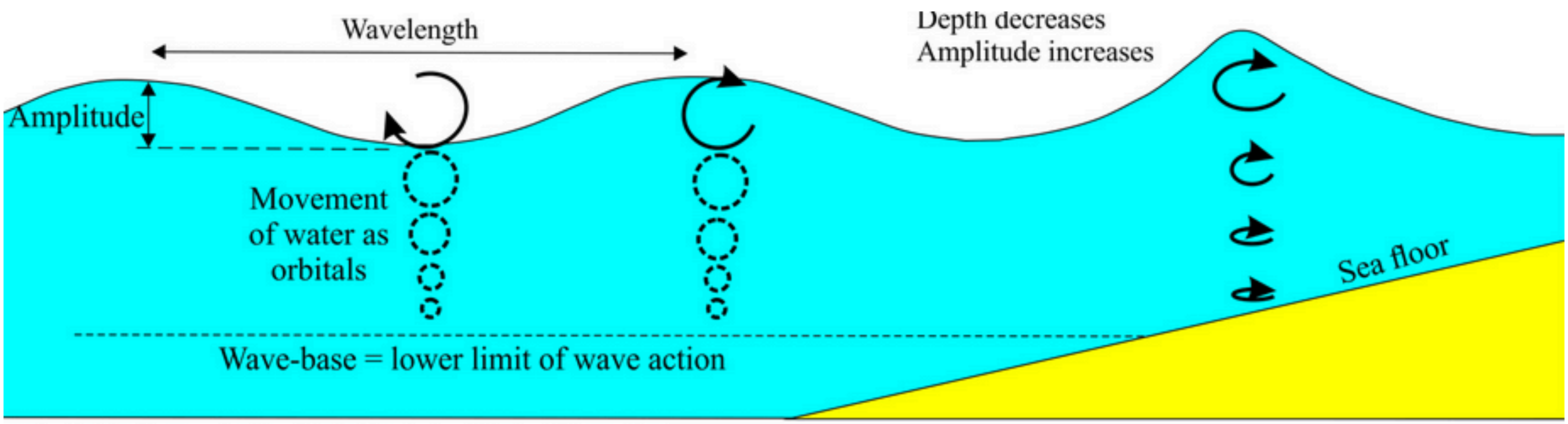
Water waves

Water waves

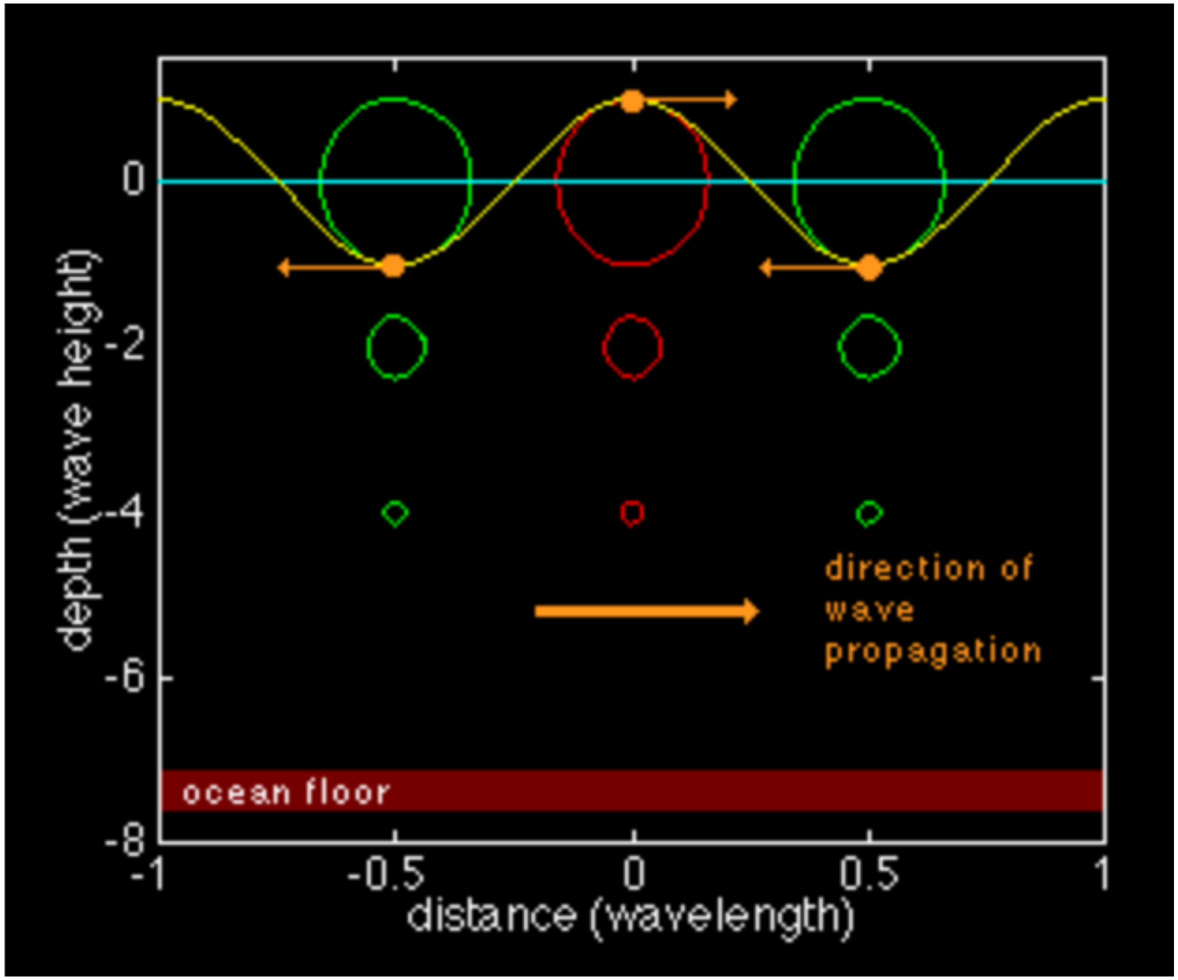


ırface wave characteristics in deep and shallow water. The orbitals (arrowed circles) depict the motion of water ırticles as each wave crest and trough passes. The orbitals become increasingly elliptical as the sea bed shoals ıd waves interact with the sea floor (towards the beach).

