Phys 4C Fall 2019

Chapter 10 and 11 Solutions to Exercises

10.16. **Leyden jar**

Assume that the jar is cylindrical, with the height being twice the diameter d (the result will depend somewhat on the proportions assumed). Then the volume is $\pi(d/2)^2 \cdot (2d)$. Setting this equal to $10^{-3}\,\text{m}^3$ gives $d = 0.086\,\text{m}$. The area of the capacitor (assuming it has no top) is $A = \pi(d/2)^2 + \pi d(2d) = 9\pi d^2/4 = 0.052\,\text{m}^2$. So the capacitance is

$$C = \frac{\kappa\epsilon_0 A}{s} = \frac{(4)\big(8.85 \cdot 10^{-12}\,\frac{\text{s}^2\,\text{C}^2}{\text{kg}\,\text{m}^3}\big)(0.052\,\text{m}^2)}{0.002\,\text{m}} = 9.2 \cdot 10^{-10}\ \text{F}. \qquad (641)$$

If we had chosen the height to instead be four times the diameter, then the capacitance would be about 20% larger. As long as the jar isn't too squat (in which case it would be better called a tray) or too tall (in which case it would be better called a tube), the dependence of the capacitance on the exact dimensions is fairly weak. (If the height is $h = nd$, then you can show that the capacitance is proportional to $(n + 1/4)/n^{2/3}$.)

The capacitance of a sphere is $4\pi\epsilon_0 r$, so a sphere will have a capacitance of $9.2\cdot 10^{-10}$ F if $r = 8.3\,\text{m}$. The diameter is then $16.6\,\text{m}$, or about 54 feet.

10.30. **Polarized hydrogen**

Since volume is proportional to r^3, the negative charge inside radius Δz is $q = -e(\Delta z/a)^3$. Gauss's law therefore gives the field due to the inner part of the electron cloud as

$$\int \mathbf{E}_e \cdot d\mathbf{a} = \frac{q}{\epsilon_0} \quad\Longrightarrow\quad E_e \cdot 4\pi(\Delta z)^2 = \frac{-e(\Delta z)^3}{\epsilon_0 a^3} \quad\Longrightarrow\quad E_e = \frac{-e\Delta z}{4\pi\epsilon_0 a^3}\,. \qquad (671)$$

This field pulls the proton downward. In equilibrium, it must be equal and opposite to the applied field E that pushes the proton upward. Hence Δz is given by

$$\Delta z = \frac{4\pi\epsilon_0 E a^3}{e}\,, \qquad (672)$$

which agrees with Eq. (10.27). The hydrogen atom won't actually remain spherically symmetric, but that won't affect the rough size of Δz.

10.42. **Energy density in a dielectric**

With a dielectric present, the capacitance of a parallel-plate capacitor is $C = \kappa\epsilon_0 A/s \equiv \epsilon A/s$. The energy stored is still $C\phi^2/2$, because it equals $Q\phi/2$ for all the same reasons as in the vacuum case (imagine a battery doing work in transferring charge from one plate to the other). So the energy density is

$$\frac{\text{energy}}{\text{volume}} = \frac{1}{2}C\phi^2\frac{1}{V} = \frac{1}{2}\frac{\epsilon A}{s}(Es)^2\frac{1}{As} = \frac{\epsilon E^2}{2}\,, \tag{690}$$

as desired. Since $\epsilon \equiv \kappa\epsilon_0$, this energy density is κ times the $\epsilon_0 E^2/2$ energy density without the dielectric. Basically, since C is κ times larger, so is the energy, and hence the energy density. To see physically why the energy is larger, consider the case of induced dipole moments, discussed in Section 10.5. The stretched atoms and molecules are effectively little springs that are stretched, so they store potential energy. This makes the total energy larger than it would be for the same equivalent charge on/at the capacitor plates (free charge plus bound-charge layer).

In an electromagnetic wave in a dielectric, the energy density of the magnetic field is still $B^2/2\mu_0$. (It would be $B^2/2\mu$ in a magnetized material, but we're assuming that the material here is only electrically polarizable.) But from Eq. (10.83) the amplitudes of the E and B fields are related by $B_0 = \sqrt{\mu_0\epsilon}\,E_0$. So $B^2/2\mu_0 = \epsilon E^2/2$. The E and B energy densities are therefore equal, just as they are in vacuum.

11.20. **Force between a wire and a loop**

At an arbitrary point on the wire, the magnetic field from the square-loop dipole has both an upward vertical (z) component and a horizontal component along the wire. But the latter produces no force on the current in the wire, so we care only about the z component. Since the current in the wire in Fig. 6.47 points into the page, the right-hand rule gives the magnetic force on the wire as pointing to the right.

Equation (11.14) gives the z component of the dipole field, with m equal to $m = I_2\ell^2$. The rightward force on a little piece dx of the wire equals $I_1 B_z\,dx$. With θ measured away from the vertical axis, dx is given by the usual expression, $dx = z\,d\theta/\cos^2\theta$. (See the reasoning in the paragraph following Eq. (1.37).) Also, the distance r to the little piece is $r = z/\cos\theta$. Integrating over the entire infinite wire, we find the total rightward force on it to be

$$\begin{aligned} F &= \int_{-\infty}^{\infty} I_1 B_z\,dx = \int_{-\infty}^{\infty} I_1\left(\frac{\mu_0 I_2\ell^2}{4\pi}\frac{3\cos^2\theta - 1}{r^3}\right)dx \\ &= \frac{\mu_0 I_1 I_2\ell^2}{4\pi}\int_{-\pi/2}^{\pi/2}\frac{3\cos^2\theta - 1}{(z/\cos\theta)^3}\frac{z\,d\theta}{\cos^2\theta} \\ &= \frac{\mu_0 I_1 I_2\ell^2}{4\pi z^2}\int_{-\pi/2}^{\pi/2}(3\cos^3\theta - \cos\theta)\,d\theta = \frac{\mu_0 I_1 I_2\ell^2}{2\pi z^2}\,. \end{aligned} \tag{707}$$

The integral here equals 2, as you can check with *Mathematica* or the integral table in Appendix K. This force is consistent with the magnitude of the leftward force on the square loop we found in Eq. (458) in Exercise 6.54, because $z \approx R$ for large z.