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19-9 Adiabatic Expansion of an Ideal Gas

Learning Objectives

19.44 On a *p*-*V* diagram, sketch an adiabatic expansion (or contraction) and identify that there is no heat exchange *Q* with the environment.

19.45 Identify that in an adiabatic expansion, the gas does work on the environment, decreasing the gas's internal energy, and that in an adiabatic contraction, work is done on the gas, increasing the internal energy.

19.46 In an adiabatic expansion or contraction, relate the initial pressure and volume to the final.

19.47 In an adiabatic expansion or contraction, relate the initial temperature and volume to the final temperature and volume.

19.48 Calculate the work done in an adiabatic process by integrating the pressure with respect to volume.

19.49 Identify that a free expansion of a gas into a vacuum is adiabatic but no work is done and thus, by the first law of thermodynamics, the internal energy and temperature of the gas do not change.

19-9 Adiabatic Expansion of an Ideal Gas

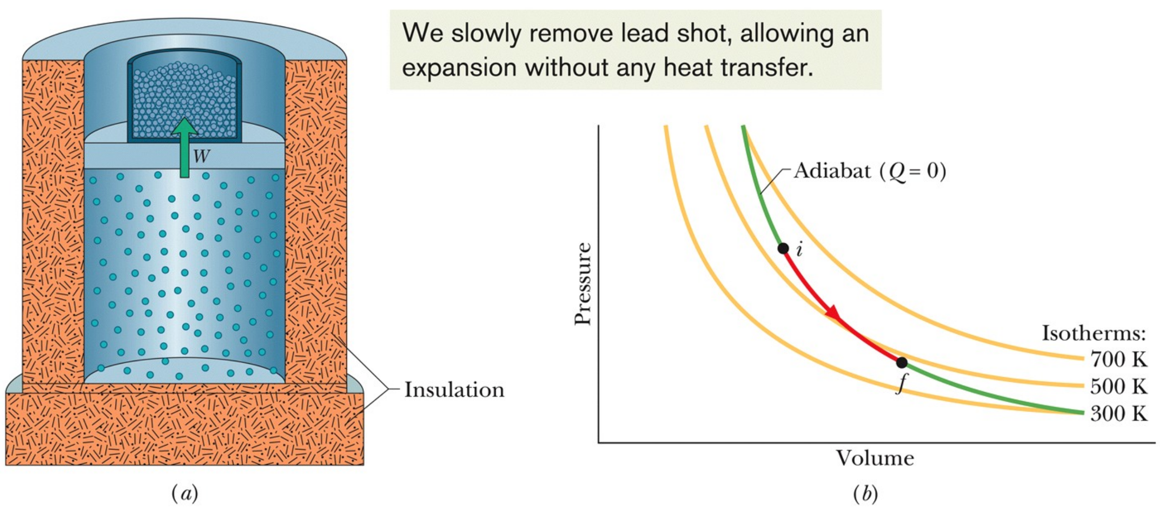
When an ideal gas undergoes a slow adiabatic volume change (a change for which $Q=0$),

$$pV^{\gamma} = \text{a constant} \qquad \text{(adiabatic process),}$$

$PV^{\gamma} = \text{const}$

$TV^{\gamma-1} = \text{const}$

in which $\gamma (= C_p/C_v)$ is the ratio of molar specific heats for the gas.



(a) The volume of an ideal gas is increased by removing mass from the piston. The process is adiabatic ($Q = 0$).

(b) The process proceeds from i to f along an adiabat on a p-V diagram.

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pV^{γ} = a constant (adiabatic process),

Adiabatic expansion

$PV = nRT$

$C_P = C_V + R$

$T = \frac{PV}{nR}$

$\frac{C_P}{C_V} = 1 + \frac{R}{C_V} = \gamma$

$\gamma = \frac{R}{C_V} + 1$

$TV^{R/C_V} = const$;

$\boxed{PV^{\gamma} = const}$

$C_V = \frac{3}{2}R$

mono

$TV^{\gamma-1} = const$

$\gamma = C_P / C_V$

Work? $= -\frac{R}{C_V} \ln V_f + C$

$pV^{\gamma} =$ a constant (adiabatic process),

Work done by gas in adiabatic expansion

$PV = nRT$

$$W = \int P\,dV = \int_{V_i}^{V_f} \frac{const}{V^{\gamma}}\,dV$$

$$\boxed{E_{int} = n C_v T}$$

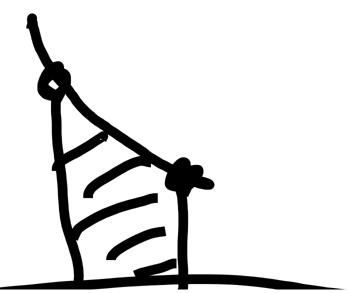
$$= const\left(\frac{V_f^{-\gamma+1} - V_i^{-\gamma+1}}{1-\gamma}\right) = \cancel{const}\,\frac{(P_f V_f - P_i V_i)}{1-\gamma}$$