Water waves

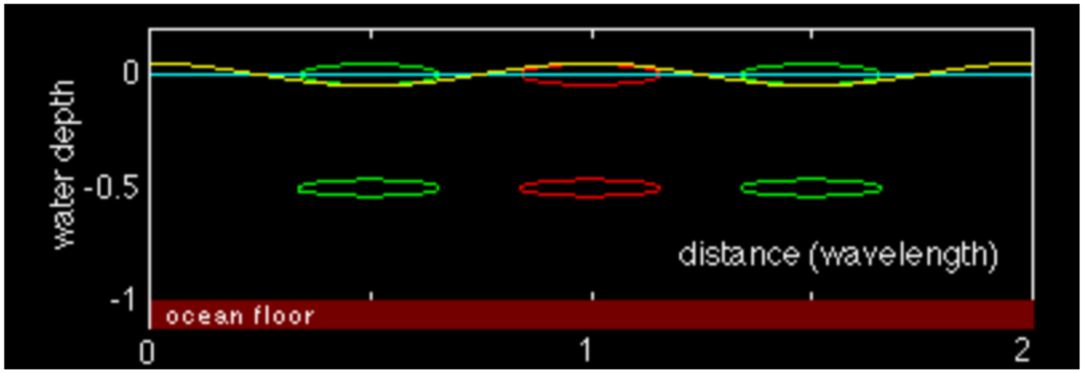
We distinguish between **deep-water waves** and **shallow-water waves**. The distinction between deep and shallow water waves has nothing to do with absolute water depth. It is determined by the ratio of the water's depth to the wavelength of the wave.

The water molecules of a deep-water wave move in a circular orbit. The diameter of the orbit decreases with the distance from the surface. The motion is felt down to a distance of approximately one wavelength, where the wave's energy becomes negligible.



Water waves

The orbits of the molecules of shallow-water waves are more elliptical.



The change from deep to shallow water waves occurs when the depth of the water, d, becomes less than one half of the wavelength of the wave, λ. When d is much greater than λ/2 we have a **deep-water wave or a short wave**. When d is much less than λ/2 we have a **shallow-water wave or a long wave**.

The speed of deep-water waves depends on the wavelength of the waves. We say that deep-water waves show **dispersion**. A wave with a longer wavelength travels at higher speed. In contrast, shallow-water waves show no dispersion. Their speed is independent of their wavelength. It depends, however, on the depth of the water.

