Physics 235

Lecture II

Stochastic Fields: Kubo #, Collisionless and Collisional Heat Transport

Recall:

→ Chirikov criterion for onset Hamiltonian chaos

→ $Ku = \delta r / \Delta_r \sim l_{ac} \frac{\delta B}{B_0} / \Delta_r$

$(\to \tilde{\nabla} T_{ac} / \Delta)$

→ $Ku < 1$

$$D_M = \sum_{n} \left| \frac{\delta B_{n}}{B_0} \right|^2 \pi \delta(k_{\parallel})$$

$$= \left(\sum_{n} \left| \frac{\delta B_{r\,n}}{B_0} \right|^2 \right) l_{ac}$$

More on Kubo

For radial excursion!

$$dr/dz = \tilde{B}_r/B_0$$

so
$$\delta r \cong \int_0^{\ell} (\tilde{B}_r/B_0)\, dz$$

Now, line trajector de-coheres from perturbation for $\ell > \ell_{ac}$

$\hookrightarrow$ autocorrelation length

What do the symbols mean?

$\ell_{ac} \cong 1/|\Delta(k_\parallel)|$ i.e. inverse spatial bandwidth

$\therefore$ $\boxed{\delta r \cong \ell_{ac}\, \tilde{B}_r/B_0}$ $\rightarrow$ {line excursion in 1 ℓ_{ac}

Can identify $\Delta_r \equiv$ scatterer radial correlation length (i.e. spatial spectral width)

then:

$$Ku \cong \delta r/\Delta_r \cong \frac{\ell_{ac}}{\Delta_r}\tilde{B}_r/B_0 \rightarrow \text{Kubo \#}$$

and can then posit:

$\rightarrow$ $Ku < 1$ $\Rightarrow$ many kicks in coherence length

$\Rightarrow$ diffusion process

$$\boxed{Ku \sim 1 \rightarrow \text{B.B.K. "natural state of EM turbulence"}}$$
$Ku \sim 1 \rightarrow$ critical balance.

$\rightarrow Ku > 1 \rightarrow$ more than one Δ_r in $K_1\tilde{b}_r$
$\rightarrow$ strong scattering $\leftrightarrow$ percolation.

QLT

Here $Ku \lesssim 1$, at first. So, proceed via Quasilinear theory.

$$\Gamma_M = \left\langle \frac{\tilde{B}_r}{B_0} \tilde{f} \right\rangle$$

$$= \sum_{\underline{k}} \frac{\tilde{B}_{r-\underline{k}}}{B_0} \tilde{f}_{\underline{k}}$$

$$-i\left(k_z - k_\theta \frac{B_\theta}{B_0}\right) \tilde{f}_{\underline{k}} = -\tilde{B}_{r_{\underline{k}}} \frac{\partial \langle f \rangle}{\partial r}$$

so

$$\Gamma_M = -D_M \, \partial \langle f \rangle / \partial r$$

$$D_M = \sum_{\underline{k}} \left| \frac{\tilde{B}_{r\underline{k}}}{B_0} \right|^2 \pi \delta(k_z - k_\theta B_\theta / B_0)$$

magnetic diffusivity

$$= \sum_{\underline{k}} \left| \frac{\tilde{B}_{r\underline{k}}}{B_0} \right|^2 \pi \delta(k_\parallel)$$

(RST Z 86)

$$\sim \left\langle \left(\frac{\delta B_r}{B_0} \right)^2 \right\rangle l_{ac}$$

What is l_{ac}?

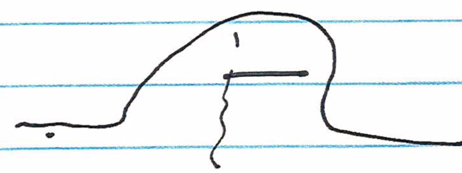
l_{ac}

N.B.: $\sum_{\underline{n}} = \sum_{m,n}$ spatial spread scatterers $\rightarrow l_{ac}$

$n = \frac{m}{q}$, $dn = \frac{m}{q^2} q'/dx$

$\Rightarrow$ spatial scale of spectral width (Δr) sets $|k_{\parallel}| \sim \left|\frac{k_\theta \Delta r}{L_s}\right|$

$$\boxed{l_{ac} \sim L_s/|k_\theta \Delta r|}$$



Lines then diffuse as:

$\langle \delta r^2 \rangle \sim D_M z$

Broader Perspective $\rightarrow$ Orbit averaging

N.B. Line Liouville eqn can be obtained by reducing / simplifying DKE

(*) $\rightarrow v_D$ large,

$$\cancel{\frac{\partial f}{\partial t}} + v_\parallel \hat{n}_0 \cdot \nabla f + \cancel{\underline{v}_D} \cdot \nabla f - \frac{c}{B_0} \nabla \phi \times \hat{z} \cdot \nabla f$$

$$\cancel{+} \; v_\parallel \frac{\delta \underline{B}_\perp}{B_0} \cdot \nabla f \cancel{- \frac{|e|}{m_e} E_\parallel \frac{\partial f}{\partial v_\parallel}} = \cancel{C(f)}$$

$$\Rightarrow \quad \mathbf{b}_0 \cdot \nabla f + \frac{\delta B_\perp}{B_0} \cdot \nabla f = 0$$

Scales

Now, scales:

$l_{ac} \to$ (scatters)
$\to$ field line memory length
self-coherence of scattering field.

$l_c \to$ line decorrelation length
(length over which line scattered from its unpo)

i.e. $$r \frac{d\theta}{dz} = \frac{B_\theta(r)}{B_0}$$

but r scattered, $\Rightarrow$

$$\frac{dy}{dz} = B_\theta(r_0) + \frac{B_\theta'(r_0)}{B_0} \delta r$$

so

$$\frac{d}{dz} \delta y \cong \frac{B_\theta'(r_0)}{B_0} \delta r$$

$$\langle \delta y^2 \rangle = \left\langle \left(\int^z \left(\frac{B_\theta'}{B_0} \right) \delta r \, dz' \right)^2 \right\rangle$$

⇒

$$\langle \delta y^2 \rangle \sim \frac{B_\theta'^2}{B_0^2} z^2 \langle (\delta r)^2 \rangle$$

$$\sim \frac{B_\theta'^2}{3 B_0^2} D_M z^3$$

akin

$$\langle \delta x^2 \rangle \sim D_v \frac{T^3}{3} \quad \text{in 1D}$$

For orbit decorrelation length:

$$k_\theta^2 \langle \delta y^2 \rangle \sim k_\theta^2 \frac{B_\theta'^2}{3 B_0^2} D_M z^3$$

⇒

$$l_{c,k} \sim \left(k_\theta^2 \frac{B_\theta'^2}{3 B_0^2} D_M \right)^{-1/3}$$

$$\sim \left(\frac{k_\theta^2}{L_s^2} \frac{D_M}{3} \right)^{-1/3}$$

analogous to shear dispersion $\left(\frac{k_\theta^2 V'^2 D}{3} \right)^{1/3}$

Also: orbit exponentiation length (separation) → $\boxed{\text{stretching}}$

show via 2pt. $\langle \delta f(1)\, \delta f(2) \rangle$

chaotic → stretching

→ for QL regime validity:

$$\boxed{l_{ac} < l_c} \rightarrow Ku < 1$$ (show!) ($l_c < l_{ac}$ → ? prop.)

and another (particle) length: l_{mfp}.

$\Rightarrow$ $l_{ac} < l_c < l_{mfp}$ → so called "collisionless regime"

$l_{ac} < l_{mfp} < l_c$ → collisional

which brings us to: something physical

⇒ (Electron) Heat Transport

Theme: - waves - interacting process

N.B. - nobody cares about "line" diffusion

why do this?

- people (i.e. experimentalists) do care about:
  - → heat
  - → particle
  - → momentum

  } transport

Rechester & Rosenbluth PRL '78 a MUST!

∴ lets begin with heat transport!

→ consider $l_{ac} < l_c < l_{mfp}$;

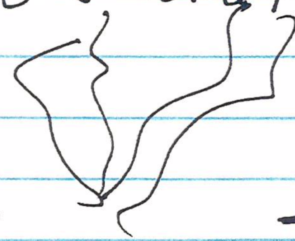
heat diffusivity

what is $\chi_\perp$?

⇒ "of course its $\chi_\perp \sim V_{th} D_M$"

→ but is it so simple?

— lines wander



Recall — thought prblm.