$$= \frac{nR(T_f - T_i)}{1-\gamma} = \boxed{n C_v (T_i - T_f) = W}$$

$$\Delta E_{int} = \cancel{Q} - W$$

$$E_{int}(T_f) - E_{in}(T_i)$$

$$\boxed{-\Delta E_{int} = W}$$



Free expansion

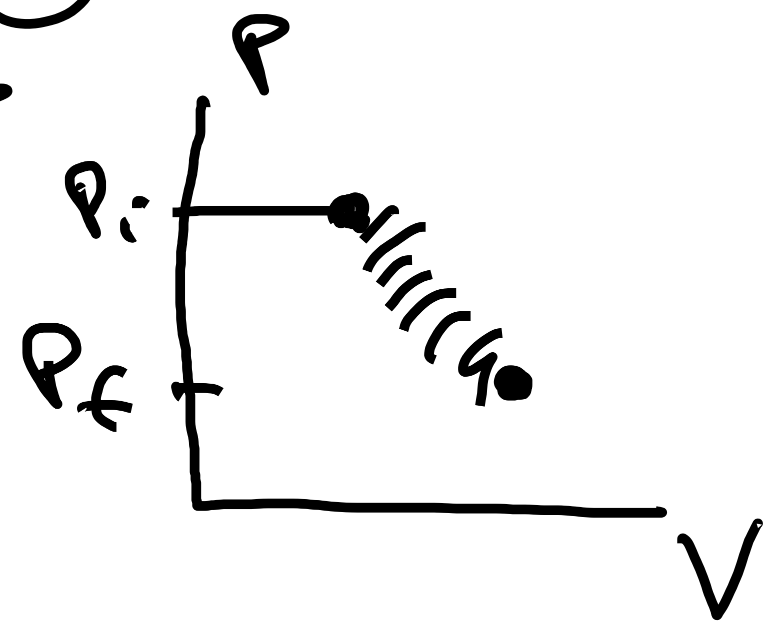
$$Q = 0, \quad W = 0$$

$$\Delta E_{int} = 0$$

for ideal gas:

$$\Delta T = 0$$

$$P_i V_i = P_f V_f$$



Examples of problems

In an isothermal expansion, an ideal gas does 25J work in expanding from volume V to volume 2V

How much work does it do in expanding from 2V to 3V?

How much heat does it absorb in expanding from 2V to 3V?

$$PV = nRT$$

$$W = \int_{V_0}^{2V_0} P\,dV = \int_{V_0}^{2V_0} \frac{nRT}{V}\,dV = nRT\ln(2) = 25J$$

$$W' = \int_{2V_0}^{3V_0} = nRT\ln\left(\frac{3}{2}\right) =$$

$$= nRT\ln(2)\cdot\frac{\ln\left(\frac{3}{2}\right)}{\ln(2)} = 25J\,\frac{\ln(3/2)}{\ln(2)}$$