λ much larger than the water depth h, travel with the phase velocity

$$c_p = \sqrt{gh} \qquad \text{(shallow water)},$$

Water waves

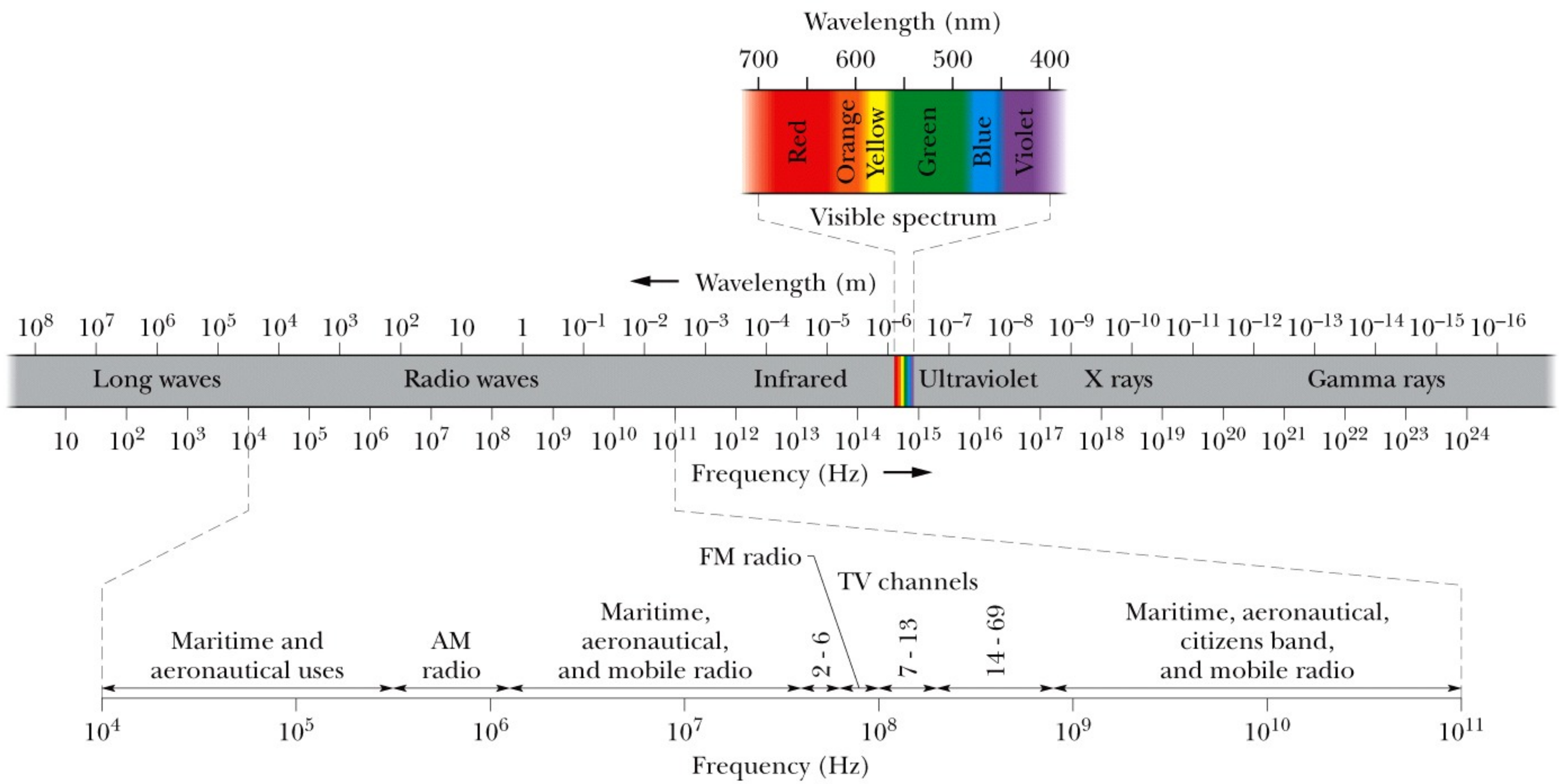
quantity	symbol	units	deep water ($h > \frac{1}{2}\lambda$)	shallow water ($h < 0.05\lambda$)	intermediate depth (all λ and h)
dispersion relation	$\Omega(k)$	rad / s	$\sqrt{gk} = \sqrt{\frac{2\pi\, g}{\lambda}}$	$k\sqrt{gh} = \frac{2\pi}{\lambda}\sqrt{gh}$	$\sqrt{gk\,\tanh(kh)}$ $= \sqrt{\frac{2\pi g}{\lambda}\tanh\left(\frac{2\pi h}{\lambda}\right)}$
phase velocity	$c_p = \frac{\lambda}{T} = \frac{\omega}{k}$	m / s	$\sqrt{\frac{g}{k}} = \frac{g}{\omega} = \frac{g}{2\pi}T$	$\sqrt{gh}$	$\sqrt{\frac{g}{k}\tanh(kh)}$
group velocity	$c_g = \frac{\partial\Omega}{\partial k}$	m / s	$\frac{1}{2}\sqrt{\frac{g}{k}} = \frac{1}{2}\frac{g}{\omega} = \frac{g}{4\pi}T$	$\sqrt{gh}$	$\frac{1}{2}c_p\left(1 + \frac{2kh}{\sinh(2kh)}\right)$

Chapter 33

Electromagnetic Waves

WILEY

Maxwell's Rainbow



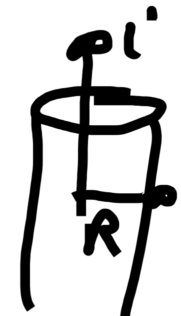
In Maxwell's time (the mid 1800s), the visible, infrared, and ultraviolet forms of light were the only electromagnetic waves known. Spurred on by Maxwell's work, however, Heinrich Hertz discovered what we now call radio waves and verified that they move through the laboratory at the same speed as visible light, indicating that they have the same basic nature as visible light. As the figure shows, we now know a wide spectrum (or range) of electromagnetic waves: Maxwell's rainbow.

