types of AEs present in the plasma eliminates this phenomenon. The system is found to be multifractal: possibly bifractal due to the present AEs.

I. INTRODUCTION

Alfvén eigenmodes (AE) play an important role in the transport of energetic particles (EP) in magnetic fusion devices. Not only does EP transport affect the confinement and efficiency of a reactor, but AEs can cause significant losses to vessel walls: damaging components and even puncturing holes through a machine[1]. Diagnostics like the Fast Ion Loss Detector (FILD) are used to help study the transport caused by these modes.

Intermittency in EP transport due to AEs is seen to occur in theory and experiment when there are multiple modes present[1, 2]. Here, the transport from one mode alters the EP distribution at the location of another mode, causing an avalanche when the second mode becomes unstable. Recently, a study at DIII-D on EP transport reported a mix of toroidicity induced Alfvén eigenmodes (TAEs) and reverse shear Alfvén eigenmodes (RSAEs) caused unfavorable fast ion transport that disappeared when the RSAEs were suppressed[3].

Section II presents a set of tools useful for characterizing intermittent time series describing EP transport. Analysis of FILD data from the current ramp phase of experiments comparing shots representative of both the intermittent case with TAEs and RSAEs as well as the RSAE suppressed cases with electron cyclotron heating (ECH) near q_{min} is performed in section III. Section IV serves as a summary.

II. DATA ANALYSIS TOOLS

Several methods of describing intermittency in a data series exist, the simplest of which is an examination of higher order moments of the system[4]. This is based on the growth of higher order moments of systems that exhibit randomness outside of the usual Gaussian nature. More enlightening is calculation of the Hurst parameter, which describes the correlations between points in a series.[5].

Several methods exist for calculating the Hurst parameter; however, one of the simplest to use is rescaled range analysis[5, 6]. This method compares the rescaled range of blocks in the series against segment length. This is most easily accomplished by dividing the time series into a set of $2N_s$ equally long pieces: one set starting at the beginning and the other from the end. Here s stands for the size of the block. Calculation starts by finding the cumulative deviation

$$Z(n) = \sum_{i=1}^{n} x(i) - m,$$

where $x(i)$ is the ith value in the series, and m is the arithmetic mean. The range is defined as

$$R_s = \max(Z(n)) - \min(Z(n)),$$

the largest difference between points in a block. Rescaling is done by dividing the range by the standard deviation, and an average of R/S values over blocks of the same length is taken. To approximate the Hurst parameter, a line is fit to a log-log plot of $(R/S)_s$ vs s, due to their relation:

$$(R/S)_s \propto s^H.$$

For a more detailed analysis of the Hurst exponent, a method called Detrended Fluctuation Analysis (DFA) can be used to calculate an exponent for multifractal system[7]. In DFA, the cumulative deviant is calculated prior to splitting the series into $2N_s$ segments. For each block, a least squares fit ζ is made to the deviate and the variance is calculated:

$$\sigma_s^2 = \frac{1}{s} \sum_{n=1}^{s} [Z_s(n) - \zeta_s(n)]^2.$$

The fluctuation of order q is then defined as

$$F_q(s) = \left[\frac{1}{2N_s} \sum_{i=1}^{2N_s} (\sigma_s^2)^{q/2} \right]^{1/q}.$$

This has the same relation to the multifractal generalization of the Hurst parameter

$$F_q(s) \propto s^{H(q)}.$$

In a monofractal system, $H(q)$ is constant; however, in a multifractal system, negative orders $q < 0$ often relate to larger $H(q)$ while positive orders correspond to smaller $H(q)$[7].

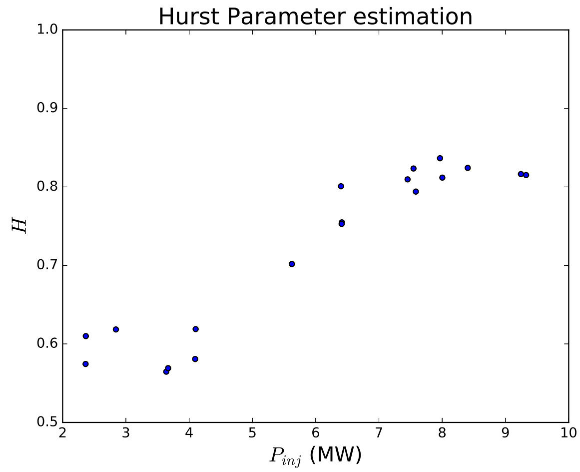

FIG. 1. Approximate Hurst parameters for shots with varying NBI power.

