35-5 Michelson's Interferometer

An interferometer is a device that can be used to measure lengths or changes in length with great accuracy by means of interference fringes.

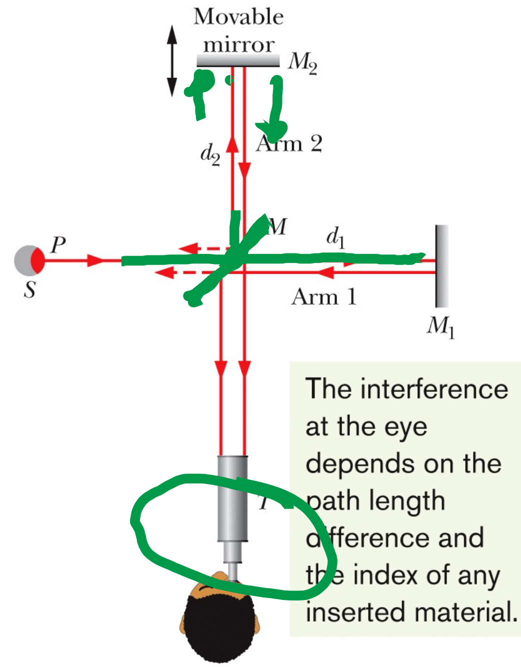
In Michelson's interferometer, a light wave is split into two beams that then recombine after traveling along different paths.

The interference pattern they produce depends on the difference in the lengths of those paths and the indexes of refraction along the paths.

If a transparent material of index n and thickness L is in one path, the phase difference (in terms of wavelength) in the recombining beams is equal to

$$\text{phase difference} = \frac{2L}{\lambda}(n-1),$$

where λ is the wavelength of the light.



Michelson's interferometer, showing the path of light originating at point P of an extended source S. Mirror M splits the light into two beams, which reflect from mirrors M_1 and M_2 back to M and then to telescope T. In the telescope an observer sees a pattern of interference fringes.

35 Summary

Huygen’s Principle

- The three-dimensional transmission of waves, including light, may often be predicted by Huygens’ principle, which states that all points on a wavefront serve as point sources of spherical secondary wavelets.

Wavelength and Index of Refraction

- The wavelength λ_n of light in a medium depends on the index of refraction n of the medium:

$$\lambda_n = \frac{\lambda}{n},$$ **Eq. 35-6**

in whicl e wavelength in vacuum.

Young’s Experiment

- In Young’s interference experiment, light passing through a single slit falls on two slits in a screen. The light leaving these slits flares out (by diffraction), and interference occurs in the region beyond the screen. A fringe pattern, due to the interference, forms on a viewing screen.
- The conditions for maximum and minimum intensity are

$$d \sin \theta = m\lambda, \quad \text{for } m = 0, 1, 2, \ldots \quad \text{(maxima—bright fringes)},$$ **Eq. 35-14**

$$d \sin \theta = (m + \tfrac{1}{2})\lambda, \quad \text{for } m = 0, 1, 2, \ldots \quad \text{(minima—dark fringes)},$$ **Eq. 35-16**

35 Summary

Coherence

- If two light waves that meet at a point are to interfere perceptibly, both must have the same wavelength and the phase difference between them must remain constant with time; that is, the waves must be coherent.

Intensity in Two-Slit Interference

- In Young's interference experiment, two waves, each with intensity I_0, yield a resultant wave of intensity I at the viewing screen, with

$$I = 4I_0 \cos^2 \tfrac{1}{2}\phi, \qquad \text{where } \phi = \frac{2\pi d}{\lambda} \sin\theta.$$

Eqs. 35-22 & 23

Thin-Film Interference

- When light is incident on a thin transparent film, the light waves reflected from the front and back surfaces interfere. For near-normal incidence, the wavelength conditions for maximum and minimum intensity of the light reflected from a *film of index n_2 in air* are

$$2L = (m + \tfrac{1}{2})\frac{\lambda}{n_2}, \qquad \text{for } m = 0, 1, 2, \ldots \quad \text{(maxima—bright film in air)},$$ **Eq. 35-36**

$$2L = m\frac{\lambda}{n_2}, \qquad \text{for } m = 0, 1, 2, \ldots \quad \text{(minima—dark film in air)},$$ **Eq. 35-37**

Michelson's Interferometer

- In Michelson's interferometer a light wave is split into two beams that, after traversing paths of different lengths, are recombined so they interfere and form a fringe pattern.