— but, lets assume parallel collisions (only) happen. (Particle stays on line!).

so motion along line is diffusive

$$\delta z^2 \sim D_{\parallel} t \sim \chi_{\parallel} t$$

$D_{\parallel} \sim V_{th}^2/\nu$

$\chi_{\parallel}$: parallel thermal diffusion

→ so: for slug heat:

$$\langle \delta r^2 \rangle \sim D_M z \sim D_M (\chi_{\parallel} t)^{1/2}$$

radial scatter

so: $\chi_\perp = d\langle \delta r^2 \rangle / dt \sim D_M (\chi_{\parallel})^{1/2} / t^{1/2}$

→ 0 !

Point: → line may wander
but

→ particle kicked back ~~of~~ along line

→ even though $l_c < l_{mfp}$,

no net radial wander as particle kicked back

Lessons:

→ collisions control irreversibility

⊕ → $\boxed{\text{need get kicked off field line}}$

→ Need:

— ⊥ coarse graining

— FLR → ρ_e

— x_L

— drifts.

minimum resolution scale

⇒ applied every l_{max}

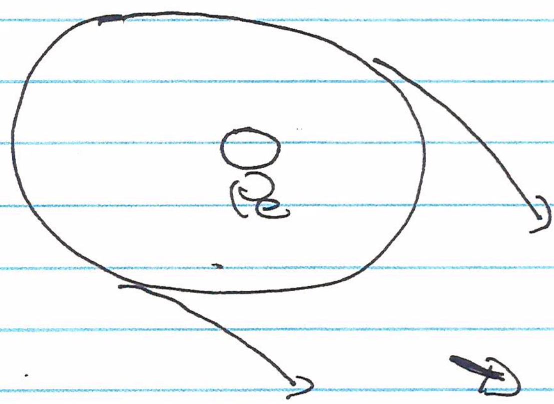
smear particle location over a resolution cell.

⇒ coarse graining ~~reses~~ resets "active volume".

so

→ consider the following argument:

① Consider disk of $r \sim \rho_e$



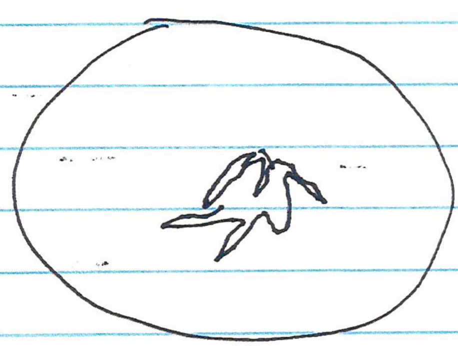
② Map disk forward, noting that $\nabla \cdot B = 0$
⇒ map is area preserving

after ~ l_{mfp}

1 $h_L > 0$

1 $h_L < 0$

($h \to$ Lyapunov Exp)

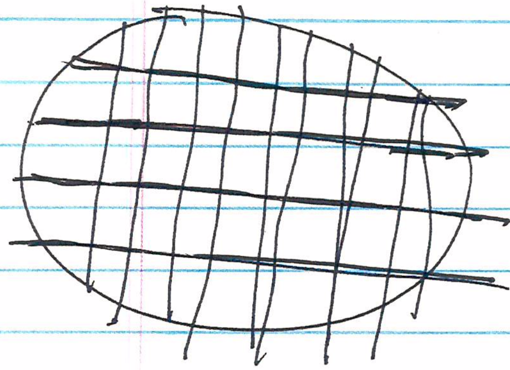


width

$w \sim \rho_e e^{-l_{mfp}/l_c}$

$(\ell \sim \rho_e e^{+l_{mfp}/l_c})$

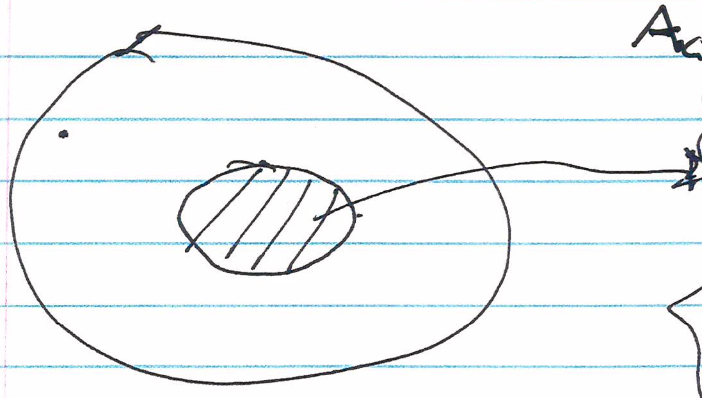
③ but coarse graining occurs at l_{mfp}



["particle/contour ~~re-sets~~/smeared" to nearest grid site

~~so~~

④



$A_{cg}\bar{f} = A_0 f_0$
$(\bar{f} < f_0)$

coarse graining of structure from previous V.G.

and so on continue...

