A Brief Look at Other Directions

→ Primary focus on transport, secondary on Self-Organization (i.e. profiles?!)

→ Principal Application → Magnetic Confinement.

Other directions:

- Kinematic Waves, Traffic Flow
  - → Whitham
  - → phenomenological macroscopics
  - → car-following (CA → fluids)

Application: Time-Delay induced jams. (N_A ?)
(cf: Kosuga, P.D., Gurcan PRL '13).

— Flocking / "Boids" (ML for Flocking Rule?)

- Patterns in swarm of self-propelled agents.

c.f. { Toner and Tu PRE '98
Marchetti, et al. RMP '13 }

— Phenomenological (compressible) Hydro.
ρ, v

Broken G-I → Flock in dissipative media.
⟷ dissipative structure

{ Very similar to car-following,
Modes, structures. }

i.e. Flocking, etc. is big area in biophysics.

Flock vs Avalanch?

→ MFE

- more on Avalanching:
  see { Hahm, P.D. '18
        { OV talk

- Zonal Flow → staircase

  → Dif-P '10 PRE,
  → Ashourvan, P.D. '17
and
  → Dritschel - McIntyre '08

ZF → outer scale

Open issue: Scale selection struggle.

- Kinetics → EPM, etc.

  - Clump emission → avalanche
    ↓
    Cerenkov.

  - Phase space avalanches ¿ ?

Kinetics → Mean Field Theory

→ Thoughts on Avalanches

Mean Field Theory - DW. $\rightarrow$ Transport

Consider DKE $\rightarrow$ simplicity

$$\frac{\partial f}{\partial t} + v_z \frac{\partial}{\partial z} f - \frac{c}{B_0} \nabla\phi \times \hat{z} \cdot \nabla f - \frac{|e|}{m_e} E_z \frac{\partial f}{\partial v_z} = 0$$

(i.e. electrons) $\rightarrow \underline{B_0} = B_0 \hat{z}$ strt.

then

$$\partial_t \langle f \rangle + \partial_r \underset{\substack{\uparrow \\ \text{transport}}}{\langle \tilde{v}_r \delta f \rangle} + \partial_{v_z} \underset{\substack{\uparrow \\ \text{heating}}}{\langle \tilde{a}_z \delta f \rangle} = 0$$

n.b. $\delta f = \frac{|e|\phi}{T}\langle f \rangle +$ [redacted] $\hat{h}$

In linear theory, with Maxwellian:

$$\partial_t \left(\frac{|e|}{T}\phi\langle f \rangle + \tilde{h} \right) + v_z \partial_z \left(\cancel{\frac{|e|}{T}\phi\langle f \rangle} + \tilde{h} \right)$$

$$- \frac{c}{B_0} \nabla\tilde{\phi} \times \hat{z} \cdot \nabla \langle f_0 \rangle - \frac{c}{B_0} \nabla\tilde{\phi} \times \hat{z} \cdot \nabla \left[\cancel{\frac{|e|\tilde{\phi}}{T}\langle f \rangle} + \tilde{h} \right]$$

$$- \frac{|e|}{m_e} E_z \partial_{v_z} \left[\cancel{\langle f \rangle} + \frac{|e|\tilde{\phi}}{T} \langle f \rangle + \tilde{h} \right] = 0$$

ignoring parallel nonlinearity;

$$\partial_t \tilde{h} + v_z \partial_z \tilde{h} - \frac{c}{B} \nabla\phi \times \hat{z} \cdot \nabla \tilde{h} - \frac{|e|}{m_e} E_z \partial_{v_z} \tilde{h}$$

$$= - \partial_t \frac{|e|\phi}{T} \langle f \rangle - \frac{c}{B_0} \nabla\phi \times \hat{z} \cdot \nabla \langle f_0 \rangle$$

$$(\omega - \omega_*)$$

better,

$$\boxed{\begin{aligned} \delta f^{(1)} &= \frac{\phi_k}{\omega - k_z v_z} L_k \langle f \rangle \\ L_k &= - \frac{c k_\theta}{B_0} \frac{\partial}{\partial r} + \frac{|e|}{m_e} k_z \frac{\partial}{\partial v_z} \end{aligned}}$$