Chapter 36

Diffraction

WILEY

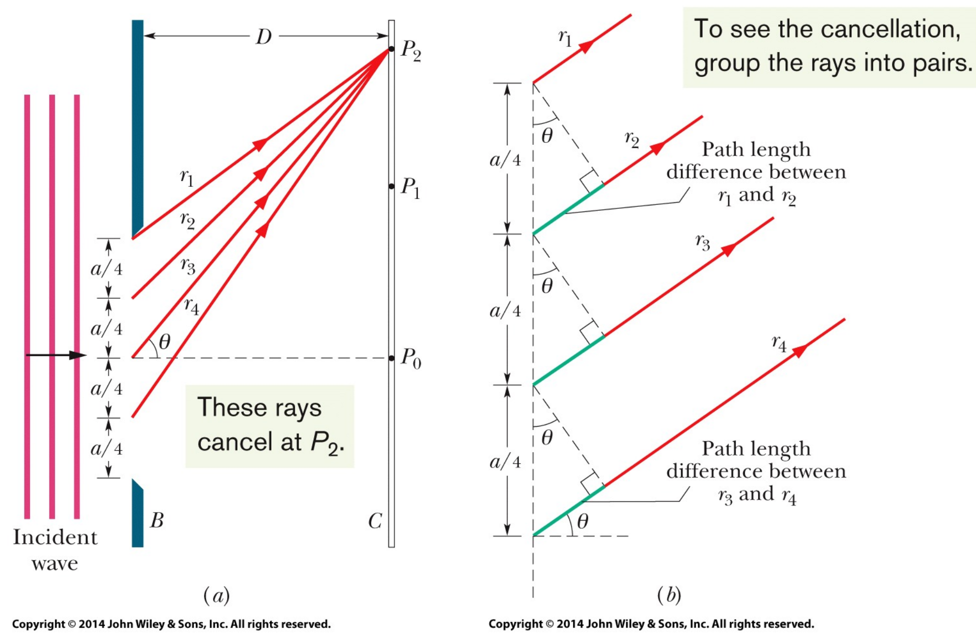
Single-Slit Diffraction

When waves encounter an edge, an obstacle, or an aperture the size of which is comparable to the wavelength of the waves, those waves spread out as they travel and, as a result, undergo interference. This type of interference is called diffraction.

