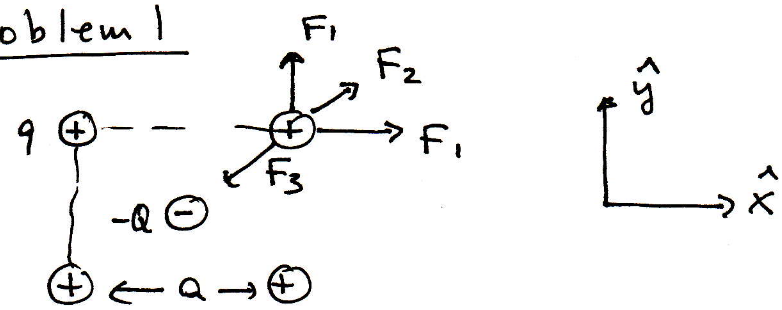
Physics 4C — Quiz 1 Solutions — Fall 2019

## Problem 1




(a) There are forces $F_1$, $F_2$, $F_3$ on positive charge.

(i) $F_1$ from the positive charges at distance $a$

$$F_1 = k\frac{q^2}{a^2}$$ point along $\hat{x}$ and along $\hat{y}$ directions

(ii) $F_2$ from positive charge at opposite corner, distance $\sqrt{2}\,a$

$$F_2 = \frac{kq^2}{(\sqrt{2}a)^2} = \frac{kq^2}{2a^2}$$

(iii) $F_3$ from negative charge, at distance $\frac{\sqrt{2}}{2}a = \frac{1}{\sqrt{2}}a$

$$F_3 = \frac{kqQ}{a^2}\cdot 2$$

The net force is along the diagonal direction:

$$F_{net} = 2F_1\cos 45^\circ + F_2 - F_3 =$$

$$= 2\frac{kq^2}{a^2}\frac{\sqrt{2}}{2} + \frac{kq^2}{2a^2} - \frac{kqQ\cdot 2}{a^2} = \frac{kq^2}{a^2}\left[\sqrt{2} + \frac{1}{2} - 2\frac{Q}{q}\right]$$

Setting $F_{net} = 0 \Rightarrow$ $\boxed{\frac{Q}{q} = \frac{1+2\sqrt{2}}{4} = 0.96}$

(b) The charge $-Q$ interacts with 4 positive charges.

The interaction energy is:

$$U = -\frac{kqQ}{\frac{1}{\sqrt{2}}a}\cdot 4 = -\frac{kq^2}{a}\cdot 4\cdot\sqrt{2}\cdot\frac{Q}{q} = -\frac{kq^2}{a}\sqrt{2}\,(1+2\sqrt{2}))$$

So to take the charge $-Q$ to infinity we need to do <u>positive work</u>

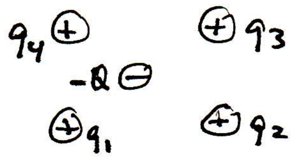
$$\boxed{W = -U = \frac{kq^2}{a}\left(4+\sqrt{2}\right) = 5.41\,\frac{kq^2}{a}}$$

(c) <u>Short way</u>: the force on all the charges is zero initially. We can simply stretch the square pulling the positive charges along the diagonals and the forces remain zero, so we do zero work to take them to infinity.

Therefore, $\boxed{U=0}$

<u>Long way</u>:

$$U = \sum_{i<j} \frac{k q_i q_j}{r_{ij}}$$




$$U = k\left(\frac{q_1q_2+q_2q_3+q_3q_4+q_4q_1}{a} + \frac{q_1q_3+q_2q_4}{\sqrt{2}\,a} - 4\,\frac{qQ}{a/\sqrt{2}}\right) = \frac{kq^2}{a}\left[4+\sqrt{2}-4\sqrt{2}\,\frac{Q}{q}\right] =$$

$$= \frac{kq^2}{a}\left[4+\sqrt{2}-\cancel{4}\sqrt{2}\,\frac{(1+2\sqrt{2})}{\cancel{4}}\right] = \frac{kq^2}{a}\left[4+\sqrt{2}-\sqrt{2}-4\right] \boxed{= 0}$$