scattering operator

$$\boxed{\begin{aligned} \partial_t \langle f \rangle = {} & \overset{1}{\partial_r D_{r,r} \partial_r \langle f \rangle} + \overset{2}{\partial_r D_{r,v} \partial_{v_z} \langle f \rangle} \\ & + \overset{3}{\partial_{v_z} D_{v_z,r} \partial_r \langle f \rangle} + \overset{4}{\partial_{v_z} D_{v,v} \partial_{v_z} \langle f \rangle} \end{aligned}}$$

$$D_{r,r} = \text{re} \sum_{k} \frac{c^2}{B_0^2} k_\theta^2 |\phi_k|^2 \frac{i}{\omega - k_z v_z}$$

symmetry owing to

$$D_{r,v} = D_{v,r} = \text{re} \sum_{k} \frac{c}{B_0} \frac{|e|}{m_e} \overbrace{k_\theta k_z} |\phi_k|^2 \frac{i}{\omega - k_z v_z}$$

$$D_{v,v} = \text{re} \sum_{k} \frac{e^2}{m_e^2} k_z^2 |\phi_k|^2 \frac{i}{\omega - k_z v_z}$$

1 + 2 $\rightarrow$ radial transport

i.e. $\sim \partial_r \Gamma_{v,r}$

4 $\rightarrow$ parallel heating

Illuminating to examine momentum / flow:

$$\partial_t \int \langle f \rangle v_z = \partial_r \Big[\int dv_z \; \overset{1}{v_z D_{r,r} \partial_r \langle f \rangle} + \int dv_z \; \overset{2}{v_z D_{r,v_z} \partial_{v_z} \langle f \rangle} \Big]$$

$$- \int dv_z \; D_{v_z,r} \partial_r \langle f \rangle$$

$$- \int dv_z \; D_{v_z,v_z} \partial_{v_z} \langle f \rangle$$

① → χ_ϕ, etc. ; Π_{resid}

$\partial_r\langle f\rangle \to \partial_r\langle T\rangle$, etc.

② → Π_{resid}, also

i.e. $\Pi_{resid} = \int dv_z \; v_z \sum_k \frac{c}{B}\frac{|e|}{m} k_\theta k_z |\phi_k|^2 \frac{\partial}{\omega - k_z v_z} \partial_{v_z}\langle f\rangle$

③, ④ → Turbulent acceleration
(aka' Li Wang, P.D.)

③ $= -\int dv_z \; D_{v_z,r} \, \partial_r\langle f\rangle$

$= -\int dv_z \sum_k \frac{c}{B}\frac{|e|}{m} \underbrace{k_\theta k_z}_{\text{symmetry}} |\phi_k|^2 \frac{\partial}{\omega - k_z v_z} \partial_r\langle f\rangle$

and $\underline{\underline{\text{not}}} \sim \nabla\cdot\Pi$, $\partial_r\langle f\rangle \to$ driver flow.

④ → another piece turbulent source

→ Correlation Times

→ Useful here to explore correlation times, for different cases, and compare to 1D

PKU Lecture III

→ "QL" Theory for Simple Drift Wave Turbulence, cont'd

- correlation times, Kubo # ⟺ relation to structure.
- relaxation – transport ⟺ heating constraint
  - avalanche &c

→ PV Conservation, PV Dynamics

— —

→ Taylor Identity, and Charney-Drazin (Momentum) Theorem.

→ Relation Modulational Theory & ZF Formation

Recall:

$$\frac{\partial f}{\partial t} + v_z \frac{\partial f}{\partial z} - \frac{c}{B_0} \nabla\phi \times \hat{z} \cdot \underline{\nabla} f - \frac{|e|}{m_e} E_z \frac{\partial f}{\partial v_z} = 0$$

$\frac{|e|}{m_i} \rightarrow$ ions

$$\delta f_k^c = \frac{\phi_k}{\omega - k_z v_z} \underline{L_k} \langle f \rangle$$

$$\underline{L_k} = -\frac{c}{B_0} k_\theta \frac{\partial}{\partial r} + \frac{|e|}{m_e} k_z \frac{\partial}{\partial v_z}$$

scattering operator.

so

$$\partial_t \langle f \rangle = \partial_r D_{r,r} \partial_r \langle f \rangle + \partial_r D_{r,v} \partial_v \langle f \rangle + \partial_v D_{v,r} \partial_r \langle f \rangle + \partial_v D_{v,v} \partial_v \langle f \rangle$$

1, 2 $\rightarrow$ Momentum flux, including Π_{resid}, V

3, 4 $\rightarrow$ turbulent acceleration.