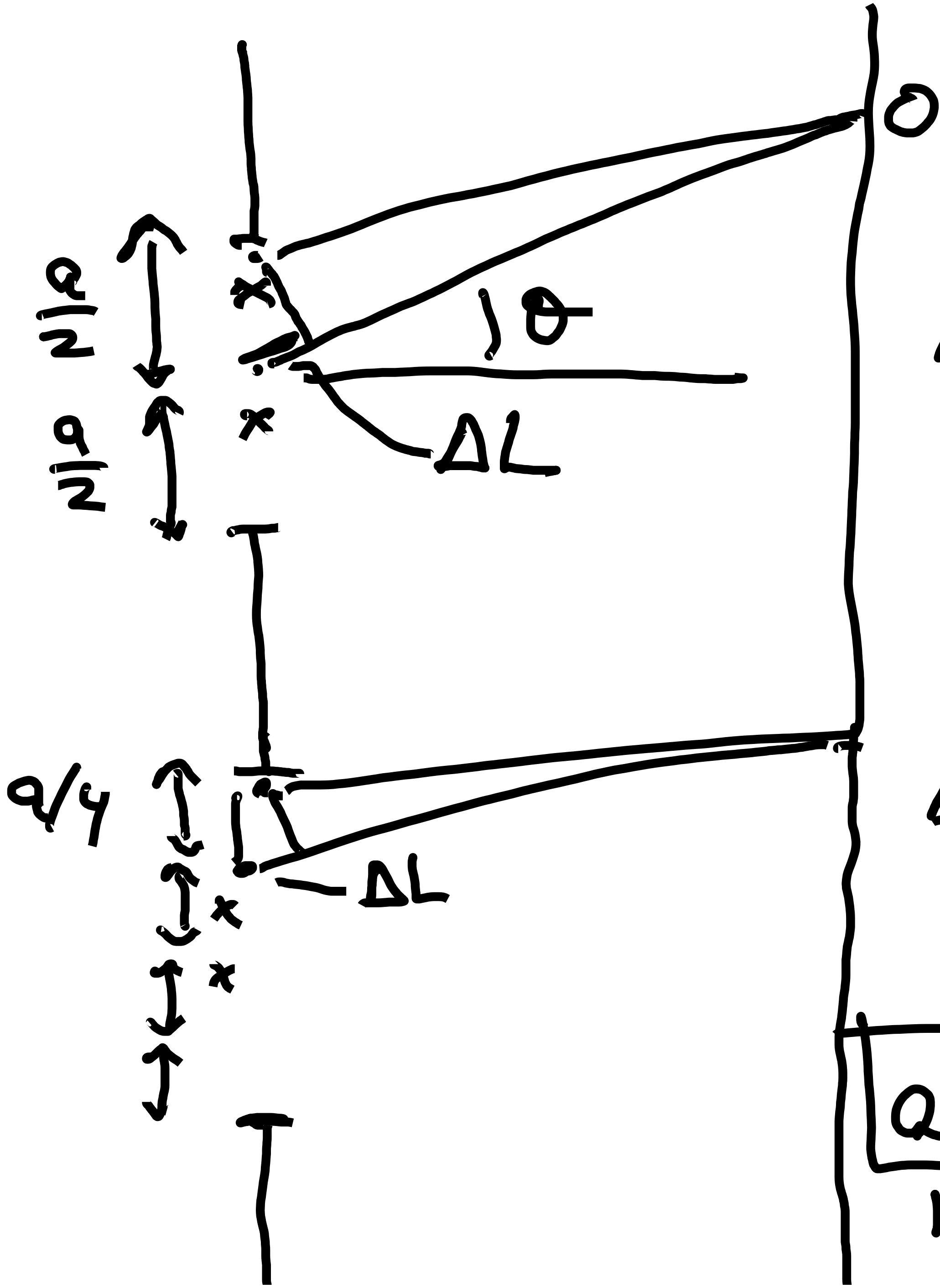
Waves passing through a long narrow slit of width *a* produce, on a viewing screen, a single-slit diffraction pattern that includes a central maximum (bright fringe) and other maxima. They are separated by minima that are located relative to the central axis by angles ϑ:

$$a \sin\theta = m\lambda, \qquad \text{for } m = 1, 2, 3, \ldots$$

The maxima are located approximately halfway between minima.



(a) Waves from the top points of four zones of width a/4 undergo fully destructive interference at point P_2. (b) For *D* >> *a*, we can approximate rays r_1, r_2, r_3, and r_4 as being parallel, at angle ϑ to the central axis.



$$\Delta L = \frac{\lambda}{2}$$

$$\Delta L = \frac{a}{2}\sin\theta = \frac{\lambda}{2}$$

$$a\sin\theta = \lambda$$

$$\Delta L = \lambda/2$$

$$\Delta L = \frac{a}{4}\sin\theta = \frac{\lambda}{2}$$

$$a\sin\theta = 2\lambda$$

$$\boxed{a\sin\theta = m\lambda}$$ minima

m = integer

Intensity in Single-Slit Diffraction

The intensity of the diffraction pattern at any given angle ϑ is

$$I(\theta) = I_m\left(\frac{\sin\alpha}{\alpha}\right)^2,$$

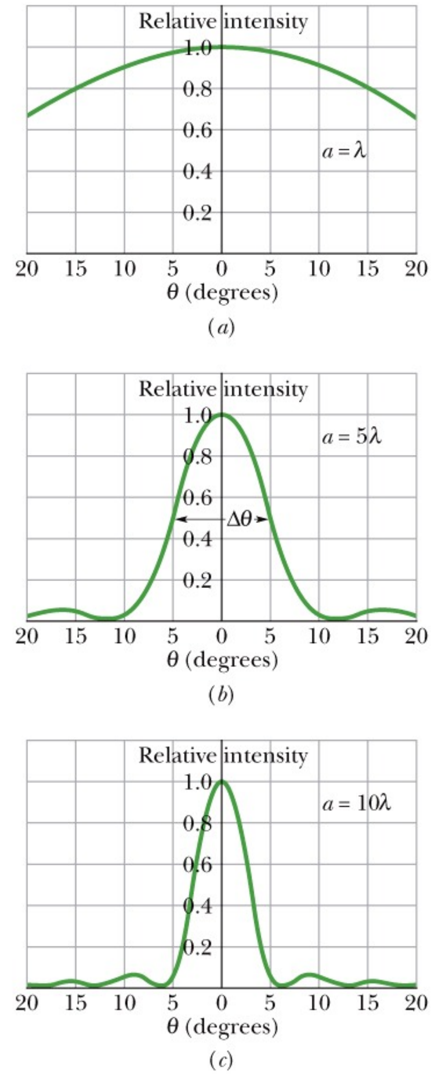
where, I_m is the intensity at the center of the pattern and