$$\varepsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$$

$$F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$$

$$\mu_0 = 4\pi \cdot 10^{-7} \frac{T.m}{A}$$

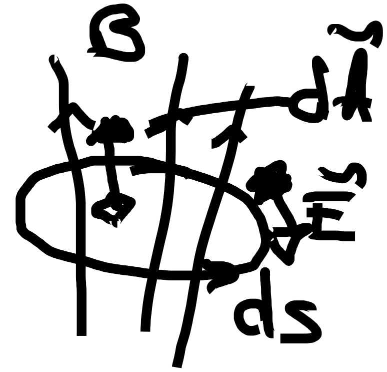
$$B = \frac{\mu_0 \, i}{2\pi R}$$



$$\oint \vec{E} \cdot d\vec{A} = q_{enc}/\varepsilon_0$$

$$F_B = q \vec{v} \times \vec{B}$$

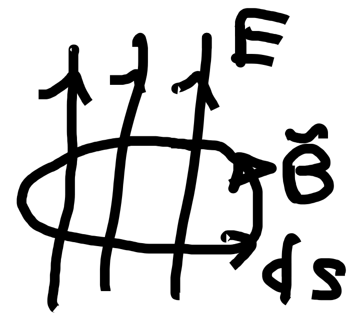
$$\oint \vec{B} \cdot d\vec{A} = 0$$



Faraday: $\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Maxwell



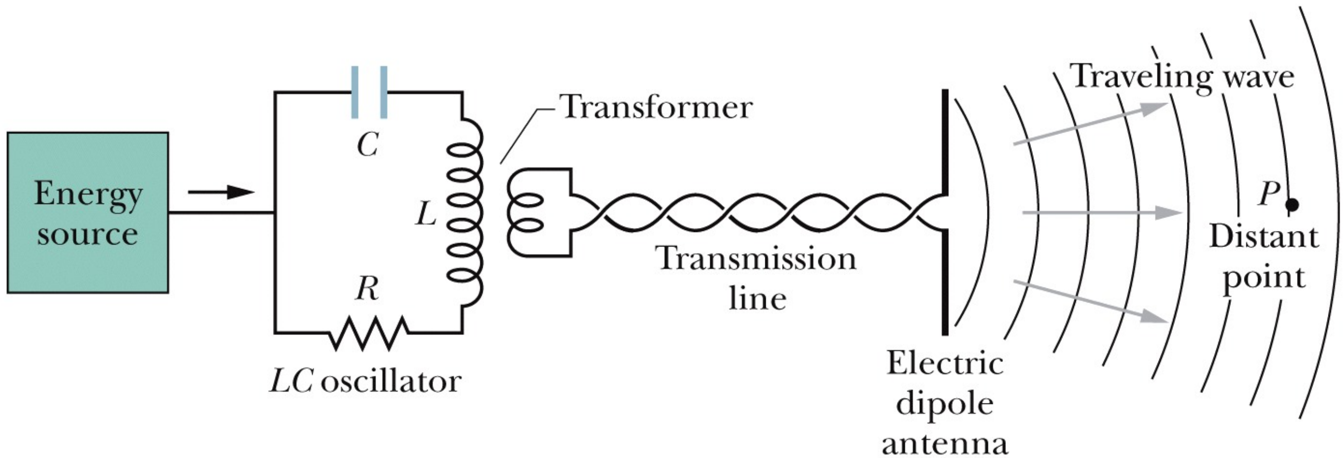
$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

$$C = \sqrt{\frac{1}{\mu_0 \varepsilon_0}}$$

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

$$\omega = \sqrt{\frac{1}{LC}}$$

Travelling Electromagnetic Wave



An arrangement for generating a traveling electromagnetic wave in the shortwave radio region of the spectrum: an LC oscillator produces a sinusoidal current in the antenna, which generates the wave. P is a distant point at which a detector can monitor the wave traveling past it.

Travelling Electromagnetic Wave

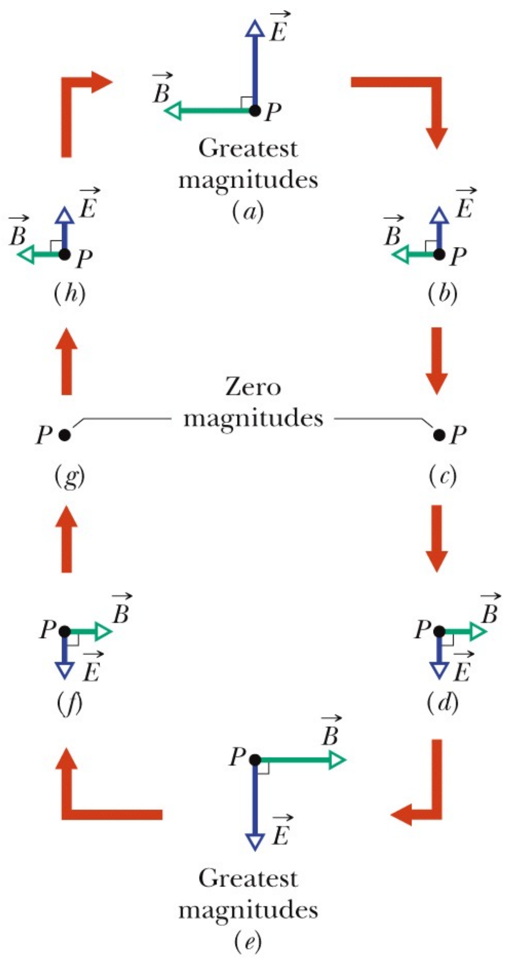
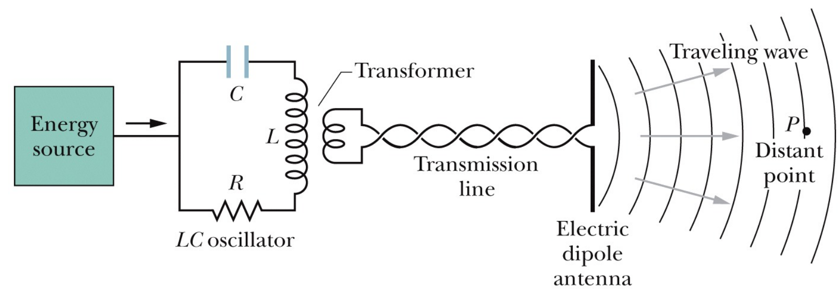


Figure 1

Figure 2



Electromagnetic Wave. Figure 1 shows how the electric field ***E*** and the magnetic field ***B*** change with time as one wavelength of the wave sweeps past the distant point P of Fig. 2 ; in each part of Fig. 1, the wave is traveling directly out of the page. (We choose a distant point so that the curvature of the waves suggested in Fig. 2 is small enough to neglect. At such points, the wave is said to be a plane wave, and discussion of the wave is much simplified.) Note several key features in Fig. 2; they are present regardless of how the wave is created:

Travelling Electromagnetic Wave

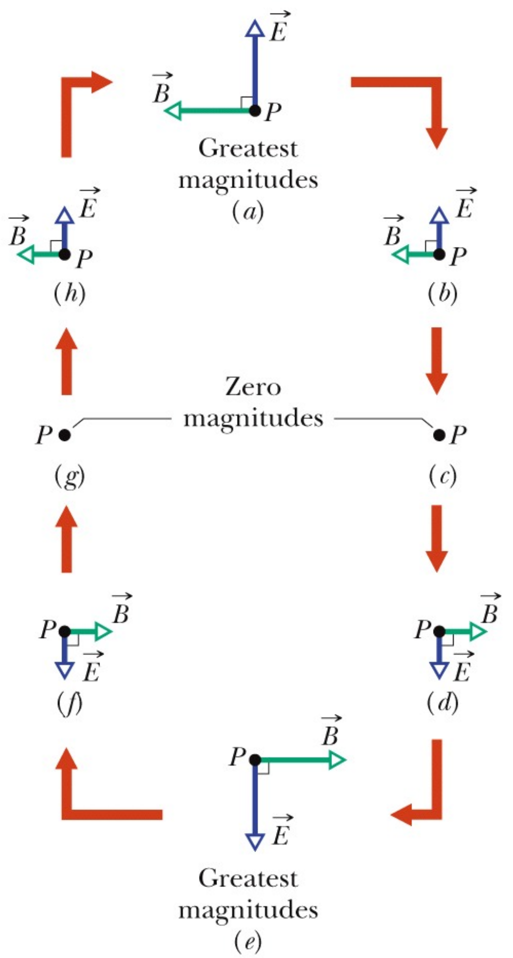
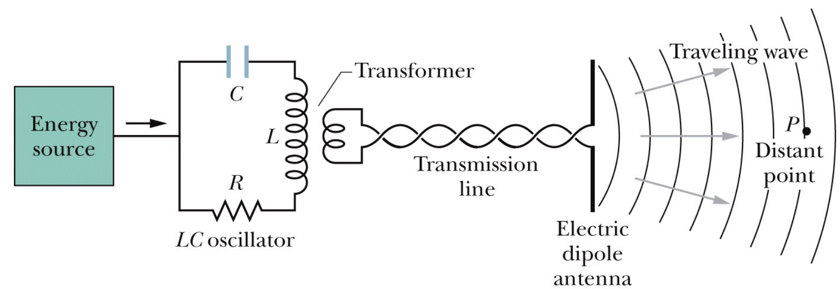


Figure 1

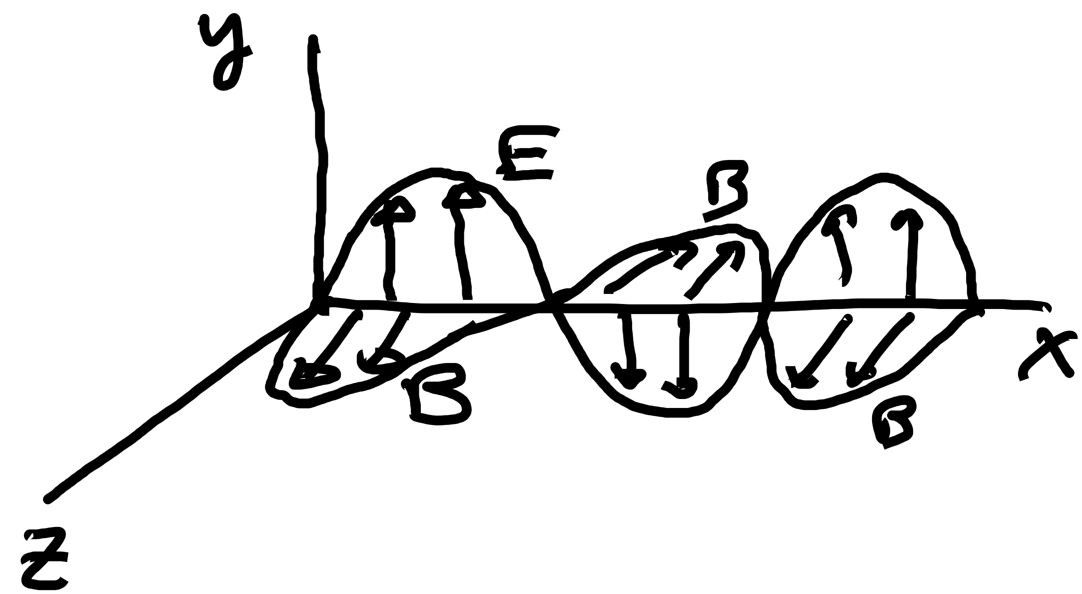
Figure 2



1. The electric and magnetic fields ***E*** and ***B*** are always perpendicular to the direction in which the wave is traveling. Thus, the wave is a transverse wave, as discussed in Chapter 16.
2. The electric field is always perpendicular to the magnetic field.
3. The cross product ***E*** × ***B*** always gives the direction in which the wave travels.
4. The fields always vary sinusoidally, just like the transverse waves discussed in Chapter 16. Moreover, the fields vary with the same frequency and in phase (in step) with each other.

$$\vec{E} = E_m \sin(kx - \omega t),\hat{y}$$

$$\vec{B} = B_m \sin(kx - \omega t),\hat{z}$$



$$\frac{\omega}{k} = c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$c = 3 \times 10^8$ m/s in vacuum

