Phys 4C Fall 2019

Chapter 8 Solutions to Exercises

8.16. **Voltages and energies**

At $t = 0$ the voltage across the capacitor is $V_0 \cos(0) = V_0$. So the voltage across the inductor must be $-V_0$, because the net voltage change around the loop is zero. The charge on the (top plate of the) capacitor is $CV = CV_0 \cos \omega t$. The clockwise current is then $I(t) = -dQ/dt = \omega CV_0 \sin \omega t$. This is zero at $t = 0$, so none of the energy is stored in the $LI^2/2$ in the inductor. All of the energy is stored in the $CV^2/2$ in the capacitor. This energy equals $CV_0^2/2$.

When $\omega t = \pi/2$ the voltage across the capacitor is $V_0 \cos(\pi/2) = 0$. So the voltage across the inductor must also be zero. The current at $\omega t = \pi/2$ is $I = \omega CV_0 \sin(\pi/2) = \omega CV_0$. Since the voltage across the capacitor is zero, none of the energy is stored in the $CV^2/2$ in the capacitor. All of the energy (which we know equals $CV_0^2/2$) is stored in the $LI^2/2$ in the inductor. As a double check, this energy is $L(\omega CV_0)^2/2 = \omega^2 LC^2 V_0^2/2$. Using $\omega = 1/\sqrt{LC}$, this becomes $CV_0^2/2$, as expected. The results are summarized in this table:

	ΔV_C	ΔV_L	U_C	U_L
$t = 0$	V_0	$-V_0$	$CV_0^2/2$	0
$t = \pi/2\omega$	0	0	0	$CV_0^2/2$

Note: At $t = 0$ you can also work out the voltage across the inductor directly, to double check that it equals $-V_0$. Using above form of $I(t)$, the voltage across the inductor is $-L(dI/dt) = -\omega^2 LCV_0 \cos \omega t$. With $\omega = 1/\sqrt{LC}$, this becomes $-V_0 \cos \omega t$, which equals $-V_0$ at $t = 0$. However, it's risky to trust this minus sign. The magnitude is certainly correct, but it's best to check the sign by thinking about things physically. At $t = 0$ the current is zero but is increasing in the clockwise direction. The voltage above the inductor must therefore be higher than the voltage below; this difference is what causes the current to increase.

8.17. **Amplitude after Q cycles**

After Q cycles, the angle ωt equals $2\pi Q$. But from Eq. (8.13) we know that $Q = \omega/2\alpha$. So after Q cycles, the angle ωt equals $2\pi(\omega/2\alpha) = \pi(\omega/\alpha)$. The time t is therefore given by $t = \pi/\alpha$. The exponential factor $e^{-\alpha t}$ that appears in $I(t)$ and $V(t)$ therefore equals $e^{-\pi}$ as desired.

8.24. **RC circuit with a voltage source**

This exercise is a special case of the general RLC circuit we solved in Section 8.3. The loop equation here is

$$RI(t) + \frac{Q(t)}{C} = \mathcal{E}_0 \cos\omega t. \tag{549}$$

Let us replace $\cos\omega t$ with $e^{i\omega t}$, and then guess an exponential solution of the form $\tilde{I}(t) = \tilde{I}e^{i\omega t}$. If $\tilde{I}e^{i\omega t}$ satisfies the equation with an $e^{i\omega t}$ on the right side, then taking the real part of the entire equation tells us that $\text{Re}(\tilde{I}e^{i\omega t})$ satisfies the equation with a $\cos\omega t$ on the right side.

If $\tilde{I}(t) = \tilde{I}e^{i\omega t}$, then $\tilde{Q}(t)$, which is the integral of $\tilde{I}(t)$, equals $\tilde{I}e^{i\omega t}/i\omega$. (There is no need for a constant of integration because we know that $Q(t)$ oscillates around zero.) So we obtain

$$R\tilde{I}e^{i\omega t} + \frac{\tilde{I}e^{i\omega t}}{i\omega C} = \mathcal{E}_0 e^{i\omega t} \quad \Longrightarrow \quad \tilde{I} = \frac{\mathcal{E}_0}{R + 1/i\omega C}. \tag{550}$$