$I(\alpha) = 0$ for $\alpha = m\pi$, $m = 1, 2, 3, \ldots$

$$\alpha = \tfrac{1}{2}\phi = \frac{\pi a}{\lambda}\sin\theta.$$

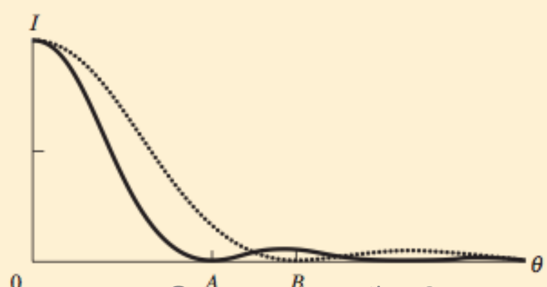
$\frac{\pi}{\lambda} a \sin\theta = m\pi$

The plots show the relative intensity in single-slit diffraction for three values of the ratio a/λ. The wider the slit is, the narrower is the central diffraction maximum.

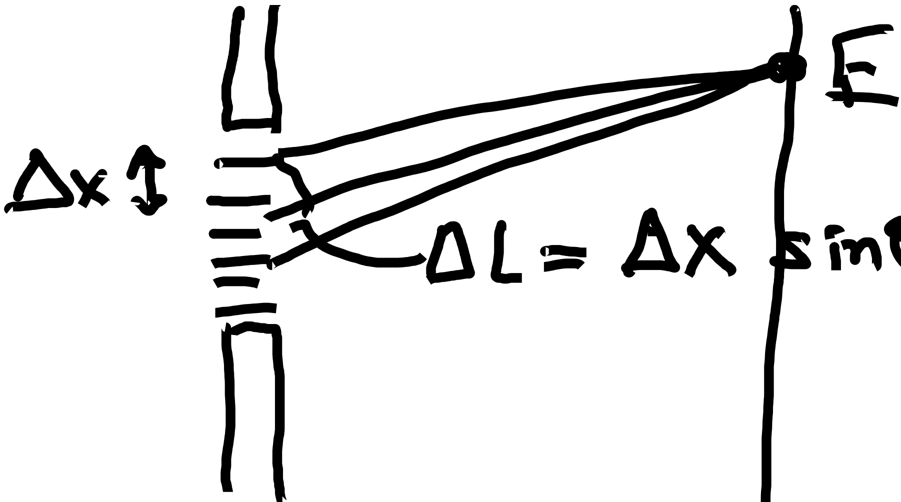


Checkpoint 3

Two wavelengths, 650 and 430 nm, are used separately in a single-slit diffraction experiment. The figure shows the results as graphs of intensity I versus angle θ for the two diffraction patterns. If both wavelengths are then used simultaneously, what color will be seen in the combined diffraction pattern at (a) angle A and (b) angle B?



Answer
(a) 650 nm
(b) 430 nm



$$\Delta L = \Delta x \sin\theta$$

$$\Delta\phi = 2\pi \frac{\Delta L}{\lambda}$$

$$\Delta\phi = \frac{2\pi}{\lambda} \Delta x \sin\theta$$

$$\Delta x = \frac{a}{N}$$

$$E = E_0 \left(1 + e^{i\Delta\phi} + e^{i2\Delta\phi} + e^{i3\Delta\phi} + \dots + e^{i(N-1)\Delta\phi}\right)$$