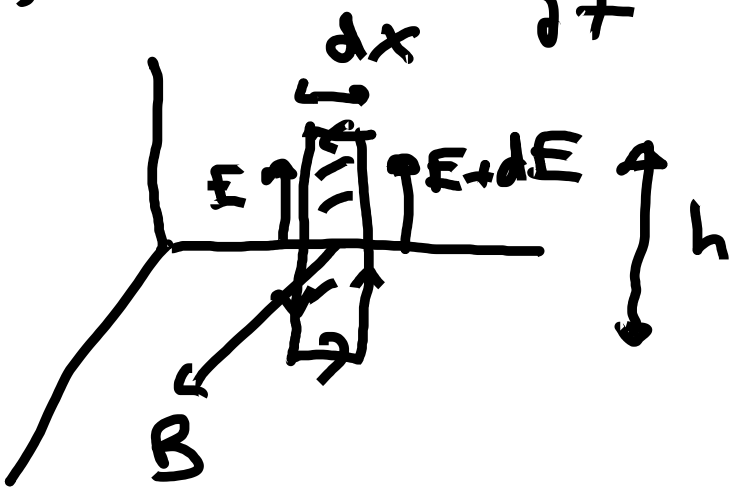
$$\frac{E_m}{B_m} = c$$

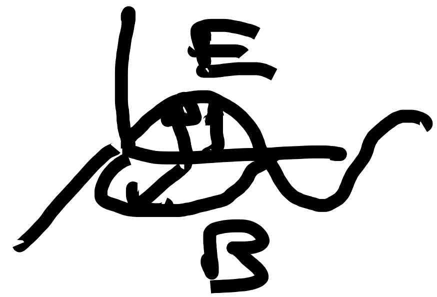
$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

$$E = E_m \sin(kx - \omega t) \; ; \; B = B_m \sin(kx - \omega t)$$

$$\oint \vec{E} \cdot d\vec{s} = -\frac{\partial \phi_B}{\partial t} \qquad \underbrace{\phi_B = \int \vec{B} \cdot d\vec{A}}$$





$$\oint \vec{E} \cdot d\vec{s} = (E + dE)h - E \cdot h$$

$$= dE \cdot h$$

$$\phi_B = B \cdot h\,dx$$

$$\frac{dE}{dx} \cdot \cancel{h} = -\frac{\partial B}{\partial t} \cdot \cancel{h} \cdot \cancel{dx}$$

$$\frac{\partial E}{\partial x} = k E_m \cos(kx - \omega t)$$

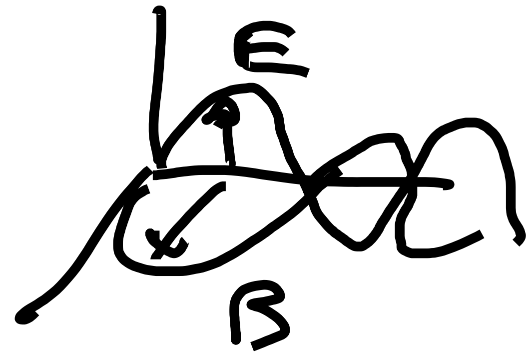
$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

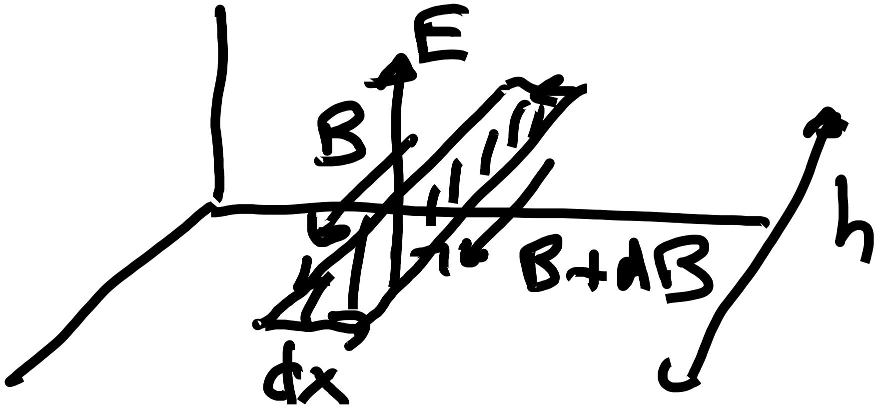
$$\frac{\partial B}{\partial t} = -\omega B_m \cos(kx - \omega t)$$

$$k E_m = \omega B_m \Rightarrow \frac{E_m}{B_m} = \frac{\omega}{k} = c$$

$E = E_m \sin(kx - \omega t) \qquad B = B_m \sin(kx - \omega t)$

$\oint \vec{B}\cdot d\vec{S} = \mu_0 \varepsilon_0 \frac{\partial \Phi_E}{\partial t} \; ; \; \Phi_E = \int \vec{E}\cdot d\vec{A}$





$\frac{\partial B}{\partial x} = k B_m \cos(\quad)$

$\oint \vec{B}\cdot d\vec{S} = Bh - (B+dB)h = -\frac{\partial B \cancel{h}}{dx} = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} \cancel{h\,dx}$

$\Phi_E = E \cdot h \cdot dx \qquad \frac{\partial E}{\partial t} = -\omega E_m \cos(\quad)$

$\frac{\partial B}{\partial x} = -\mu_0 \varepsilon_0 \frac{\partial E}{\partial t} \Rightarrow k B_m = \mu_0 \varepsilon_0 \omega \frac{E_m}{B_m} = \mu_0 \varepsilon_0 \omega \cdot C$

$1 = \mu_0 \varepsilon_0 C \omega / k \qquad C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