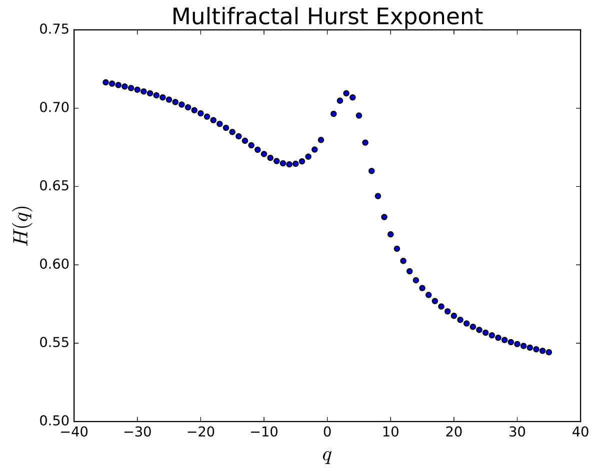

FIG. 2. Multifractal generalized Hurst exponent for a 5.6 MW shot as a function of order q.

III. APPLICATION TO DIII-D LOSSES

The clearest examples of the intermittent losses in FILD data from DIII-D are those when the signal is not dominated by prompt losses from neutral beams. The Hurst exponent for a series of shots sweeping a range of NBI power is calculated here. The increase in H with P_{inj} in Fig 1 agrees with the theory that intermittent transport is in part caused by avalanches due to overlapping AEs; that is, the injected power increases mode amplitude which should increase transport. Shots at the lowest injected powers still show Hurst parameters well above the Gaussian noise level of 0.5, suggesting a baseline of $H \sim 0.6$ for losses without significant AE overlap.

Looking at a single shot ($P_{inj} = 5.6\,\mathrm{MW}$), a multifractal analysis shows that the generalized Hurst parameter is clearly not constant. It is possible that — due to the existence of two separate types of AEs involved (TAEs and RSAEs) — this is a bifractal system, but it cannot be monofractal.

Data with ECH is more difficult to describe, as it was obtained when prompt losses have a sizable effect on the FILD data; however, similar shots without ECH can still be used to make comparisons. While the calculated Hurst parameters might be misleading in comparison to data without prompt losses, comparing higher order moments of the time series is still possible. Figure 3 shows calculated fifth order moments for shots sweeping a range of P_{inj}. There is no real distinction between the two sets at low beam power, where the AE amplitudes would be smallest, but at increased strengths, there is a clear gap between shots that have suppressed RSAEs and those that have not.

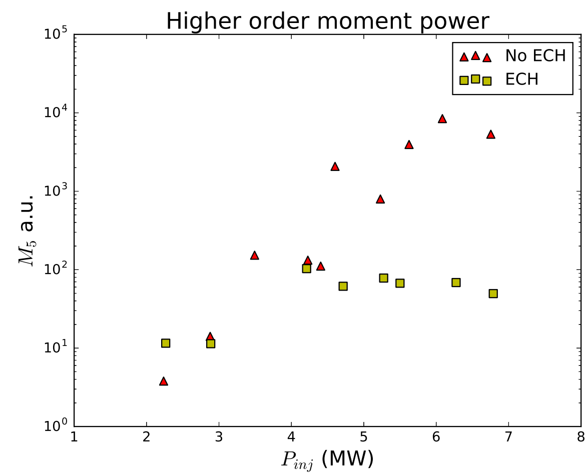

FIG. 3. Comparison of fifth order moments between shots that used ECH near q_{min} to suppress RSAEs and those without

IV. SUMMARY

Fast ion losses in the DIII-D tokamak show evidence of intermittent transport due to avalanching caused by averlapping AEs. Increasing mode amplitude via neutral beams tends to increase the Hurst exponent relating to the measured losses with the largest increase ($\Delta H \sim 0.2$) between NBI powers of 4 and 8 MW. Application of R/S analysis to more shots can help fill in gaps around 5 MW and potentially probe higher injected powers. Perhaps a more useful future analysis would be to compare the Hurst parameter against TAE and RSAE amplitudes directly from fluctuation measurements.

The generalized Hurst parameter for a representative shot shows clear signs of being a multifractal system.The overall trend outside of the peak is decreasing H with order q, characteristics noted in [7]. The nature of these experiments and the existing AEs suggests a good next step to take would be determining if this really is a bifractal system, or if there are other sources of multifractality.

Study of high order moments of shots with and without AE overlap show a clear gap between the two sets over ranges of neutral beam power consistent with increased Hurst exponents ($P_{inj} \gtrsim 4\,\mathrm{MW}$). A more in depth analysis of this gap would require removing the effects of pulsed prompt losses from the signals and is left to future work.

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