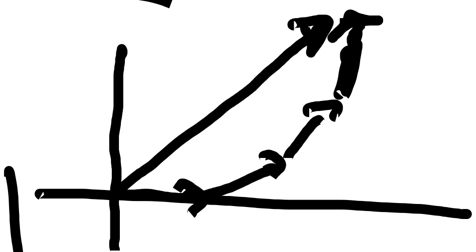
$$1 + \omega + \omega^2 + \omega^3 + \dots + \omega^{N-1} = \frac{1-\omega^N}{1-\omega}$$

$$(1 + \omega + \omega^2 + \omega^3 + \dots \omega^{N-1})(1-\omega) =$$

$$= 1 - \cancel{\omega} + \cancel{\omega} - \omega^2 + \omega^2 \dots - \omega^N$$

$$\omega = e^{i\Delta\phi}$$

$$\left|\frac{1-w^N}{1-w}\right| = \left|\frac{1-e^{+iN\Delta\phi}}{1-e^{i\Delta\phi}}\right| =$$



$$= \left|\frac{e^{i\frac{N}{2}\Delta\phi}}{e^{i\Delta\phi/2}}\right| \left|\frac{e^{-i\frac{N}{2}\Delta\phi} - e^{i\frac{N}{2}\Delta\phi}}{e^{-i\Delta\phi/2} - e^{i\Delta\phi/2}}\right| =$$

$$\frac{N\Delta\phi}{2} = \frac{\pi a}{\lambda}\sin\theta$$

$$= \frac{\sin\frac{N}{2}\Delta\phi}{\frac{\Delta\phi}{2}} = (\text{square})$$

$$= \left|2N\frac{\sin\left(\frac{\pi a}{\lambda}\sin\theta\right)}{\frac{\pi a}{\lambda}\sin\theta}\right|^2 = I_m\frac{\left(\sin\left(\frac{\pi a}{\lambda}\sin\theta\right)\right)^2}{\left(\frac{\pi a}{\lambda}\sin\theta\right)^2}$$

Diffraction by a Circular Aperture

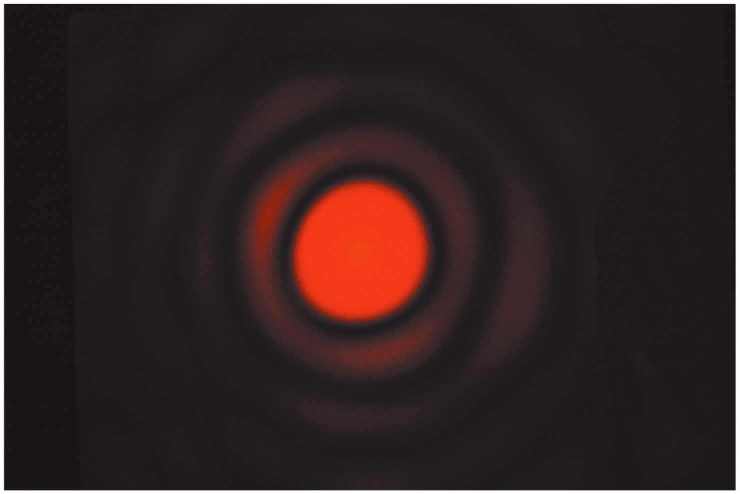
Diffraction by a circular aperture or a lens with diameter *d* produces a central maximum and concentric maxima and minima, given by

$$\sin\theta = 1.22\frac{\lambda}{d} \quad \text{(first minimum—circular aperture).}$$

The angle ϑ here is the angle from the central axis to any point on that (circular) minimum.

$$\sin\theta = \frac{\lambda}{a} \quad \text{(first minimum—single slit),}$$

which locates the first minimum for a long narrow slit of width a. The main difference is the factor 1.22, which enters because of the circular shape of the aperture.