N.B. show $\Gamma_e = \Gamma_i$, in QLT.

Now, seek examine τ_{ac} for Drift Waves,

Spectral structure ⟺ Geometry ⟺ Kubo # Connection

$$D_{\perp,\perp} = \sum_{\underline{n}} \frac{c^2}{B_0^2} |\phi_{\underline{n}}|^2 k_\theta^2 \frac{i}{\omega - k_z v_z}$$

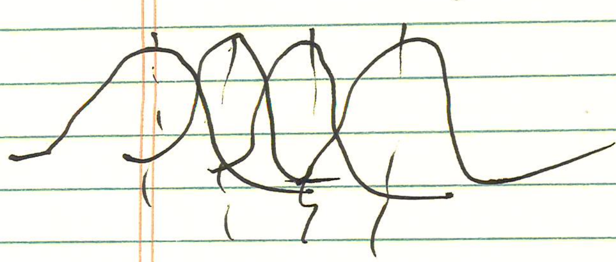
assume trial spectrum structure:

$$|\phi_{\underline{n}}|^2 = |\phi_0|^2 \left[\frac{\Delta k_\theta}{(k_\theta - \bar{k}_\theta)^2 + (\Delta k_\theta)^2} \right] \left[\frac{\Delta k_z}{k_z^2 + (\Delta k_z)^2} \right]$$

centroid k_θ — spread — spread k_z

→ ad-hoc but useful to extract time scales.

For sheared system:



$q' > 0$

$$m = nq$$

$$dn = \frac{m}{q^2} q' \, dx$$

$$= \frac{k_\theta r q'}{q^2} dx$$

$$|\phi_k|^2 = |\phi_0|^2 \, S'(k_\theta) \, F\left(\frac{r - r_n}{\Delta_s}\right)$$

↳ spectral width, spec.

$$\sum_m \sum_n \rightarrow \int dk_\theta \int \frac{k_\theta r}{q^2} \, dx$$

if overlap

sum over modes sweeps spectral width

⇒ Δk_z corresponds to spectral width integration

→ can extend to ballooning space. extent mode along line ⟺ coupling.

Now,

$$D_{\perp,\perp} \approx \frac{i}{\omega_k - k_z v_z + i/\tau_{ac}}$$

$$1/\tau_{ac} = \left| \Delta k_\theta \frac{d\omega}{dk_\theta} - v_z \Delta k_z \right|$$

→ have assumed $d\omega / dk_{\parallel} \to 0$

Recall 1D:

$$1/\tau_{ac} = \left| \frac{d\omega}{dk} - v \right| \Delta k$$

resonant

$$= \left| \frac{d\omega}{dk} - \frac{\omega}{k} \right| \Delta k$$

i.e. $\boxed{\text{wave - particle auto-correlation time depends on particle speed.}}$

Can see:

- electrons / ions near Maxwell $\left\{ \begin{array}{l} \text{resonant} \end{array} \right.$ — ITG
  $\omega / k_{\parallel} \to \omega / k_{\parallel} \leq v_{thi}$

$$1/\tau_{ac} = \left| \Delta k_{\theta} \frac{d\omega}{dk_{\theta}} - \frac{\omega}{k_{\parallel}} |\Delta k_{\parallel}| \right|$$

$$\simeq \left| \Delta k_{\theta} \frac{d\omega}{dk_{\theta}} - v_{thi} |\Delta k_{\parallel}| \right|$$

$1/\tau_{c0} \sim |v_{The}\,\Delta k_{z0}|$ → edw.

$1/\tau_{c0} \sim |v_{Thi}\,\Delta k_{z}|$ → ITG, near marginal (resonant case).

$1/\tau_{c0} \sim |\Delta k_0\, d\omega/dk_0|$ → ITG above marginal.