⑤ Ludwig Boltzmann assures us no memory between steps ($1\ l_{mfp}$/collision time)

so initial spot expands, with random walk, as

$\langle \delta r^2 \rangle \sim D_{cl}\, l_{mfp}$. | in l_{mfp}

~~18.~~

i.e. $\boxed{\text{coarse graining interval sets } \langle \delta r^2 \rangle \text{ step!}}$

$\Rightarrow$

⑥ then, for $\chi_\perp$ (in l_{mfp}): $1/\tau_c \sim \nu_c$

$$\chi_\perp \sim \langle \delta r^2 \rangle / \tau_c \sim D_M \frac{l_{mfp}}{\tau_c}$$

$$\sim v_{th} D_M.$$

$$\Rightarrow \boxed{\chi_\perp \sim v_{th} D_M}$$

→ collisionless stochastic field heat diffusivity

→ manifestly independent of collisionality

→ yet clearly dependent on collisions and coarse graining!

Lesson: Coarse graining essential to irreversibility

~~[illegible]~~

Collisions → arrow of time.

or Coarse graining essential to kick particle off field line, or else collisional back-scatter reverses wander.

Stoch Fields, cont'd

Exercises (suggested):

i.) Derive the magnetic diffusivity with magnetic drifts. How do these modify D_M? Explain why high energy particles (runaways) are confined longer than thermals.

ii.) Formulate the theory of diffusion due stochastic fields in toroidal geometry using ballooning mode formalism for the fluctuations.

iii.) What happens to net cross field transport in a standing spectrum of e.s. and magnetic perturbations. When might transport vanish? Why?

→ Collisional Regime — More challenging

Here: $l_{ac} < l_{mfp} < l_c$
(short mean free path)

Point: → $l_{mfp} < l_c$ ⇒ particle random walks parallel and undergoes many kicks in l_c. So parallel motion is diffusive.

what is going on?

→ perpendicular motion is continuous coarse graining/spreading, at
$D_\perp \sim \rho_e^2 \nu_{ee} \sim \rho_e^2 \frac{V_{The}}{l_{mfp}}$

So, can write:

$\langle dr^2 \rangle \sim D_M l_{c,d}$

↓

parallel correlation length
(d signifies diffusive regime)

but also note that parallel motion is diffusive, so:

but time set by:

$$\chi_\parallel / l_{c,\sigma}^2 \sim 1/t \qquad \triangleq$$

$$\Rightarrow \frac{\langle \delta r^2 \rangle}{t} \sim \frac{\chi_\parallel}{l_{c,\sigma}^2} D_M l_{c,\sigma}$$

$$\sim D_M \frac{\chi_\parallel}{l_{c,\sigma}} \sim D_M \chi_\parallel / l_{c,\sigma}$$

$$\boxed{\chi_\perp = D_M \frac{\chi_\parallel}{l_{c,\sigma}}} \rightarrow$$

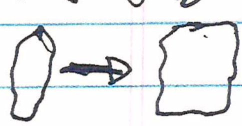
perpendicular heat conductivity in collisional regime.

$$\chi_\perp = v_{th} D_M \frac{l_{mfp}}{l_{c,\sigma}}$$

Now, what is $l_{c,\sigma}$?

Notice $l_{c,\sigma}$ is set by competition between 2 processes:

(1) width of δ increases due diffusion (coarse graining)



so $(d\delta)^2 \sim (D_\perp dt)$

$$d\delta \sim (D_\perp dt)^{1/2}$$

but

$$\chi_\parallel/(dL)^2 \sim 1/dt$$

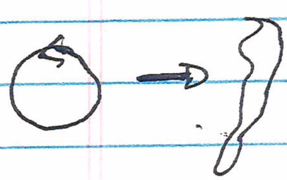
$\Rightarrow$

$$d\delta \sim \left(\frac{D_\perp (dL)^2}{\chi_\parallel}\right)^{1/2}$$

$$\boxed{d\delta \sim (D_\perp/\chi_\parallel)^{1/2}\, dL}$$

$$\underbrace{\frac{\chi_\parallel}{L^2} \sim \frac{D_\perp}{\Delta^2}}$$

(2) Width shrinks, due stochastic instability and area conservation:



$$d\delta/dL = -\delta/l_c \qquad \text{(exponential decay)}$$

then balance at:

$$\underbrace{d\delta \sim (D_\perp/\chi_\parallel)^{1/2} dL}_{\text{smearing}} \sim \underbrace{\frac{\delta}{l_c} dL}_{\text{thinning}}$$

so

$$\boxed{\delta \sim l_c (D_\perp/\chi_\parallel)^{1/2}}$$

N.B.: Can select δ from:

$$\partial_t T - \chi_\parallel \nabla_\parallel^2 T - D_\perp \nabla_\perp^2 T = 0$$

$$\Rightarrow \frac{\chi_\parallel}{l_c^2} \sim \frac{D_\perp}{\delta^2} \qquad \boxed{\delta \sim l_c (D_\perp/\chi_\parallel)^{1/2}}$$

Finally, need correlation length $l_{c\delta}$ for chunk size δ. Assume set by k_θ

$$k_\theta^{-1} \sim \delta e^{z/l_c}\Big|_{l_{c\delta}} \sim \delta e^{l_{c\delta}/l_c}$$

$$\boxed{l_{c\delta} \sim l_c \ln\left(1/k_0\delta\right)}$$

$$\boxed{l_{c\delta} \sim l_c \ln\left((\chi_\parallel/D_\perp)^{1/2}\right)}$$

$$\boxed{l_{c\delta} \sim l_c \ln\left((\chi_\parallel/D_\perp)^{1/2}/k_0 l_c\right)}$$

$$\Rightarrow \boxed{\chi_\perp \sim D_M\, \chi_\parallel / l_{c\delta}}$$

Apart from log factor:

$$\boxed{\chi_\perp \sim v_{Th}\, D_M \underbrace{\left(l_{mfp}/l_c\right)}_{<1}}$$

$\Rightarrow$ reduced relative to collisionless value.

Lesson: — collisions reduce ($l_{mfp} < l_c$) reduce χ_{eff} relative to "collisionless case"

— interplay of perp and parallel diffusion

— again, critical to knock particle off field line.

Now, the above calculation requires thought. It's much more convenient to crank mindlessly.

⇒ Hydro approach: Kadomtsev and Pogutse (not mindless but systematic)

Consider heat flux along wiggling fields i.e.

$$\underline{q} = -\chi_\parallel \nabla_\parallel T \, \hat{b} - \chi_\perp \underline{\nabla}_\perp T$$

$-\chi_\parallel \nabla_\parallel T \, \hat{b}$: parallel conduction

$-\chi_\perp \underline{\nabla}_\perp T$: perp. conduction

tricky in codes

$\chi_\parallel \gg \chi_\perp$

Here: $\underline{b} = \underline{b}_0 + \underline{\tilde{b}}$

$\underline{b}_0$ ↓ unperturbed

$\underline{\tilde{b}}$ ↦ fluctuating

$$\nabla_{\parallel} = \partial_z + \underline{\tilde{b}} \cdot \underline{\nabla}_{\perp}$$

$\underline{\tilde{b}} \cdot \underline{\nabla}_{\perp}$ ↓ piece along wiggling line

So, seek mean radial heat flux

$$\begin{aligned}\langle q_r \rangle = & -\chi_{\parallel} \langle \tilde{b}_r^2 \rangle \partial_r \langle T \rangle \quad (1) \\ & -\chi_{\parallel} \langle \tilde{b}_r \partial_z \tilde{T} \rangle \quad (2) \\ & -\chi_{\parallel} \langle \tilde{b}_r \tilde{b}_r \partial_r \tilde{T} \rangle \quad (3) \\ & -\chi_{\perp} \nabla_r \langle T \rangle\end{aligned}$$

(1), (2): usual quadratic

(3) → cubic

Now $\dfrac{(3)}{(2)} \sim \dfrac{\cancel{\chi_{\parallel}}\, \cancel{\tilde{b}_r} \tilde{b}_r \cancel{\tilde{T}} / \Delta_r}{\cancel{\chi_{\parallel}}\, \cancel{\tilde{b}_r} \cancel{\tilde{T}} / l_{ac}}$

$$\sim \frac{\tilde{b}_r\, l_{ac}}{\Delta_r} \sim Ku$$

so cubic nonlinearity dominates for $K_u > 1$.