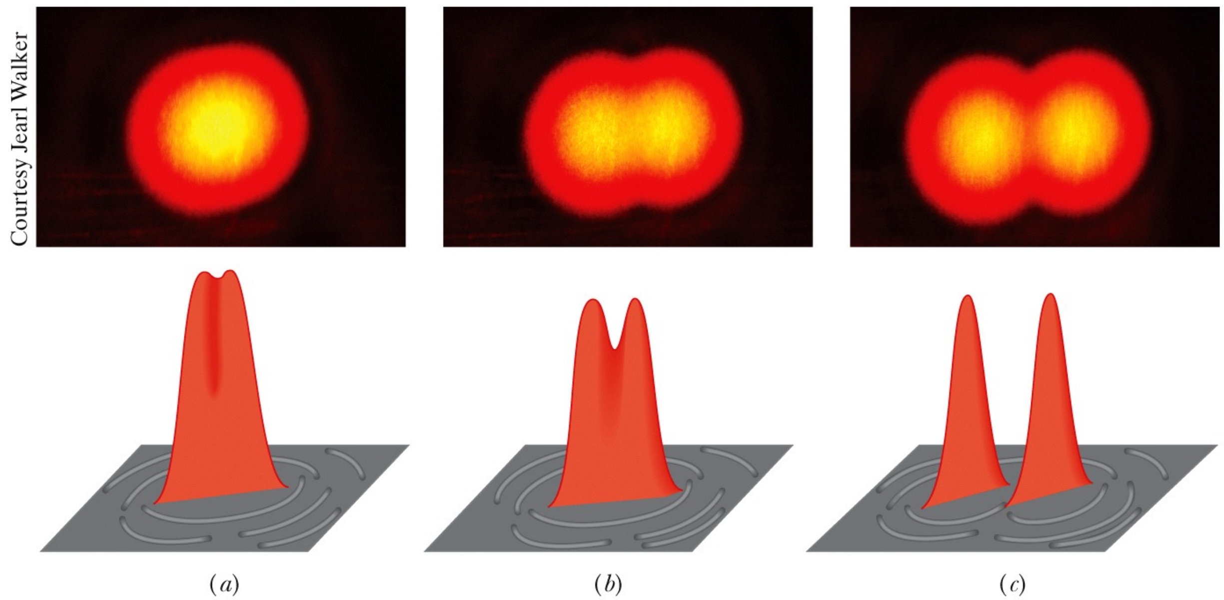


Courtesy Jearl Walker

The diffraction pattern of a circular aperture. Note the central maximum and the circular secondary maxima. The figure has been overexposed to bring out these secondary maxima, which are much less intense than the central maximum.

Diffraction by a Circular Aperture

Resolvability



The images of two point sources (stars) formed by a converging lens. At the bottom, representations of the image intensities. In (a) the angular separation of the sources is too small for them to be distinguished, in (b) they can be marginally distinguished, and in (c) they are clearly distinguished. Rayleigh's criterion is satisfied in (b), with the central maximum of one diffraction pattern coinciding with the first minimum of the other.

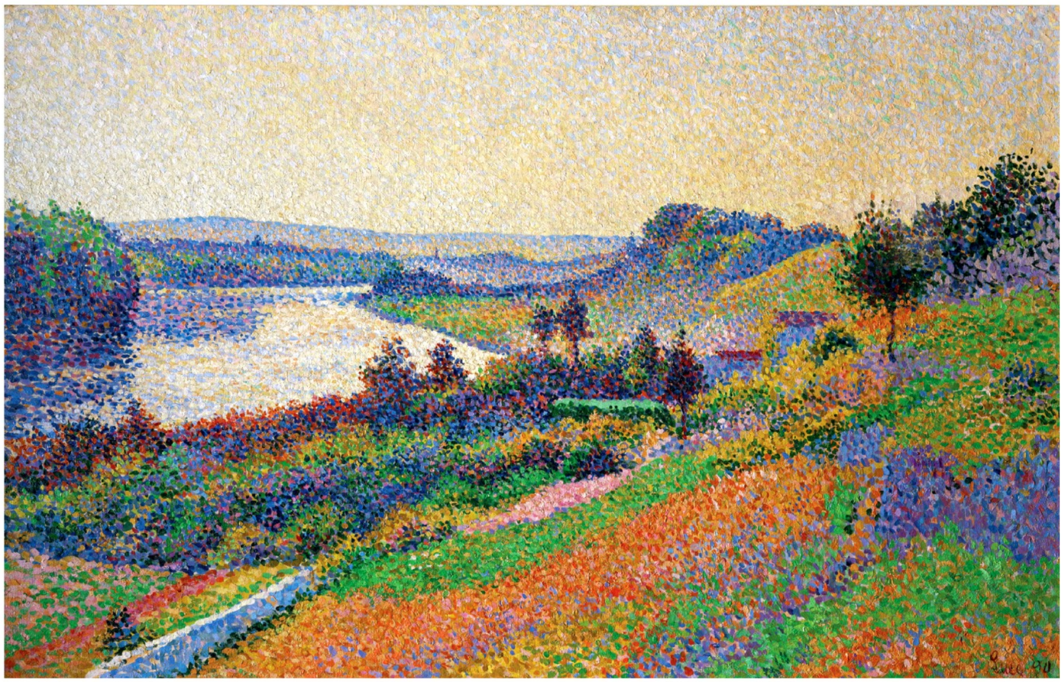
Rayleigh's criterion suggests that two objects are on the verge of resolvability if the central diffraction maximum of one is at the first minimum of the other. Their angular separation can then be no less than

