Physics 235

→ Notes 3: Transport Channels, Particle Transport, Ambipolarity

→ Channels { heat, particles, momentum → e, i }

- Particles { why speak of heat transport? → test particle diffusivity calculated; why not particle transport? }

Note: The 'point' of stochastic fields is
$\chi_\perp \sim v_{the} D_M$
↳ electron speed.

Useful to:

→ ↗ consider kinetic eqn.

↘ distinguish between:
a) self-consistent case
$\tilde{B}$ produced by $\tilde{J}_{\parallel}$ in plasma
(i.e. EM instabilities - Ampère's Law)
b) $\tilde{B}$ produced by external means (coil) - i.e. RMP, though plasma response (i.e. screening) significant.

Now,

$$\frac{\partial f}{\partial t} + v_{\parallel} \nabla_{\parallel 0} f + v_{\parallel} \frac{\delta B}{B_0} \cdot \nabla_{\perp} f + \cdots = 0$$

then integrating;

$$\frac{\partial \langle n_e \rangle}{\partial t} + \nabla_\perp \cdot \left\langle \frac{\delta B_r}{B_0} \frac{\tilde{J}_{\parallel e}}{(-n_0|e|)} \right\rangle + \cdots = 0$$

but

$$\nabla_\perp^2 A_\parallel = -\frac{4\pi}{c} \tilde{J}_\parallel$$

$$= -\frac{4\pi}{c} \left(\tilde{J}_{\parallel e} + \tilde{J}_{\parallel i} \right)$$

$$\tilde{J}_{\parallel e} = -\frac{c}{4\pi} \nabla_\perp^2 A - J_{\parallel i}$$

$$= -\frac{c}{4\pi} \nabla_\perp^2 A_\parallel - n_0|e| \hat{V}_{\parallel i}$$

ion flow

so

$$\frac{\partial \langle n_e \rangle}{\partial t} + \nabla_\perp \cdot \left\langle \frac{\delta B_r}{B_0} \left(-\frac{c}{4\pi} \nabla_\perp^2 \hat{A}_\parallel - n_0|e| \hat{V}_{\parallel i} \right) \right\rangle + \cdots$$

$$= 0$$

$$\delta B_r = \nabla_\theta \hat{A}_\parallel$$

7)

$$\frac{\partial \langle n_e \rangle}{\partial t} + \frac{\partial}{\partial r} \left\langle \nabla_\theta \hat{A}_\parallel \left(-\frac{c}{4\pi} \nabla_\perp^2 \hat{A}_\parallel - n_{0} e \, V_{\parallel c} \right) \right\rangle$$

$$+ \quad = 0$$

8)

$$\frac{\partial \langle n_e \rangle}{\partial t} = \frac{c}{4\pi} \frac{\partial}{\partial r} \overset{①}{\left\langle (\nabla_\theta \hat{A}_\parallel)(\nabla_\perp^2 \hat{A}_\parallel) \right\rangle}$$

$$+ \; n_0 e \overset{②}{\left\langle \delta B_r \hat{V}_{\parallel c} \right\rangle}$$

① : Taylor identity calculation (akin $\langle v_r \nabla^2 \phi \rangle$)

$$\langle (\nabla_\theta \hat{A}_\parallel)(\partial_r^2 A_\parallel + \underbrace{\cancel{\nabla_\theta^2 \hat{A}_\parallel}}_{\text{odd}}) \rangle$$

$$= \langle (\nabla_\theta A_\parallel)(\partial_r^2 A_\parallel) \rangle$$

$$= \langle \partial_r (\nabla_\theta A_\parallel)(\partial_r A_\parallel) \rangle - \overset{\text{odd}}{\cancel{\langle (\nabla_\theta \partial_r A_\parallel)(\partial_r A_\parallel) \rangle}}$$

$$= \partial_r \langle (\nabla_\theta \hat{A}_\parallel)(\partial_r A_\parallel) \rangle$$

$$= -\partial_r \langle (\delta B_r)(\delta B_\theta) \rangle$$

[magnetic stress.

$$\frac{\partial \langle n_e \rangle}{\partial t} = -\frac{c}{4\pi} \partial_r \langle (\delta B_r)(\delta B_\theta) \rangle$$

↳ magnetic stress

$$+ \, n_0 |e| \langle \delta B_r \hat{V}_{\parallel i} \rangle$$

↳ magnetic "flutter" of parallel ion flow.

Points:

— no explicit dependence on small electron inertia; i.e. $1/M_e$

— relation of magnetic stress to electron particle transport due stochastic fields

i.e. suggests effect on zonal flows as zonal flows are fundamentally charge transport effect.