N.B. Ion diffusion on EDW is non-resonant.

Also:

→ Geometry / spectral structure reduces sensitivity (to dispersion), of τ_{c0}.

→ $Ku \sim \frac{\tau_{c0}\tilde{V}}{\Delta} < 1$ ⇒ validates ~~[illegible]~~ stochastic transport modelling.

if: $\tau_{ac}\,\omega_* < 1$

→ $Ku < 1$ → EDW
→ ~ marg. ITG

$Ku \sim 1$ for fluid ITG.

Precession Resonance: An Interesting Twist

$$D_{\perp,\perp} = \frac{c^2}{B_0^2}\sum_n \frac{n^2}{r^2} q^2 \left|\bar{\phi}_n\right|^2 \frac{i}{\omega - \bar{\omega}_D \epsilon}$$

bounce average.

as before;

$$\frac{i}{\omega - \bar{\omega}_D \epsilon} \Rightarrow \frac{1}{\left|\frac{d\omega}{dk_\theta}\Delta k_\theta - v_D \epsilon \Delta k_\theta\right|}$$

$\epsilon = \omega/\bar{\omega}_D$ for resonant particles

$$\sim \frac{1}{|\Delta k_\theta| \left[\left|\frac{d\omega}{dk_\theta} - \frac{\omega}{k_\theta}\right|\right]}$$

$$\frac{1}{\tau_{ac}} \equiv |\Delta k_\theta| \left[\left|\frac{d\omega}{dk_\theta} - \frac{\omega}{k_\theta}\right|\right]$$

→ valid 1D → dispersion sensitive

→ precession resonance modes (CTEM, EAM) test n.h

Other cases

→ precession:

$$D = \frac{c^2}{B_0^2} \sum_n \frac{n^2 q^2}{r^2} \overline{|\phi_n|^2} \frac{i}{\omega - \omega_D}$$

as before

$$\frac{i}{\omega - \omega_D} \sim \frac{\cancel{i}}{\cancel{i} \left| \frac{d\omega}{dk_\theta} \Delta k_\theta - \omega_D \epsilon \Delta k_\theta \right|}$$

$\epsilon = \omega / \omega_D$

$\epsilon \sim \frac{\omega}{\omega_D}$

$$\sim \frac{1}{\left| \frac{d\omega}{dk_\theta} \Delta k_\theta - \cancel{\omega_D} \Delta k_\theta \frac{\omega}{k_\theta \cancel{\omega_D}} \right|}$$

$$\boxed{\tau_{ac} \sim \frac{1}{|\Delta k_\theta| \left| \left[\frac{d\omega}{dk_\theta} - \frac{\omega}{k_\theta} \right] \right|}}$$

→ 'ala' 1D → dispersion sensitive.

→ CTEM tend to run $k_\theta \uparrow$, as

$\tau_{ac} \uparrow$.

N.B.: Precession resonance driven turbulence has long wave-particle correlation times.

large Ku.

⇒ ? How treat ?

— granulations ⇒ strongly correlated clusters ?
y.k. – Ph.D., et seq.,

— but still statistical ??

⇒ Useful to consider single, few coherent CTEM.

BGK solution ?

⇒ HW: Effect $E \times B$ shear on
γ_{ec} – EDW
– Precession

$\boxed{\text{What is QL saturation for simple drift wave?}}$

$$\frac{|e|}{m} \frac{k_z}{} \to \frac{|e|}{M_0} \frac{\omega}{v_z}$$

→ **Relaxation**

from resonance

$$L_k \langle f \rangle = \frac{k_\theta}{\Omega_0} \frac{\partial}{\partial r} \langle f \rangle + \frac{\omega_k}{v_z} \frac{\partial}{\partial v_z} \langle f \rangle$$

and

$$\partial_t \langle f \rangle = \sum_{\underline{k}} \left[L_{\underline{k}} \, |\phi_k|^2 \pi \delta(\omega - k_z v_z) \, L_{\underline{k}} \langle f \rangle \right]$$

relaxn: what is QL stationary state?