$$\theta_R = 1.22\frac{\lambda}{d} \quad \text{(Rayleigh's criterion).}$$

in which *d* is the diameter of the aperture through which the light passes.

Diffraction by a Circular Aperture

Pointillism



Maximilien Luce, *The Seine at Herblay*, 1890. Musée d'Orsay, Paris, France. Photo by Erich Lessing/Art Resource

Rayleigh's criterion can explain the arresting illusions of color in the style of painting known as pointillism. In this style, a painting is made not with brush strokes in the usual sense but rather with a myriad of small colored dots. One fascinating aspect of a pointillistic painting is that when you change your distance from it, the colors shift in subtle, almost subconscious ways. This color shifting has to do with whether you can resolve the colored dots.

Checkpoint 4

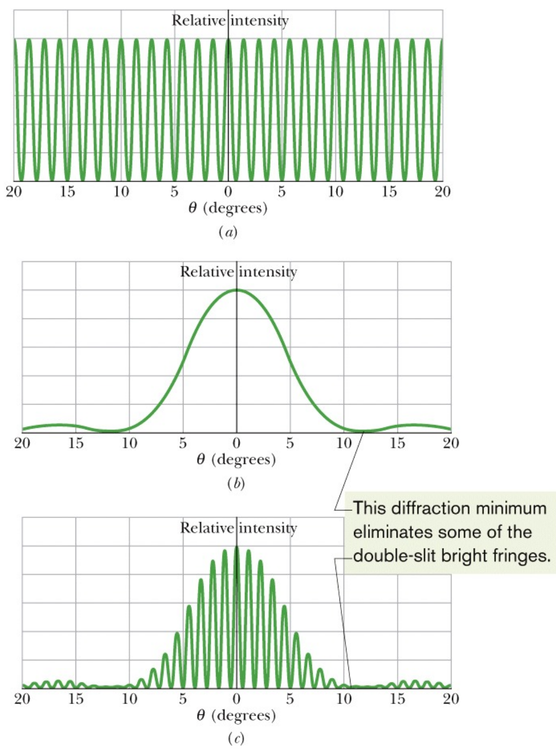
Suppose that you can barely resolve two red dots because of diffraction by the pupil of your eye. If we increase the general illumination around you so that the pupil decreases in diameter, does the resolvability of the dots improve or diminish? Consider only diffraction. (You might experiment to check your answer.)