$K_u < 1 \Rightarrow$ drop cubic.

To compute $\langle Q_r \rangle$, need

- retain ① (usual), and ②
- iterate for $\tilde{T}$ using

$\underline{\nabla} \cdot \underline{Q} = 0$ i.e. ala QLT.

Thinking (gasp!) first:

$$\langle Q_r \rangle \cong -\nu_{\parallel}\left[\langle \tilde{b}_r^2 \rangle \partial_r T + \langle \tilde{b}_r \partial_z \tilde{T} \rangle \right] - \nu_{\perp} \nabla_r \langle T \rangle$$

$$\cong -\nu_{\parallel}\left[\langle \tilde{b}_r \, \overbrace{b \cdot \nabla T} \rangle \right] - \nu_{\perp} \nabla_r \langle T \rangle$$

linearization:

$\tilde{b}_r \partial_r \langle T \rangle + \partial_z \tilde{T}$

$\therefore$

Point: - need non-zero $\tilde{b} \cdot \nabla T$ fluctuation to drive heat flux

- $\boxed{\text{i.e. temperature can't be constant along line, to drive parallel heat flux}}$

- $\underline{\nabla} \cdot \underline{q} = 0 \Rightarrow$ result must imply $\chi_\perp$ dependence!

$\chi_\perp$ to balance

(b)

$$\langle q_r \rangle = -\chi_{\parallel} \left[\langle \tilde{b}_r^2 \rangle \partial_r \langle T \rangle + \langle \tilde{b}_r \partial_z \tilde{T} \rangle \right] - \chi_\perp \nabla_\perp \langle T \rangle$$

$$\nabla \cdot \underline{q} = 0$$

$$\Rightarrow \quad \nabla_{\parallel} \tilde{q}_{\parallel} + \nabla_\perp \cdot \tilde{q}_\perp = -\chi_{\parallel} \partial_z \tilde{b} \, \partial \langle T \rangle / \partial r$$

i.e.

 ~~1/2~~

$$\underline{q} = -\chi_{\parallel}\left[(\partial_z + \underline{\tilde{b}}\cdot\underline{\nabla})(T_0 + \tilde{T})(\underline{b}_0 + \underline{\tilde{b}}) \right] - \chi_{\perp}\underline{\nabla}_{\perp} T$$

so

$$-\chi_{\parallel}\partial_z^2\tilde{T} - \chi_{\perp}\nabla_{\perp}^2\tilde{T} = -\chi_{\parallel}\partial_z\tilde{b}\frac{\partial\langle T\rangle}{\partial r}$$

$\Rightarrow$

$$\tilde{T}_{\underline{k}} = \frac{-\chi_{\parallel} i k_z \tilde{b}_{\underline{k}} \, \partial\langle T\rangle/\partial r}{\left(\chi_{\parallel}k_z^2 + \chi_{\perp}k_{\perp}^2\right)}$$

so

$$-\chi_{\parallel}\langle\tilde{b}^2\rangle\frac{\partial\langle T\rangle}{\partial r} - \chi_{\parallel}\langle\tilde{b}\,\partial_z\tilde{T}\rangle$$

$$= -\chi_{\parallel}\sum_{\underline{k}}\left(-\frac{\chi_{\parallel}k_{\parallel}^2|\tilde{b}_{\underline{k}}|^2}{\chi_{\parallel}k_z^2 + \chi_{\perp}k_{\perp}^2} + |\tilde{b}_{\underline{k}}|^2\right)\frac{\partial\langle T\rangle}{\partial r}$$

$$= -\chi_{\parallel}\frac{\partial\langle T\rangle}{\partial r}\sum_{\underline{k}}\left(\frac{-\cancel{\chi_{\parallel}k_{\parallel}^2}}{\chi_{\parallel}k_z^2 + \chi_{\perp}k_{\perp}^2} + \frac{\cancel{\chi_{\parallel}k_{\parallel}^2} + \chi_{\perp}k_{\perp}^2}{\chi_{\parallel}k_z^2 + \chi_{\perp}k_{\perp}^2}\right)$$

80

$$\boxed{\langle q_r \rangle_{NL} = -\chi_\parallel \frac{\partial \langle T\rangle}{\partial r} \sum_n \frac{\chi_\perp k_\perp^2 |b_n|^2}{\chi_\parallel k_\parallel^2 + \chi_\perp k_\perp^2}}$$

Note explicit dependence on $\chi_\perp$!