$$L_k \to 0$$

$$-\frac{k_\theta}{\Omega_0} \frac{\partial}{\partial r} \langle f \rangle + \frac{\omega}{v_z} \frac{\partial}{\partial v_z} \langle f \rangle \to 0$$

$$\left(-\frac{k_\theta}{\Omega_0} \frac{1}{\Delta r} + \frac{\omega / v_z}{\Delta v_z} \right) \langle f \rangle \to 0$$

$$\boxed{\Delta r - \Delta \frac{v_z^2}{2} \frac{k_\theta}{\Omega_0 \omega_k} = 0}$$

$$\Delta \left(r - \frac{v_z^2}{2} \frac{k_\theta}{\Omega_0 \omega_k} \right) = 0$$

constraint for relaxed state

Formally:

$$\partial_t \langle f \rangle = \left[\sum_k L_k |\phi_k|^2 \pi \delta(\omega - k_z v_z) L_k \right] \langle f \rangle$$

$$\partial_t \int dr \int dv_z \frac{\langle f \rangle^2}{2} = -\int dr\, dv_z \sum_k |\phi_k|^2 \pi \delta(\omega - k_z v_z) * \left(L_k \langle f \rangle \right)^2$$

$L_k \langle f \rangle = 0 \Rightarrow$ QW "plateau"

N.B. - Note this is not simply flattening of density

- $L_k \rightarrow 0$ before $\frac{\partial \langle f \rangle}{\partial r} \rightarrow 0$, as heating Δv_z^2 occurs.

$$\boxed{\begin{aligned} \Delta r &= \frac{\Delta V_z^2}{2}\frac{k_\theta}{\Omega_e \omega_{\parallel}} \\ r - \frac{V_z^2}{2}\frac{k_\theta}{\Omega_e \omega_{\parallel}} &= \text{const.} \end{aligned}}$$

⇒ i.e. { any $\Delta r > 0$ displacement ⇒ heating ($\Delta V_z^2 > 0$) }

how QL saturate without $Dn \to 0$?!

→ DW expends energy on heating via Landau damping ⇒ route for non-trivial QL state.

$\boxed{\omega \sim \omega - \omega_{*e}}$

⇒ $\Delta V_z^2 \sim \Delta r \dfrac{\cancel{\Omega_e}\,\omega_{\parallel}}{\cancel{k_\theta}} \dfrac{\cancel{k_\theta}\,\rho_e V_{Te}^2}{L_n \cancel{\Omega_e}}$

$$\boxed{\frac{\Delta V_z^2}{V_{The}^2} \sim \frac{\Delta r}{L_n} \frac{\cancel{V_{Te}^2}}{}}$$

→ [heating – transport relation

Lesson

– DW's do distort distribution function

– relevant to ITG near marginal.

– heating often overlooked!

Comment:

→ How include source / sink ⇒ headed toward

Flux driven

$$\partial_t \langle f \rangle = L\, D\, L \langle f \rangle + S_0(r, t)$$

dynamics couples $\left\{ \begin{array}{l} \text{transport} \\ \text{heating} \end{array} \right.$

→ What of avalanching?

- avalanche in <u>phase space</u>
- must evolve in r (as usual) and v_z i.e. heating penalty for step Δr.
- "bi-variate Burgers" with constraint on radial, $v_\parallel$ scattering?

and <u>Energetics</u>:

- Now DKE / QL:

$$\partial_t \langle E_{kin} \rangle + \underbrace{\frac{\partial}{\partial r} Q_e}_{\text{radial transport}} - \langle E_z J_z \rangle = 0$$

and P-Thm:

$$\partial_t W_{DW} + \underbrace{\partial_r S_r}_{\text{radial wave energy density flux}} + \langle E_z J_z \rangle_R = 0$$

$$\Rightarrow \langle E_z J_z \rangle_R = \partial_t \langle E_{kin} \rangle_R + \partial_r Q_{e\,R}$$

$\therefore$

$$\boxed{\underbrace{\partial_t \langle E_{kin} \rangle_R + \partial_t W_{DW}}_{\text{RP + wave as before}} + \partial_r(\underbrace{Q_{e,R}}_{\text{transport}} + \underbrace{S_r}_{\text{wave transport}}) = 0}$$

"coupling"

n.b. $S_r \longleftrightarrow Q_{e,R}$

$S_r \sim V_{gr} \mathcal{E}_{DW}$. $\rightarrow$ <u>ZF generation</u>

— Dominant Balance:

$$\partial_t \langle E_{kin} \rangle_R + \partial_r Q_{e_R} = 0$$

N.B. show: $\Gamma_e = \Gamma_i$
res NR.