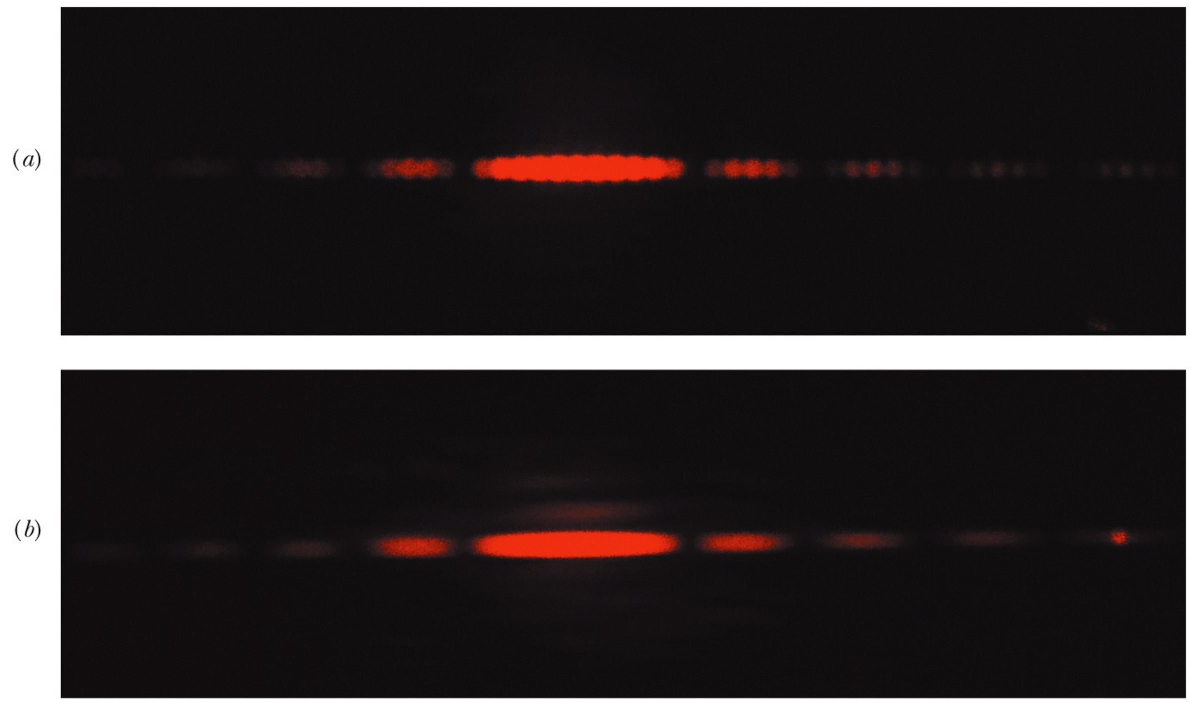
Answer:

Resolvability improves.

Diffraction by a Double Slit

Waves passing through two slits produce a combination of double-slit interference and diffraction by each slit.



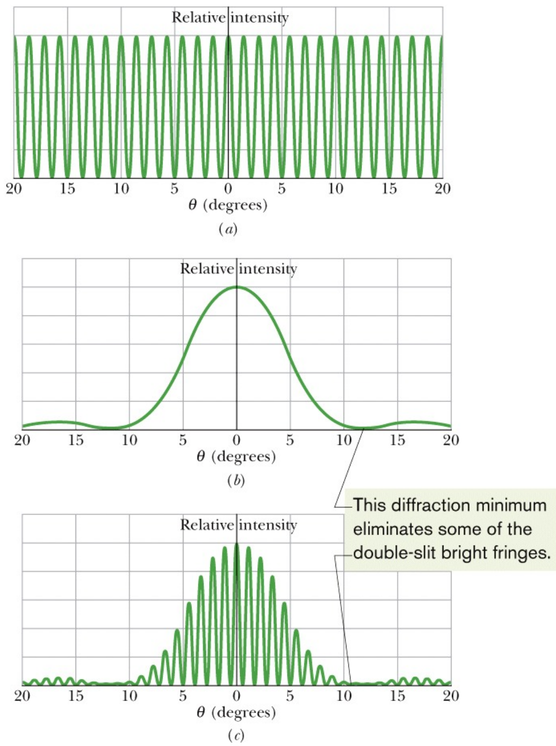


Courtesy Jearl Walker

(a) The intensity plot to be expected in a double-slit interference experiment with vanishingly narrow slits. (b) The intensity plot for diffraction by a typical slit of width a (not vanishingly narrow). (c) The intensity plot to be expected for two slits of width a. The curve of (b) acts as an envelope, limiting the intensity of the double-slit fringes in (a). Note that the first minima of the diffraction pattern of (b) eliminate the double-slit fringes that would occur near 12° in (c).

Diffraction by a Double Slit

Waves passing through two slits produce a combination of double-slit interference and diffraction by each slit.



For identical slits with width a and center-to-center separation *d*, the intensity in the pattern varies with the angle ϑ from the central axis as

$$I(\theta) = I_m(\cos^2 \beta)\left(\frac{\sin \alpha}{\alpha}\right)^2 \quad \text{(double slit)},$$

in which

$$\beta = \frac{\pi d}{\lambda} \sin \theta$$

and

$$\alpha = \frac{\pi a}{\lambda} \sin \theta.$$

Note carefully that the right side of double slit equation is the product of I_m and two factors. (1) The interference factor $\cos^2\beta$ is due to the interference between two slits with slit separation *d*. (2) The diffraction factor $[(\sin a)/a]^2$ is due to diffraction by a single slit of width *a*.