Travelling Electromagnetic Wave

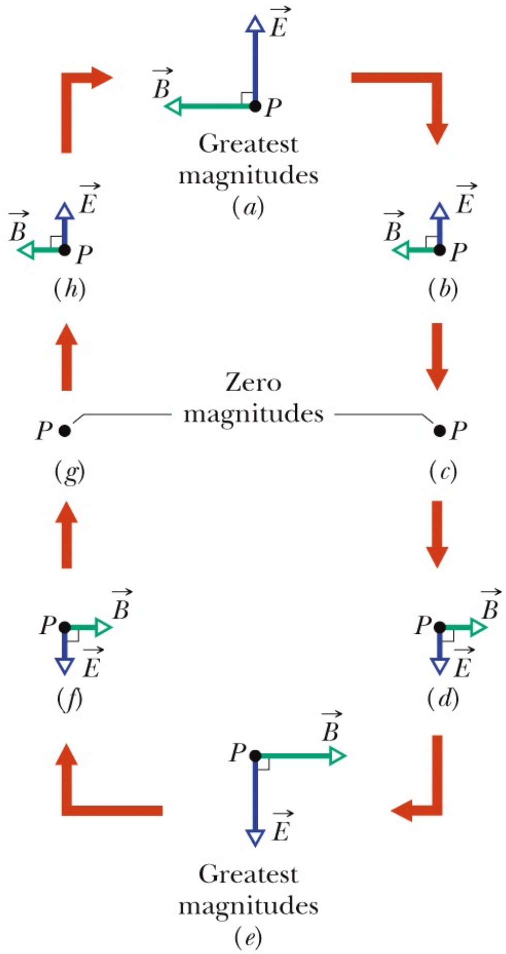
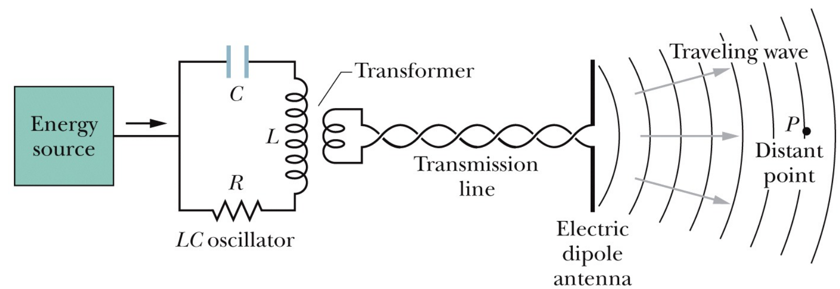


Figure 1

Figure 2



In keeping with these features, we can deduce that an electromagnetic wave traveling along an x axis has an electric field ***E*** and a magnetic field ***B*** with magnitudes that depend on x and t:

$$E = E_m \sin(kx - \omega t),$$

$$B = B_m \sin(kx - \omega t),$$

where E_m and B_m are the amplitudes of ***E*** and ***B***. The electric field induces the magnetic field and vice versa.

Travelling Electromagnetic Wave

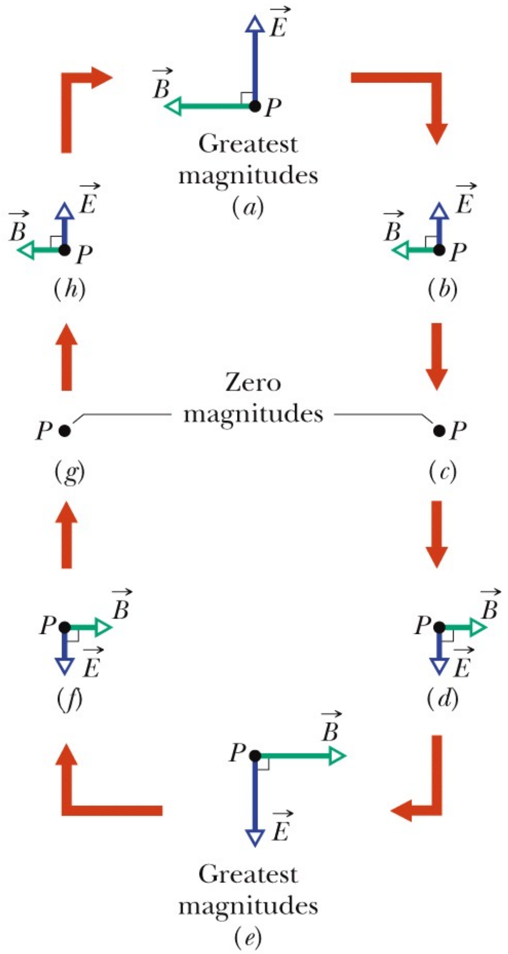
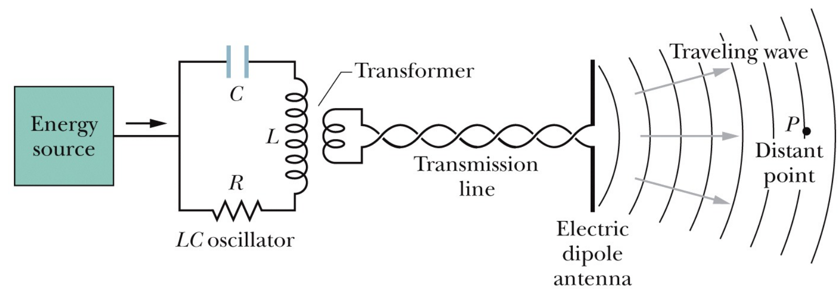


Figure 1

Figure 2



Wave Speed. From chapter 16 (Eq. 16-13), we know that the speed of the wave is ω/k. However, because this is an electromagnetic wave, its speed (in vacuum) is given the symbol *c* rather than *v* and that c has the value given by

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \quad \text{(wave speed)},$$

which is about 3.0×10^8 m/s. In other words,

All electromagnetic waves, including visible light, have the same speed *c* in vacuum.