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Examples of problems

$$P_{cond} = \frac{Q}{t} = kA\frac{T_H - T_L}{L}$$

$$C_V = \frac{5}{2}R$$

$$t = \frac{Q}{kA(T_H - T_L)} \cdot L$$

$$Q = m \cdot C_V \, 5^\circ C$$

m_{air}

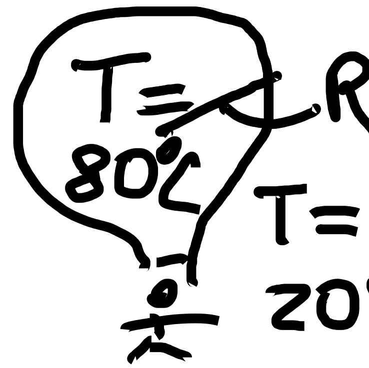
$$\rho = 1.29\ kg/m^3$$

$$PV = nRT$$

$T_H = 80^\circ C$

$T_L = 20^\circ C$

$$V = \frac{4}{3}\pi R^3$$



$T = 20^\circ C$

$L = 2cm$

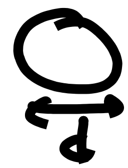
$$k = 0.024 \frac{W}{mK}$$

$R = 2m$

$A = 4\pi R^2$

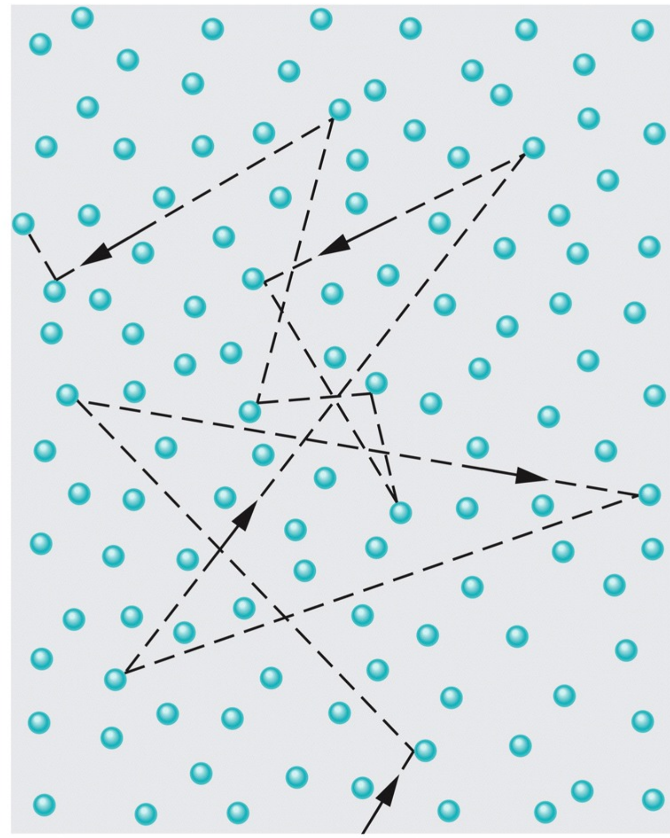
19-5 Mean Free Path

The mean free path λ of a gas molecule is its average path length between collisions and is given by



$$\lambda = \frac{1}{\sqrt{2}\pi d^2 N/V}$$

where N/V is the number of molecules per unit volume and d is the molecular diameter.



In the figure a molecule traveling through a gas, colliding with other gas molecules in its path. Although the other molecules are shown as stationary, they are also moving in a similar fashion.

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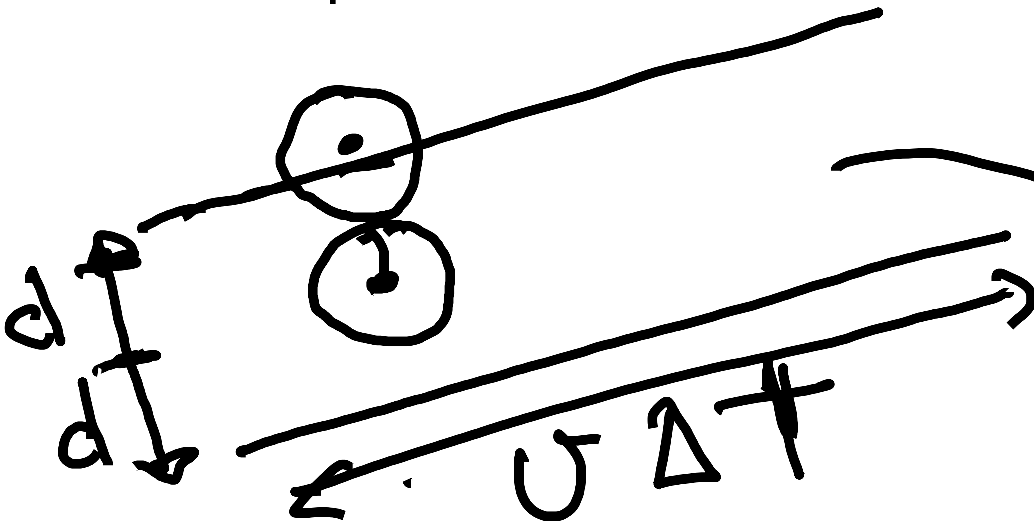
$$\lambda = \frac{1}{\sqrt{2}\pi d^2 N/V}$$

Mean free path

time Δt
between collision



volume $=$
$\pi d^2 \cdot v\Delta t$

$$\frac{N}{V}\pi d^2 v\Delta t = \#\text{ of collisions in }\Delta t$$

$$\lambda = \frac{\text{length of path in }\Delta t}{\#\text{ of coll}} = \frac{\cancel{v\Delta t}}{\frac{N}{V}\pi d^2 \cancel{v\Delta t}}$$

λ for air $\sim 0.1\,\mu m$

$= 10^{-7} m = 1000\,\text{Å}$ at sea level

at 300 km, $\lambda \sim 20\,km$

19-6 The Distribution of Molecular Speed

Learning Objectives

19.24 Explain how Maxwell's speed distribution law is used to find the fraction of molecules with speeds in a certain speed range.

