Turbulent damping on cosmic ray-induced streaming instability[a]

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(Dated: June 10, 2019)

Abstract

We examine two of the few governing factors that determine the degree of galactic cosmic ray isotropy: the turbulent damping on Alfven waves and the diffusion of the cosmic rays via magnetic field wandering. The traditional wisdom is that cosmic ray streaming velocity is set by balancing streaming instability growth rate with nonlinear Landau and ambipolar dampings. In this paper, we revisit several recent studies showing that magnetic turbulent damping also suppresses the cosmic ray streaming instability. Our goal is to qualitatively compare the turbulent damping with nonlinear and ambipolar dampings. We will also briefly discuss the superballistic spread-out of cosmic rays (i.e. the diffusion perpendicular to the local magnetic field) when they stream through stochastic magnetic field lines. Both mechanisms decrease the cosmic ray anisotropy which help explain the observed isotropic galactic cosmic rays.

[a] This work is the author's term paper for PHYS 235–Nonlinear Plasma Theory (SP19) in UCSD.

I. INTRODUCTION

The observed degree of isotropy of GeV cosmic rays in the frame of local interstellar medium is of order 10^{-4}, which implies the streaming velocity v_D is $\sim 10^{-4}\ c$. This result is actually surprising given that we are talking about ions moving at relativistic speed. The lack of fast-streaming beam is due to the fact that Alfven waves are unstable when the streaming velocity is faster than the Alfven wave velocity, v_A. The instability produces Alfven waves and particles scatter on them at the pitch angle in the wave frame. The resulting quasilinear wave-particle interaction isotropizes the cosmic rays and reduces v_D to v_A.

While Alfven waves are produced through streaming instability, there are also wave dampings that suppress Alfven waves. Cosmic rays are considered self-confined when the streaming instability growth rate is balanced by wave-damping processes. Two textbook examples are ion-neutral (ambipolar) damping and nonlinear Landau damping [1]. Ion-neutral damping occurs when Alfven wave propagates in partially ionized interstellar medium (ISM). Nonlinear Landau damping occurs in fully ionized plasma and is caused by the resonance interaction between thermal ions and the beat wave of two colliding Alfven waves.

Besides the two damping processes described above, the cosmic ray-induced streaming instability may also be suppressed by sub-Alfvenic turbulent damping due to the MHD cascade [2]. We will discuss that the nonlinear Landau damping is likely less important than turbulent damping for suppressing streaming instability in the Milky Way. On the other hand, while ion-neutral damping is effective in the partially-ionized plasma, the galactic disk is certainly not all filled with partially-ionized ISM. This idea leads us to ask *whether it is ion-neutral damping or Alfvenic turbulent damping that plays the critical role for suppressing the cosmic ray-induced streaming instability.*

In addition to streaming instability, the magnetic field wandering also affects the isotropy of cosmic ray [3, 4]. The diffusion perpendicular to the mean magnetic field will spread out the cosmic rays while they propagate along magnetic field lines. We will briefly mention their implication to anisotropy but would not discuss them in detail.

II. TURBULENT DAMPING OF STREAMING INSTABILITY

The reference [2] has a great detailed discussion on the MHD cascade and turbulent damping in the sub-Alfvenic regime (with Alfven Mach number $M_A = u_L/V_A < 1$) and super-Alfvenic regime (with $M_A > 1$). In the cosmic ray steaming instability scenario, the weak and strong sub-Alfvenic turbulent dampings are mostly related in the realistic magnetized plasma in Milky Way halo. Here we quickly review them.