Energy Transport and The Poynting Vector

The Poynting Vector: The rate per unit area at which energy is transported via an electromagnetic wave is given by the Poynting vector

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

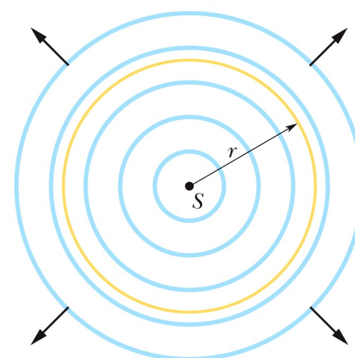
The direction of the Poynting vector $\vec{S}$ of an electromagnetic wave at any point gives the wave's direction of travel and the direction of energy transport at that point.

The time-averaged rate per unit area at which energy is transported is S_{avg}, which is called the intensity *I* of the wave:

$$I = \frac{1}{c\mu_0}E_{\mathrm{rms}}^2.$$

in which $E_{rms} = E_m/\sqrt{2}$.

The energy emitted by light source *S* must pass through the sphere of radius *r*.



A point source of electromagnetic waves emits the waves isotropically—that is, with equal intensity in all directions. The intensity of the waves at distance *r* from a point source of power P_s is

$$I = \frac{\text{power}}{\text{area}} = \frac{P_s}{4\pi r^2},$$

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} \equiv \text{Poynting vector}$$

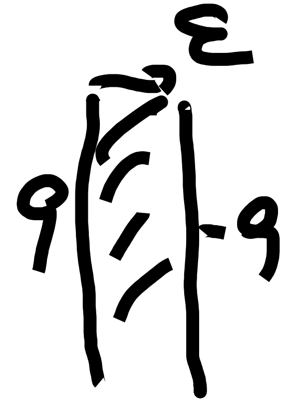
$$S = \frac{\text{power}}{\text{area}} = \frac{\text{energy/time}}{\text{area}}$$

intensity: $I = S_{av} = \frac{1}{c\mu_0} E_{rms}^2$

$$E_{rms} = \frac{E_m}{\sqrt{2}}$$

Energy density: $c^2 = \frac{1}{\mu_0 \varepsilon_0}$

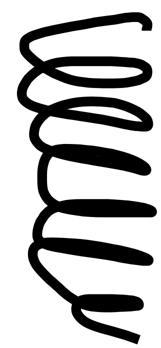
$$u = \cancel{\varepsilon}\underbrace{\frac{1}{2}\varepsilon_0 E^2} = \frac{B^2}{2\mu_0}$$

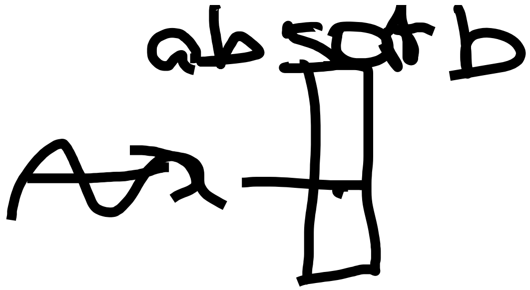


$$I = \frac{1}{c\mu_0} E_{rms}^2$$

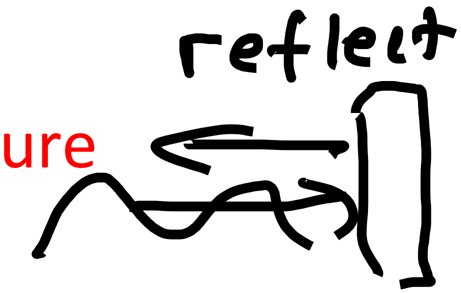
$$U = \frac{q^2}{2C}$$

$$u = \frac{I}{c} = \frac{1}{c^2 \mu_0} E_{rms}^2 = \varepsilon_0 E_{rms}^2 =$$





Radiation Pressure



When a surface intercepts electromagnetic radiation, a force and a pressure are exerted on the surface.

If the radiation is totally absorbed by the surface, the force is

$$F = \frac{IA}{c} \qquad \textit{Total Absorption}$$

in which I is the intensity of the radiation and A is the area of the surface perpendicular to the path of the radiation.

If the radiation is totally reflected back along its original path, the force is

$$F = \frac{2IA}{c} \qquad \textit{Total Reflection back along path}$$

The **radiation pressure** p_r is the force per unit area:

$$p_r = \frac{I}{c} \qquad \textit{Total Absorption}$$

and

$$p_r = \frac{2I}{c} \qquad \textit{Total Reflection back along path}$$

$E = P \cdot c$

$\Delta U =$ energy absorbed in Δt

momentum change $\Delta p = \frac{\Delta U}{c}$

units: $\frac{\text{joule}}{m} \, s = \frac{N \cdot \cancel{m}}{\cancel{m}} \cdot s = kg \cdot \frac{m}{s^{\cancel{2}}} \cdot \cancel{s} = kg \cdot \frac{m}{s}$

force $= \frac{\Delta P}{\Delta t} = \frac{\Delta U}{c \cdot \Delta t} \Rightarrow \underline{\Delta U = c \Delta P}$

$\Delta U = I \cdot A \cdot \Delta t =$

$\frac{\Delta p}{\Delta t} = \frac{\Delta U}{c} = \frac{I \cdot A \cdot \cancel{\Delta t}}{c \, \cancel{\Delta t}} = F = \frac{I \cdot A}{c}$

$P = F/A = I/c$

light bulb 100 W, 1 m away = r

$P_s = 100\,W$

$$I = \frac{P_s}{A} = \frac{100\,W}{4\pi r^2}$$

P_r = pressure on wall $= \frac{2I}{c} =$

$$= 2\,\frac{100\,W}{4\pi r^2 \cdot c} = 4 \times 10^{-8}\,Pa$$

Force on $10\,cm^2$ area $= P_r \cdot A = 10^{-13}\,N$