80.

$$\langle q_r\rangle_{NL} \cong -\cancel{\chi_\parallel} \frac{\partial\langle T\rangle}{\partial r}\int dk_\perp \int dk_z \frac{\chi_\perp k_\perp^2 \langle \tilde{b}_n^2\rangle}{\cancel{\chi_\parallel}\left(k_z^2 + \frac{\chi_\perp k_\perp^2}{\chi_\parallel}\right)}$$

$$= -\frac{\partial \langle T\rangle}{\partial r}\int dk_\perp \int dk_z \frac{\chi_\perp k_\perp^2 \langle \tilde{b}^2\rangle_n}{\left(\frac{k_z^2}{(\chi_\perp/\chi_\parallel)k_\perp^2}+1\right)\left(\frac{\chi_\perp}{\chi_\parallel}k_\perp^2\right)}$$

$$= -\frac{\partial\langle T\rangle}{\partial r}\int dk_\perp \frac{k_\perp^2(\chi_\parallel\chi_\perp)^{1/2}}{\sqrt{k_\perp^2}}\langle \tilde{b}^2\rangle_n l_{ac}$$

autocorrelation

bandwidth

autocorrelation l_{ac} enters via normalization

$\Rightarrow$

$$\boxed{\langle q_r \rangle_{NL} \cong -\sqrt{\chi_\parallel \chi_\perp}\, \underbrace{\langle \tilde{b}^2 \rangle l_{ac}}\, \underbrace{\sqrt{\overline{k_\perp^2}}}\, \frac{\partial \langle T \rangle}{\partial r}}$$

Note: - need $\nabla_\parallel \tilde{T} \neq -\tilde{b}_r \, \partial \langle T \rangle / \partial r$

$(\widetilde{B \cdot \nabla T} \neq 0)$ for $\perp$ heat flux

- $\langle \tilde{b}^2 \rangle l_{ac} \sim D_M$.

$\sqrt{\overline{k_\perp^2}} \sim 1/\Delta_\perp$

so

$$\langle q_r \rangle \cong -\chi_{\perp \mathrm{eff}}\, \partial \langle T \rangle / \partial r \quad - \chi_\perp\, \partial \langle T \rangle / \partial r$$

$$\chi_{\perp \mathrm{eff}} \cong \sqrt{\chi_\parallel \chi_\perp}\, \frac{D_M}{\Delta_\perp}.$$

$$\left(\begin{array}{l} \chi_\parallel \chi_\perp \sim \\ \frac{4\pi n_e}{\cancel{\nu}} \, \alpha_s^2 \cancel{\nu} \sim D_B \end{array} \right)$$

$$\boxed{\chi_{\perp eff} \simeq \frac{D_B D_M}{\Delta_\perp}}$$

- $\chi_{\perp eff}$ scales with Bohm, not Spitzer ($\chi_\parallel$)

- kicking off line important, again.

To compare R&R:

$$\chi_\perp \sim \frac{\sqrt{\chi_\parallel \chi_\perp} \langle \tilde{b}^2 \rangle l_{ac}}{\Delta_\perp}$$

what is $\Delta_\perp$?

Now $\frac{\chi_\parallel}{l_c^2} \sim \frac{\chi_\perp}{\Delta_\perp^2}$ {diffusion set ⊥ scale.

$$\Delta_\perp \sim l_c \sqrt{\chi_\perp / \chi_\parallel}$$

enters spectrum (small layer)

$\Rightarrow$

$$\chi_\perp \sim \sqrt{\chi_{\parallel}\chi_\perp}\; \frac{\langle \tilde{b}^2 \rangle l_{ac}}{l_0 (\chi_\perp/\chi_{\parallel})^{1/2}}$$

$$\boxed{\chi_\perp \sim \frac{\chi_{\parallel}}{l_c} D_M}$$

so – modulo $k_\parallel$, $\Delta_\perp$; agrees with RR to within log. factor

$$-\ \chi_\perp \sim V_{th} D_M \frac{l_{mfp}}{l_c}$$

$\Rightarrow$ covers diffusion in $k_\parallel$ ← stochastic fields

$\Rightarrow$ **Lesson: Take care re: irreversibility !**