Physics 4C   Quiz 3 Solutions   Winter 2018

Problem 1

Let V = volume of ice. Your feet start to get wet when the ice is totally submerged. At that point the buoyant force is

$F_b = \rho_{fluid} V g$ and has to equal the weight of the ice + you:

$$\rho_{fluid} V g = \rho_{ice} V g + m_{you} g \Rightarrow$$

$$V = \frac{m_{you}}{\rho_{fluid} - \rho_{ice}} \Rightarrow m_{ice} = \frac{\rho_{ice}}{\rho_{fluid} - \rho_{ice}} m_{you} = \boxed{805\ kg}$$

Problem 2

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2$$

Continuity: $A_1 v_1 = A_2 v_2$, $R_2 = 2R_1 \Rightarrow A_2 = 4A_1 \Rightarrow$

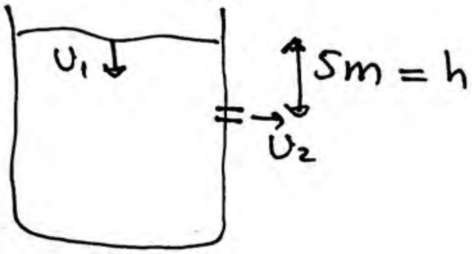
$\Rightarrow \boxed{v_2 = v_1/4}$ pressures are equal, so

$$\frac{1}{2} v_1^2 + 0 = \frac{1}{2} v_2^2 + gh \Rightarrow v_1^2 = \frac{v_1^2}{16} + 2gh \Rightarrow$$

$$\frac{15}{16} v_1^2 = 2gh \Rightarrow h = \frac{15}{32} \frac{v_1^2}{g}$$

$$\Rightarrow h = \frac{15}{32} \times \frac{25\ m^2/s^2}{9.81\ m/s^2} = \boxed{1.19\ m}$$

Problem 3



$$p_0 + \frac{1}{2}\rho v_1^2 + \rho g h = p_0 + \frac{1}{2}\rho v_2^2 + 0$$

assume $v_1 << v_2$, neglect v_1 $\Rightarrow$ $v_2 = \sqrt{2gh}$, $h = 5m$

The volume flow is $R_V = \frac{dV}{dt} = A \cdot v_2$, with $A = 10\,cm^2$

For $\Delta V = 10\,l$ of water to flow out : $v_2 = 9.9\,m/s = 99\,dm/s$

$$\Delta t = \frac{\Delta V}{A v_2} = \frac{10\,dm^3 \cdot s}{\frac{1}{10}\,dm^2 \cdot 99\,dm} = \boxed{1.01s}$$

Problem 4

$x(t) = X_m \cos(\omega t)$; $X_m = 5m$. Speed is

$\dot{x}(t) = -\omega X_m \sin(\omega t)$. maximum speed $v_m = \omega X_m = 5m/s$

$\Rightarrow \boxed{\omega = 1\,s^{-1}}$

it goes fastest between $x = -2.5m$ and $x = +2.5m$

$\Rightarrow \omega t = \frac{\pi}{3}$ ($\cos\frac{\pi}{3} = \frac{1}{2}$) and $\omega t = \frac{2\pi}{3}$ ($\cos\frac{2\pi}{3} = -\frac{1}{2}$) $\Rightarrow$

$$\omega \cdot \Delta t = \frac{2\pi}{3} - \frac{\pi}{3} = \frac{\pi}{3} \Rightarrow \boxed{\Delta t = \frac{\pi}{3\omega} = 1.05s}$$

Problem 5

$$T = 2\pi\sqrt{\frac{I}{mgh}} \qquad I = I_{CM} + mh^2 \; ; \; I_{CM} = \frac{1}{12} mL^2$$

When pivot is at end, $h = L/2 \Rightarrow$

$$I = \frac{1}{12} mL^2 + \frac{1}{4} mL^2 = \frac{1}{3} mL^2.$$

So $\frac{I}{h} = \frac{1}{3} \frac{mL^2}{L/2} = \frac{2}{3} mL$

When pivot is at distance $L/4$ from end, $h' = \frac{L}{4}$

$$I' = \frac{1}{12} mL^2 + \frac{1}{16} mL^2 = \frac{7}{48} mL^2$$

$$\frac{I'}{h'} = \frac{7}{48} \frac{mL^2}{L/4} = \frac{7}{12} mL$$

So $\frac{T_{L/4}}{T_{L/2}} = \sqrt{\frac{7/12}{2/3}} = \sqrt{\frac{7}{8}} \Rightarrow$ $\boxed{T_{L/4} = 0.935\, T_{L/2}}$

Problem 6

$$x(t) = X_m e^{-bt/2m} \cos(\omega' t + \phi)$$

energy $E = \frac{1}{2} X_m^2 e^{-bt/m}$

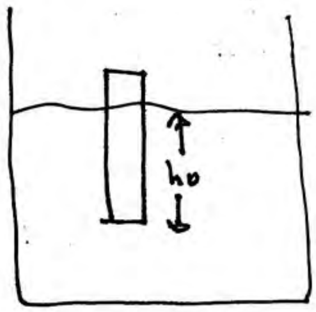
time for $E \to \frac{3}{4} E$: $e^{-bt/m} = \frac{3}{4} \Rightarrow \frac{bt}{m} = \ln\frac{4}{3} \Rightarrow t = \frac{m}{b} \ln\frac{4}{3}$

time for $E \to \frac{1}{2} E$: $t' = \frac{m}{b} \ln 2$

so additional time is $\frac{t' - t}{t} = \frac{\ln 2 - \ln 4/3}{\ln 4/3} = 1.41 \Rightarrow$

if 20 oscillations to lose 1/4, $\boxed{\text{28 oscillations}}$ to lose another 1/4

Problem 7



in equilibrium at height ho:

$\rho_{fluid} \cdot A \cdot h_0 \cdot g = m_{cyl} \cdot g$. If h increases, force on cylinder is

$F = m_{cyl} g - \rho_{fluid} \cdot A \cdot hg = -\rho_{fluid} A (h - h_0) g \equiv -k(h - h_0)$

so it undergoes harmonic motion, with frequency

$$\omega = \sqrt{\frac{k}{m_{cyl}}} = \sqrt{\frac{\rho_{fluid} A g}{m_{cyl}}} \Rightarrow T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m_{cyl}}{\rho_{fluid} A g}}$$

now $m_{cyl} = \rho_{cyl} \cdot A \cdot h_{total} = \rho_{fluid} \cdot A \cdot h_0$ (from first equation)

$$\Rightarrow T = 2\pi \sqrt{\frac{\rho_{fluid} \cdot A \cdot h_0}{\rho_{fluid} \cdot A \cdot g}} = 2\pi \sqrt{\frac{h_0}{g}}$$

$h_{total} = 10\,cm$, 90% submerged $\Rightarrow h_0 = 9\,cm \Rightarrow$

$$T = 2\pi \sqrt{\frac{9\,cm \cdot s^2}{981\,cm}} = \boxed{0.6\,s}$$