Getting the i out of the denominator, we can write $\tilde{I}$ in polar form as

$$\begin{aligned} \tilde{I} &= \frac{\mathcal{E}_0(R - 1/i\omega C)}{R^2 + 1/\omega^2 C^2} = \frac{\mathcal{E}_0}{R^2 + 1/\omega^2 C^2} \cdot (R + i/\omega C) \\ &= \frac{\mathcal{E}_0}{R^2 + 1/\omega^2 C^2} \cdot \sqrt{R^2 + 1/\omega^2 C^2}\, e^{i\phi} = \frac{\mathcal{E}_0}{\sqrt{R^2 + 1/\omega^2 C^2}}\, e^{i\phi}, \end{aligned} \tag{551}$$

where $\tan\phi = 1/R\omega C$. The actual current is then

$$I(t) = \text{Re}(\tilde{I}e^{i\omega t}) = \text{Re}\left(\frac{\mathcal{E}_0}{\sqrt{R^2 + 1/\omega^2 C^2}}\, e^{i\phi} e^{i\omega t}\right) = \frac{\mathcal{E}_0}{\sqrt{R^2 + 1/\omega^2 C^2}} \cos(\omega t + \phi). \tag{552}$$

For large ω, the amplitude of the current goes to $\mathcal{E}_0/R$, and the phase ϕ goes to zero. This makes sense, because the capacitor essentially isn't there (that is, it behaves like a short circuit) because the oscillations happen too quickly for any charge to build up on the capacitor. So we simply have a resistor in series with the voltage source.

For small ω, the amplitude of the current goes to zero, and the phase ϕ goes to $\pi/2$. In this case, the charge (which has a maximum value of $C\mathcal{E}_0$ on the capacitor) sloshes back and forth very slowly, so the current is very small. The resistor essentially isn't there (the voltage drop IR across it is very small). So we simply have a capacitor in series with the voltage source. And $\phi = \pi/2$ for such a circuit. (The current is ahead of the voltage, because the current reaches its maximum while charge is building up on the capacitor, and then a quarter cycle later the charge reaches its maximum. We are taking Q to be the charge on the top plate of the capacitor, as we did in Section 8.3.)

8.25. **Light bulb**

The normal current for a 60 watt, 120 volt light bulb is $I = P/V = (60\,\text{W})/(120\,\text{V}) = 0.5\,\text{A}$. The resistance of the filament is then $R = V/I = (120\,\text{V})/(0.5\,\text{A}) = 240\,\Omega$. (This could also be obtained from $P = V^2/R \Longrightarrow R = V^2/P$.) We want to have the same current, 0.5 A, when the bulb is connected in series with an impedance of $i\omega L$, across 240 volts. (We want the same current because the resistor has a fixed resistance, so the power is determined by the current flowing through it; and the power when operating normally is 60 watts.) The magnitude of the total impedance is $|Z| = \sqrt{R^2 + (\omega L)^2}$, so the current will equal 0.5 A if

$$0.5 = \frac{V}{|Z|} = \frac{V}{\sqrt{R^2 + (\omega L)^2}} = \frac{240\,\text{V}}{\sqrt{(240\,\Omega)^2 + (\omega L)^2}}. \tag{553}$$

(Ohm's law works with $|Z|$; see Eq. (8.77).) Solving for ωL gives $\omega L = 240\sqrt{3}\,\Omega = 416\,\Omega$. And since $\omega = 2\pi\nu = 2\pi(60\,\text{s}^{-1}) = 377\,\text{s}^{-1}$, we have $L = (416\,\Omega)/(377\,\text{s}^{-1}) = 1.10$ H.

8.32. **Finding L**

The setup is shown in Fig. 143. We can quickly determine the amplitude of the current (or the rms value, depending on what the voltmeter is calibrated to read; the final value of L won't depend on the choice). The frequency is $\omega = 2\pi(1000\,\text{s}^{-1}) = 6283\,\text{s}^{-1}$, so the $V_0 = I_0|Z|$ statement for the capacitor alone yields

$$15.5\,\text{V} = \frac{I_0}{\omega C} \implies I_0 = (15.5\,\text{V})(6283\,\text{s}^{-1})(10^{-6}\,\text{F}) = 0.0974\,\text{A}. \tag{570}$$

Since the elements are in series, this is the current through all of the components in the circuit. The $V_0 = I_0|Z|$ statement for the whole circuit then tells us that (ignoring the units)

$$I_0 = \frac{V_0}{|Z|} \implies 0.0974 = \frac{10.1}{\sqrt{(35)^2 + (\omega L - 1/\omega C)^2}} \implies \omega L - 1/\omega C = \pm 97.6\,\Omega. \tag{571}$$

Note that there are two roots. We therefore have

$$L = \frac{1}{\omega^2 C} \pm \frac{97.6}{\omega} \implies 0.0253 \pm 0.0155 \implies L = 0.041\,\text{H} \quad \text{or} \quad 0.0098\,\text{H}. \tag{572}$$

So L could be 41 mH or 9.8 mH. The amplitude of the voltage across the inductor alone is $I_0\omega L$, which gives 25.1 V and 6.0 V for the two possibilities. If we then measure the voltage across the inductor and obtain 25.4 V, the second possibility is ruled out, and we have reasonably good agreement with the computed value of 25.1 V.

