PHYSICS 230 : ADVANCED SOLID STATE PHYSICS
HW ASSIGNMENT #1

(1) Consider an electron in the presence of a magnetic field $\boldsymbol{B} = -B\hat{\boldsymbol{z}}$ and electric field $\boldsymbol{E} = E_x\,\hat{\boldsymbol{x}} + E_y\,\hat{\boldsymbol{y}}$.

(a) Using the cyclotron and guiding center ladder operators, write the Hamiltonian. What can you say about the spectrum?

(b) Work in the Landau gauge $\boldsymbol{A} = -Bx\hat{\boldsymbol{y}}$ and an infinite cylindrical geometry, where periodic boundary conditions are enforced on the y coordinate over the interval $[0, L_y]$. Find the spectrum and its density of states.

(2) Compute the holonomy (*i.e.* the geometric phase) for a LLL coherent state wavefunction $|\,\boldsymbol{R}\,\rangle$ executing some prescribed path $\mathcal{C}$ in the plane.

(3) For a 'square torus' threaded by $N_\phi = 4$ Dirac flux quanta, find the 4×4 LLL Hamiltonian in the toroidal strip basis (§1.5.2 of the Lecture Notes) for the potential $V(x, y) = V_0 \sin\big(2\pi(x + y)/L\big)$.

(4) Find a gauge for Hofstadter model on a triangular lattice with $\phi = \frac{1}{2}\pi$ per unit cell. Note that a unit cell consists of one up and one down pointing triangle. Generalize your result to the case where the flux in each up-triangle is α and in each down triangle is $\frac{1}{2}\pi - \alpha$. Make sure to include the boundary phases in your specification of A_{ij}.