— at edge;

$W_{DW} \rightarrow \mathcal{E}_{kin}$ ports

$S_r \rightarrow Q_{e,R}$.

i.e. wave radiative losses significant.

$\Rightarrow ZF \downarrow$

c.s. $\omega = \dfrac{k_\theta V_*}{1+k_\perp^2\rho_s^2}$

$V_{gr}\mathcal{E} \rightarrow$ [wave energy density flux]

$$V_{gr} = -\frac{2\boxed{k_r k_\theta}\rho_s^2 V_*}{(1+k_\perp^2\rho_s^2)^2}$$

Reynolds stress

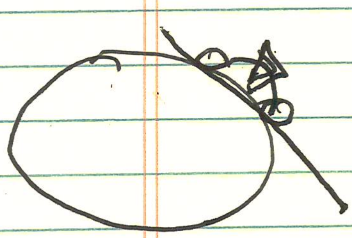
→ PV Mixing + G-D Thms — QLT for Fluid DW

Closely related to Vlasov Dynamics is β-Plane / QG

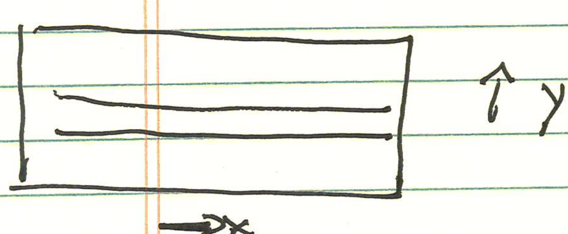
$$\frac{\partial}{\partial t}(\nabla^2\phi + \beta y) + \langle v\rangle\cdot\nabla(\nabla^2\phi + \beta y)$$
$$+ \tilde{v}\cdot\nabla(\nabla^2\phi + \beta y) - \nu\nabla^2(\nabla^2\phi + \beta y) = 0$$

$$\partial_t \nabla^2\phi + \langle V\rangle_x \partial_x \nabla^2\phi + \underline{\tilde{v}}\cdot\nabla\nabla^2\phi$$
$$-\nu\nabla^2\nabla^2\phi = -\tilde{v}_y\left(\beta + \langle\nabla^2\phi\rangle\right)$$

Mean vorticity



$$\xi = \langle\xi\rangle + \tilde{\xi}$$



obeys:

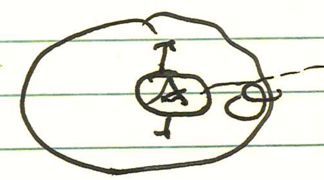
$$\partial_t \xi + \underline{v}\cdot\nabla\xi = +\nu\nabla^2\xi$$

$\int \underline{v}\cdot d\underline{l} = \int \underline{\omega}\cdot d\underline{a} = \text{const.}$

Kelvin's Thm.

$\omega \to \underline{\omega} + 2\underline{\Omega}$

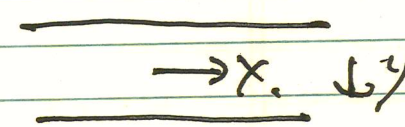
$C = \int d\underline{a}\cdot(\underline{\omega} + 2\underline{\Omega})$



$\dot{C} = 0 \Rightarrow$

$$A\frac{d\omega}{dt} = -2\Omega\sin\theta_0\frac{dA}{dt}$$

$$\frac{d\omega}{dt} = -\frac{2\Omega}{A}\sin\theta_0\frac{dA}{dt}$$



$$= -2\Omega\frac{d\theta}{dt}\sin\theta_0$$

$$= -\frac{2\Omega}{R}\left(R\frac{d\theta}{dt}\right)\sin\theta_0$$

$$= -\beta v_y$$

$\omega = \nabla^2\phi$ $\qquad$ $\underline{v} = -\frac{\nabla p}{2\Omega}\times\hat{z}$