A. strong sub-Alfvenic turbulence

The Alfven waves are distorted by shearing due to the evolving perpendicular magnetic turbulence, δB_x, of the induced perpendicular scale, x, as the waves propagate along the magnetic field lines. The distortion perpendicular to the magnetic field line is estimated as $\delta_x \approx V_A t\,(\delta B_x/B)^2$ where $\delta B_x/B$ is inversely proportional to eddy turnover time, x/u_x. The damping of the Alfven wave with wavelength λ happens at the resonance condition, $\delta_x = \lambda$. At the resonance, the turbulent damping time, t, would be the resonant eddy turnover time, x/u_x, where $u_x = V_A \left(l_\perp/L\right)^{1/3} M_A^{4/3}$ is the velocity fluctuation at the scale x in the strong sub-Alfvenic turbulence flow. As a result the strong sub-Alfvenic damping rate is estimated to be [2]

$$\Gamma_{\rm sub,s} \approx \frac{u_x}{x} \approx \frac{V_A M_A^2}{\lambda^{1/2} L^{1/2}}. \tag{1}$$

The maximum wavelength that strong sub-Alfvenic turbulence can cascade is $\lambda_{\rm max,s} = L M_A^4$. The Larmor radius, r_L, of the particles emitting Alfven waves can interact with strong turbulence if $r_L < L M_A^4$. Above this scale, the wave is cascaded through weak sub-Alfvenic turbulence.

B. weak sub-Alfvenic turbulence

In the weak sub-Alfvenic turbulence regime, the scaling relation for velocity fluctuation of scale l is $u_l \sim u_L \left(l_\perp/L\right)^{1/2}$, where u_L is the velocity of injection energy. Comparing to strong turbulence, the cascade of weak turbulence is $\left(V_A l_\perp / u_l L\right)^2$ times slower. The weak sub-Alfvenic turbulent damping is given by

$$\Gamma_{\rm sub,w} \approx \frac{V_A M_A^{8/3}}{\lambda^{2/3} L^{1/3}}, \tag{2}$$

with the gyro-frequency r_L that can be affected in the range $L M_A^4 < r_L < L M_A$.

Interestingly, in the limit of small Alfven Mach number, $M_A \ll 1$, the weak turbulent damping can affect a much larger r_L and energy range of cosmic rays than in the case of strong turbulent damping. In the following, we will see which kind of turbulence affect the cosmic ray streaming more significantly.

C. strong v.s weak

Consider coronal region of ISM in galactic disk with $n_i \approx 10^{-3}$ cm^{-3}, $T \approx 10^6$ K, and $B_0 \approx 3\ \mu$G. The Alfven speed we get is $V_A \approx 2 \times 10^7$ cm/s. The turbulence velocity, u_L. is likely much less than 10^6 cm/s, but let's use this value for estimating the upper bound of Alfven Mach number. As a result, we expect $M_A < 0.05$ in the galactic disk. If we take the injection scale to be about the scale heigh of the galactic disk, i.e. $L \approx 100$ pc, then $L M_A^4 \approx 5 \times 10^{14}$ cm.

For the relativistic cosmic ray protons, the Lorentz factor γ in the Milky Way frame is the ratio of its kinetic energy to the proton rest mass energy. The relativistic proton of the Lorentz factor γ has the Larmor radius $r_L \approx 10^{12}\gamma$ cm. Comparing r_L with $L M_A^4$, we see that the critical value is $\gamma \approx 500$. So for cosmic rays propagating in the galactic disk, we expect the cosmic rays of energy below 500 GeV undergo the strong sub-Alfvenic turbulent damping and energy above 500 GeV undergo weak sub-Alfvenic turbulent damping.

As we have mentioned earlier, the degree of anisotropy of cosmic rays is approximately the ratio of streaming velocity to the speed of light. In a steady state, the streaming velocity is determined by balancing the streaming instability growth rate with the turbulent damping rate,

$$v_D \approx V_A \left(1 + \frac{\Gamma_{\rm sub}\, n_i r_L}{\gamma c n_{\rm cr}}\right). \tag{3}$$

The strong sub-Alfvenic turbulent damping rate should be used in Eq. 3 when $\gamma < 500$, and weak sub-Alfvenic turbulent damping is used when $\gamma > 500$.