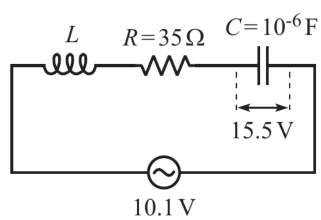

Figure 143

8.35. *RC* **circuit**

(a) The total impedance is

$$Z = R - \frac{i}{\omega C} = 2000\,\Omega - \frac{i}{(377\,\text{s}^{-1})(10^{-6}\,\text{F})} = (2000 - 2650i)\,\Omega. \qquad (578)$$

The magnitude is $|Z| = \sqrt{2000^2 + 2650^2} = 3320\,\Omega$.

(b) The rms voltage is $V = 120\,\text{V}$, so the rms current is

$$I = \frac{V}{|Z|} = \frac{120\,\text{V}}{3320\,\Omega} = 0.036\,\text{A}. \qquad (579)$$

(c) The power dissipated (across just the resistor, of course) is

$$P = I^2 R = (0.036\,\text{A})^2(2000\,\Omega) = 2.6\,\text{W}. \qquad (580)$$

There is no need for a factor of 1/2 since we are using rms values. We can alternatively use the $P = VI\cos\phi$ expression (where these are the rms values). Here $V = 120\,\text{V}$, $I = 0.036\,\text{A}$, and $\tan\phi = 2650/2000$ which yields $\cos\phi = 0.60$. These quantities yield $P = 2.6\,\text{W}$, as desired.

(d) A voltmeter connected across the resistor will read

$$V_R = IR = (0.036\,\text{A})(2000\,\Omega) = 72\,\text{V} \qquad \text{(rms)}. \qquad (581)$$

A voltmeter connected across the capacitor will read

$$V_C = \frac{I}{\omega C} = (0.036\,\text{A})(2650\,\Omega) = 95\,\text{V} \qquad \text{(rms)}. \qquad (582)$$

(e) The amplitudes of the voltages associated with the above rms values are 102 V and 134 V. The voltages across the resistor and capacitor are 90° out of phase, with the resistor ahead of the capacitor. (Remember, in general we have V_L ahead of V_R ahead of V_C.) So the pattern will be an ellipse, as shown in Fig. 146. If the plates are connected in the natural way as shown, then the ellipse is traced out counterclockwise. To see why, consider an instant when the current through the resistor is maximum downward, in which case the right plate of the tube is at a higher potential (so the electrons are deflected that way). The charge on the capacitor is 90° out of phase with the current, so there is no charge on the capacitor at this moment. The voltage across the capacitor is therefore zero, so the electrons are at the point A in the figure.

A quarter cycle later, the top plate of the capacitor will have maximum charge, in which case the top plate of the tube is at a higher potential (so the electrons are deflected that way). And the current is zero at this moment, so the voltage across the resistor is zero. The electrons are therefore at point B in the figure. We see that the curve on the screen passes point B a quarter cycle after point A. So the curve is traced out counterclockwise. On the other hand, if the connections are made in the reverse manner for either of the elements, then the curve would be traced out clockwise. If both connections are reversed, then the trace reverts back to counterclockwise. Without being told which way the connections are made, there is no way to know the direction of the trace.

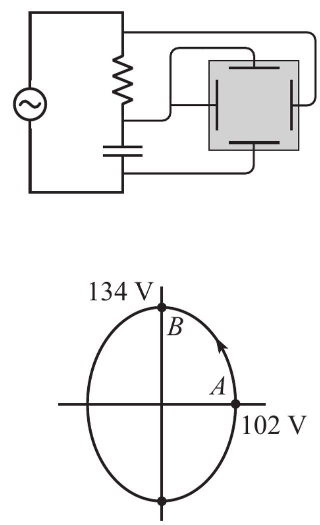

Figure 146