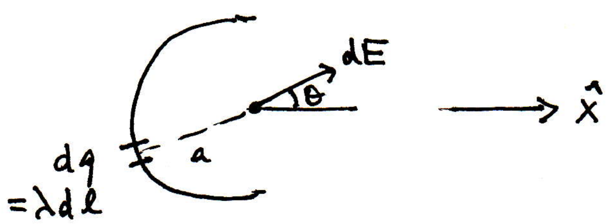
Problem 2

(b) is easy. For $d \gg a$, the semicircle looks like a point charge $q = \pi a \lambda \Rightarrow$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{d^2} = \frac{1}{4\pi\varepsilon_0}\frac{\pi a \lambda}{d^2} = \boxed{\frac{\lambda}{4\varepsilon_0}\frac{a}{d^2}}$$

(we can use $d$ or $d+a$ in denominator, answer is approximately same).

(a) We need to do an integral:




The net field will be along the horizontal $\hat{x}$ direction. So:

$$dE = \frac{dq}{4\pi\varepsilon_0 a^2} = \frac{\lambda\, dl}{4\pi\varepsilon_0 a^2}$$

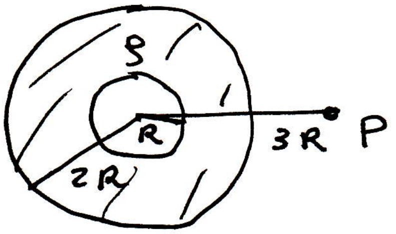
$$dE_x = dE\cos\theta = \frac{\lambda\, dl}{4\pi\varepsilon_0 a^2}\cos\theta$$

Now $dl = a\,d\theta \Rightarrow$ $\quad dE_x = \frac{\lambda}{4\pi\varepsilon_0 a}\cos\theta\, d\theta \Rightarrow$

$$\Rightarrow E_x = \int_{-\frac{\pi}{2}}^{\pi/2} dE_x = \frac{\lambda}{4\pi\varepsilon_0 a}\sin\theta\Big]_{-\frac{\pi}{2}}^{\pi/2} = \frac{\lambda}{2\pi\varepsilon_0 a}$$

$$\Rightarrow \boxed{E_x = \frac{\lambda}{2\pi\varepsilon_0 a}}$$

## Problem 3




(a) Field outside: $E = \dfrac{q}{4\pi\varepsilon_0 r^2}$ ; given $E(3R)$, find $E(2R)$

$$E(2R) = E(3R)\left(\frac{3R}{2R}\right)^2 = \frac{9}{4}E(3R) \Rightarrow \boxed{E(2R) = 45\ \frac{N}{C}}$$

(b) Use Gauss: $\oint \vec{E}\cdot d\vec{a} = \dfrac{q_{enc}}{\varepsilon_0}$

$$\Rightarrow E(r)\,4\pi r^2 = \frac{\rho}{\varepsilon_0}\frac{4}{3}\pi\left(r^3 - R^3\right) \Rightarrow \boxed{E(r) = \frac{\rho}{3\varepsilon_0}\,\frac{r^3 - R^3}{r^2}}$$

$$\Rightarrow E\left(\tfrac{3}{2}R\right) = \frac{\rho}{3\varepsilon_0}R\,\frac{\frac{27}{8}-1}{\frac{9}{4}} \Rightarrow \boxed{E\left(\tfrac{3}{2}R\right) = \frac{19}{54}\frac{\rho}{\varepsilon_0}R}$$

(c)


$dW = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q\,dq}{r}$ = work to bring $dq$ from $\infty$ to $r$

$dq = 4\pi r^2\,dr\,\rho$ ; $q = \frac{4}{3}\pi(r^3 - R^3)\rho$

$$\Rightarrow dW = \frac{\rho^2}{4\pi\varepsilon_0}\frac{4\pi}{3}(r^3 - R^3)\frac{4\pi r^2 dr}{r} = \frac{4\pi}{3\varepsilon_0}\rho^2(r^4 - R^3 r)$$

$$\Rightarrow W = U = \int_R^{2R} dW = \frac{4\pi}{3\varepsilon_0}\rho^2\left(\frac{r^5}{5} - \frac{R^3 r^2}{2}\right]_R^{2R} = \frac{94\pi}{15\varepsilon_0}\rho^2 R^5$$

$$\Rightarrow \boxed{U = \frac{94\pi}{15\varepsilon_0}\rho^2 R^5}$$