— fluctuating parallel ion flows + magnetic tilt ⇒ change electron density. (see Chernikov).

N.B. For external field, key is to calculate:

- δB_r in plasma
- $\tilde{J}_{\|0}$ in plasma.

$\Rightarrow$ δB_r in $\sim \dfrac{\delta B_{ext}}{\epsilon}$

→ Re: zonal flows:

Point: Magnetic wandering impacts zonal flows via charge

Recall: – fluctuations → DW model
i.e. H-M, etc.

– zonal mode: charge separation, radially – ambipolarity breaking

$\nabla \cdot J = 0$

usual e.s.; $\nabla_r J_{r\,pol} = 0$

$\Rightarrow$

$$\underbrace{\frac{\partial}{\partial t} \nabla_\perp^2 \phi}_{\text{linear pol. drift}} + \underbrace{\underline{V} \cdot \underline{\nabla}\, \nabla^2 \phi}_{\text{NL pol. drift}} = 0$$

so, for $\langle\ \rangle \rightarrow$ zonal symmetry

$\left[\int \text{ion gc} - \int \text{electron gc} \right.$

$$\frac{\partial}{\partial t}\langle \nabla_r^2 \phi \rangle + \partial_r \langle \tilde{v}_r \nabla_r^2 \tilde{\phi} \rangle + \cdots = 0$$

$\downarrow$ polarization charge Q_{pol}.

$\downarrow$

- flux of polarization charge (i.e. net difference ions, electrons)
- relate to Reynolds stress by Taylor identity
- compute via modulation

so, can view:

$$\partial_t \langle Q_{pol} \rangle + \partial_r \langle \tilde{v}_r Q_{pol} \rangle = 0$$

$$\rightarrow \Gamma_{charge} = \left(\langle \tilde{v}_r \tilde{n}_{i,gc} \rangle + \langle \tilde{v}_r Q_{pol} \rangle \right) |e| - \left(\langle \tilde{v}_r \tilde{n}_{e,gc} \rangle \right) |e|$$

$\hat{n}_i = \hat{n}_e \Rightarrow \Gamma_{charge} = 0$ ✓.

$n'_{i\,gc} + n_{i\,pol}$

But magnetic perturbations ~~[illegible]~~ induce new means charge transport

$\Rightarrow \underline{\nabla} \cdot \underline{J} = 0$

$$\nabla_\perp \cdot \underline{J}_{pol} + \nabla_\parallel J_\parallel = 0$$

$$\nabla_\parallel = \nabla_\parallel^{(0)} + \frac{\delta \underline{B}}{B_0} \cdot \nabla_\perp$$

$\therefore$

$$\frac{\partial}{\partial t} \langle \nabla^2 \phi \rangle + \langle \underline{\tilde{v}} \cdot \nabla \nabla^2 \tilde{\phi} \rangle = \langle \underline{\tilde{B}} \cdot \nabla \tilde{J}_\parallel \rangle$$

obvious from 2D MHD

ion pol. charge transport $\Rightarrow$ advection

current flow along tilted lines

(n.b. current: electrons ≠ ions)

so

$$\partial_t \langle \nabla^2 \phi \rangle = -\partial_r \left\{ \langle \tilde{V}_r \nabla_\perp^2 \tilde{\phi} \rangle - \langle \tilde{B}_r \tilde{J}_\parallel \rangle \right\}$$

so current flow along tilted lines acts to affect charge balance.

$\Rightarrow$ enters effect in Z.F.

Note:

- sign not clear apriori
- for (~) Alfvenic fluctuations $\Rightarrow$ damps
  i.e. acts cancel e.s.
- obviously related to electron particle transport.

Other:

→ electron momentum transport

→ "electron viscosity" / "hyper-resistivity"

i.e.

$$E_{\parallel} = \eta J_{\parallel} - \nabla_{\perp} \cdot \mu \cdot \nabla_{\perp} J_{\parallel}$$

(?!)

$$\frac{c^2}{\omega_{pe}^2} V_{Th} D_M.$$

⇒ Reconnection rate

i.e. Sweet-Parker:

$$V \sim \cancel{\quad} \quad V_A \frac{\Delta}{L}$$

$$\sim 1/S^{1/2} \, V_A$$

$$= V_A / \sqrt{R_m}$$

Electron Viscosity:

$$V \sim V_A / (R_{m_\mu})^{1/4}$$

→ Physics of Hyper-resistivity is essential to ELM crash.

→ One take: Drake '95

ITG : RSFI :: ETG : ∇J-driven

$\mu = \mu(\nabla J)$ → nonlinearity !

⇒ Good topic for further research.