III. COMPARISON WITH NONLINEAR LANDAU AND AMBIPOLAR DAMPINGS

As shown in Eq. 3, a larger damping rate will lead to a faster streaming. This is because the damping suppresses the growth of waves, thereby decreasing the wave-particle interaction rates. Since most of the relativistic cosmic rays are at $\sim$ 1 GeV energy scale, we expect that in the galactic disk environment, cosmic ray induced streaming instability may be suppressed by strong sub-Alfvenic waves. Let's first compare strong sub-Alfvenic turbulent damping with the nonlinear Landau damping in the fully ionized regions. If the nonlinear Landau damping was the only damping mechanism, then the damping rate obtained via steady state is given as

$$\Gamma_{\rm NL} \sim \Omega\sqrt{\frac{r_L n_{cr} v_i}{L_z n_i V_A}}, \tag{4}$$

where L_z is the cosmic ray scale-height. The ratio of two dampings is [1]

$$\frac{\Gamma_{\rm sub,s}}{\Gamma_{\rm NL}} \sim \frac{B^{3/2} M_A^2 n_i^{-1/4} L_z^{1/2}}{L^{1/2} n_{cr}^{1/2} T^{1/4} m_i^{1/2}} \propto M_A^2\, L_z^{1/2}. \tag{5}$$

In a weak cascade regime where M_A might be small enough that the nonlinear Landau damping might start to be comparable to sub-Alfvenic turbulent damping. However, it's important to keep in mind that nonlinear Landau damping is a self-regulated process – a suppression of instability will lead to a faster streaming, thereby increasing the scale heigh L_z. On the other hand, the turbulent damping does not have self-regulation, so it is likely always dominant in most of the environment.

The Milky Way certainly is not full with ionized plasma. Actually only $\sim 1\%$ of ISM is ionized, and a significant fraction of ISM is *partially ionized* gas. The ion-neutral damping, which has the form [5]

$$\Gamma_{in} \approx \frac{1}{4} n_n \langle\sigma v\rangle_{in}, \tag{6}$$

is efficient in partially ionized gas. An important point about Eq. 6 is that there is no self-regulation in ion-neutral damping. As a result, the streaming instability growth rate in partially-ionized ISM gas is likely balanced by both the turbulent damping and the ion-neutral damping.

The idea of turbulent damping leads us to beg the question of how good "the source to sink" model in Ref. [1] and "leaky-box" model in Ref. [6] really are. The turbulent damping is just as ubiquitous as the two classic damping mechanisms. The consideration of turbulent damping may decrease the self-confinement energy scale below 100 GeV.

IV. OTHER POSSIBLE ISOTROPIZATION

In a magnetized plasma, electrons have perpendicular diffusion due to a magnetic field wandering in a chaotic magnetic field environment [7]. This idea is used to explain the thermal conductivity in galaxy clusters [8, 9]. The same idea can be applied to the cosmic ray transport in the galactic disk. As a beam of cosmic ray propagate with distance l along the local magnetic field lines, the cosmic rays's perpendicular distance from the initial magnetic field line would be $d\,(l)$

$$d\,(l) \sim d(0)e^{l/L_k}, \tag{7}$$

where L_k is the Lyapunov length. That is, there is a (super)diffusion induced by magnetic field wandering [10]. As the cosmic rays stream along the magnetic field, they also spread out super-ballistically in the direction perpendicular to local magnetic field lines, thereby decreasing the anisotropy of the cosmic rays.

V. SIGNIFICANCE

The isotropy of cosmic rays has been well measured up to 10^6 GeV. Any damping of the steaming instability may decrease the streaming velocity of cosmic rays and alter the observed galactic cosmic ray isotropy. At low energy end ($\gamma \lesssim 1000$) of the cosmic ray spectrum, strong sub-Alfvenic turbulent damping should also be considered in the leaky-box or any self-confinement models. On the high energy end ($\gamma \gtrsim 1000$) of the cosmic ray spectrum, the weak sub-Alfvenic turbulent damping could effectively damp the streaming instability over a large range of Larmor radii r_L. Both cases have not been well-studied in the current cosmic ray community. Further studies are required to understand when and how the cosmic ray self-confinement breaks down in the galactic disk.

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