19.25 Sketch a graph of Maxwell's speed distribution, showing the probability distribution versus speed and indicating the relative positions of the average speed v_{avg}, the most probable speed v_P, and the *rms* speed v_{rms}.

19.26 Explain how Maxwell's speed distribution is used to find the average speed, the *rms* speed, and the most probable speed.

19.27 For a given temperature T and molar mass M, calculate the average speed v_{avg}, the most probable speed v_P, and the *rms* speed v_{rms}.

19-6 The Distribution of Molecular Speed

$v_{rms} = \int dv \cdot P(v) \cdot$

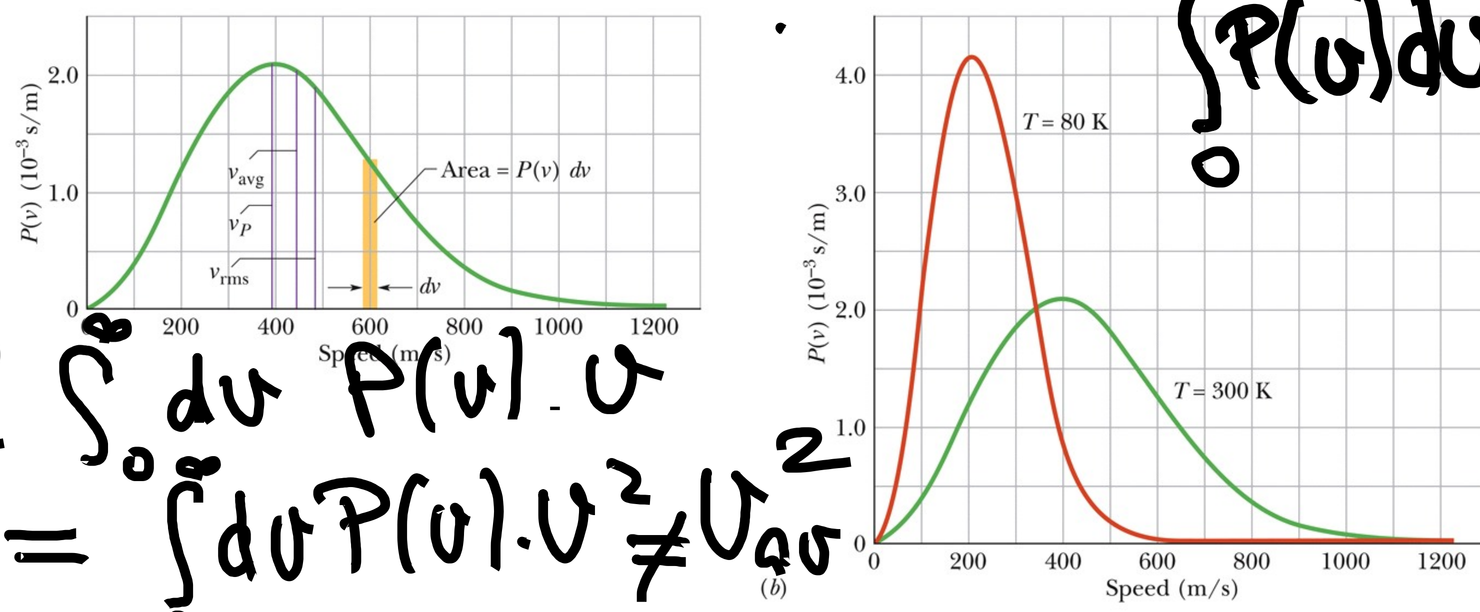
The **Maxwell speed distribution** *P(v)* is a function such that *P(v)dv* gives the fraction of molecules with speeds in the interval *dv* at speed *v*:

$v_{rms}^2 = (v^2)_{av}$

$$P(v) = 4\pi\left(\frac{M}{2\pi RT}\right)^{3/2} v^2 e^{-Mv^2/2RT}.$$

$e^{-mv^2/2kT}$

$\int_0^\infty P(v)\,dv = 1$



$v_{av} = \int_0^\infty dv\, P(v) \cdot v$

$(v^2)_{av} = \int_0^\infty dv\, P(v) \cdot v^2 \neq v_{av}^2$

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19-6 The Distribution of Molecular Speed

Three measures of the distribution of speeds among the molecules of a gas:

$$v_{\text{avg}} = \sqrt{\frac{8RT}{\pi M}} \quad \text{(average speed)},$$

$$v_P = \sqrt{\frac{2RT}{M}} \quad \text{(most probable speed)},$$

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} \quad \text{(rms speed)}.$$

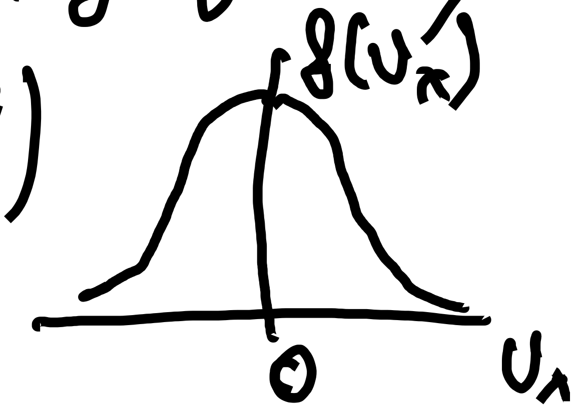
Velocity distribution

$$\vec{v} = (v_x, v_y, v_z) \qquad \langle v^2 \rangle_{av} = \frac{3kT}{m}$$

$$F(v_x, v_y, v_z) = f(v_x)\cdot f(v_y)\cdot f(v_z)$$

$$= g(v^2) = g(v_x^2 + v_y^2 + v_z^2)$$

$$f(v_x) = e^{\lambda v_x^2} = e^{-\frac{m v_x^2}{2kT}}$$



$$f(v_x)\cdot f(v_y)\cdot f(v_z) = e^{\lambda v_x^2}\, e^{\lambda v_y^2}\, e^{\lambda v_z^2} =$$

$$= e^{\lambda(v_x^2 + v_y^2 + v_z^2)}$$

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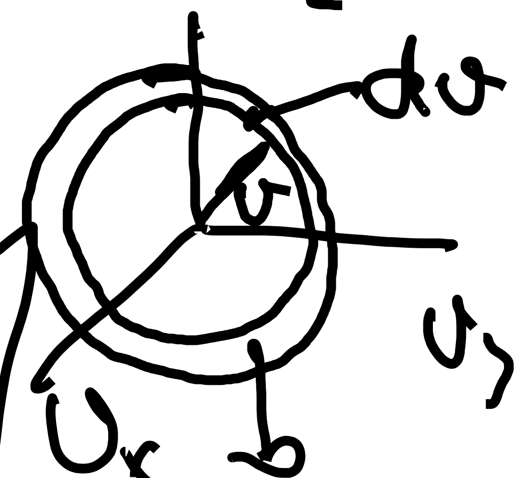
$$f(v_x) = \left(\frac{m}{2\pi kT}\right)^{1/2} e^{-\frac{m}{2kT} v_x^2}$$

$$\int_{-\infty}^{\infty} dv_x\, f(v_x) = 1$$

$$\underline{F(v_x, v_y, v_z)} = \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{m}{2kT} v^2}$$

$$P(v)\,dv = \int F(v_x, v_y, v_z) =$$

$$\boxed{P(v) = 4\pi v^2 \cdot \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{m}{2kT} v^2}}$$



$\underline{4\pi v^2 \cdot d(}$

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19 Summary

Kinetic Theory of Gases

- relates the macroscopic properties of gases to the microscopic properties of gas molecules.

Avogadro's Number

$$N_A = 6.02 \times 10^{23}\ \text{mol}^{-1}$$ **Eq. 19-1**

- Mole related to mass of a molecule

$$M = mN_A$$ **Eq. 19-4**

Ideal Gas

An ideal gas is one for which the pressure *p*, volume *V*, and temperature *T* are related by

$$pV = nRT \quad \text{(ideal gas law).}$$ **Eq. 19-5**

Temperature and Kinetic Energy

- The average translational kinetic energy per molecule of an ideal gas is

$$K_{avg} = \tfrac{3}{2}kT.$$ **Eq. 19-24**

Maxwell Speed Distribution

- The three measures of distribution of speed

$$v_{avg} = \sqrt{\frac{8RT}{\pi M}} \quad \text{(average speed),}$$ **Eq. 19-31**

$$v_P = \sqrt{\frac{2RT}{M}} \quad \text{(most probable speed),}$$ **Eq. 19-35**

$$v_{rms} = \sqrt{\frac{3RT}{M}} \quad \text{(rms speed).}$$ **Eq. 19-22**