# 16-5 Interference of Waves

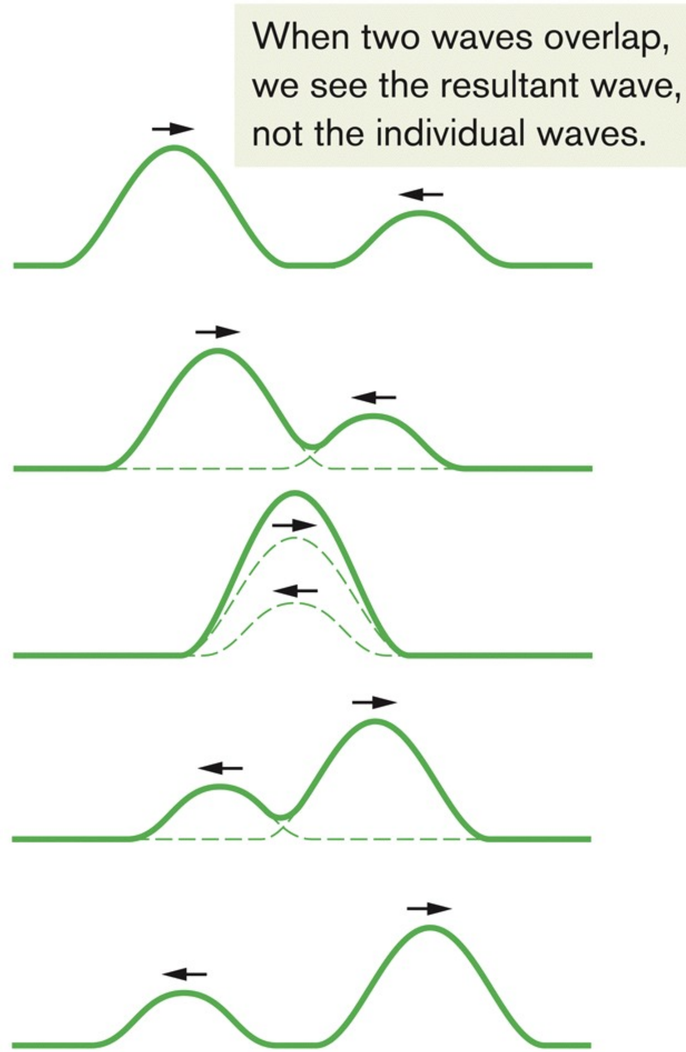
## Principle of Superposition of waves

Let $y_1(x, t)$ and $y_2(x, t)$ be the displacements that the string would experience if each wave traveled alone. The displacement of the string when the waves overlap is then the algebraic sum

$$y'(x,t) = y_1(x,t) + y_2(x,t).$$

This summation of displacements along the string means that

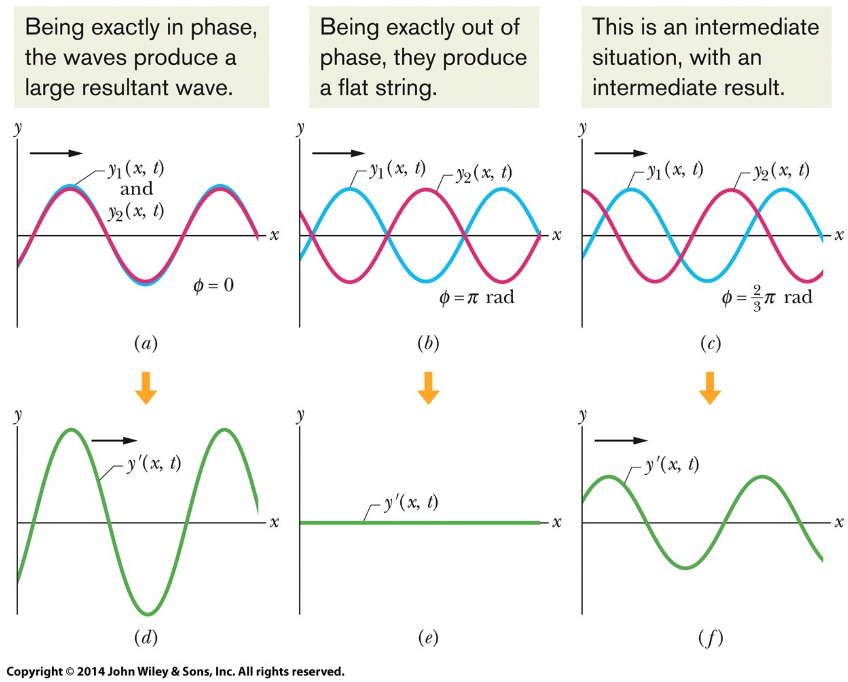
- Overlapping waves algebraically add to produce a **resultant wave** (or **net wave**).
- Overlapping waves do not in any way alter the travel of each other.



# 16-5 Interference of Waves

## Constructive and Destructive Interference

$$y'(x,t) = y_1(x,t) + y_2(x,t).$$




Two identical sinusoidal waves, $y_1(x, t)$ and $y_2(x, t)$, travel along a string in the positive direction of an *x* axis. They interfere to give a resultant wave $y'(x, t)$. The resultant wave is what is actually seen on the string. The phase difference $\Phi$ between the two interfering waves is (a) $0$ rad or $0^o$, (b) $\pi$ rad or $180^o$, and (c) $2/3\ \pi$ rad or $120^o$. The corresponding resultant waves are shown in (d), (e), and ( f ). 

$$y_1(x,t) = y_m \sin(kx - \omega t)$$

$$y_2(x,t) = y_m \sin(kx - \omega t + \phi)$$

$$y(x,t) = y_1(x,t) + y_2(x,t)$$

$$\sin\alpha + \sin\beta = 2 \sin\frac{\alpha+\beta}{2} \cos\frac{\alpha-\beta}{2}$$

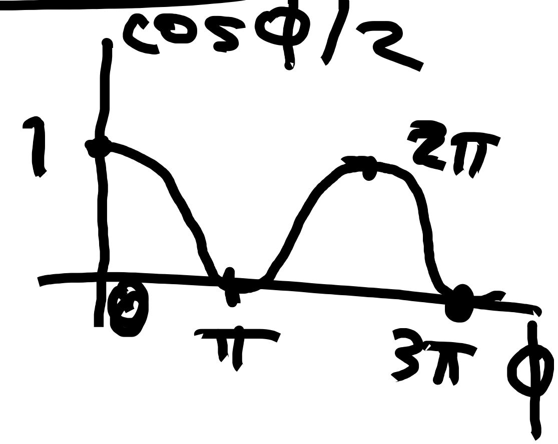
$$y(x,t) = 2\cos\frac{\phi}{2} \sin(kx - \omega t + \phi/2)$$

constructive interference

$\phi = 0, 2\pi, \ldots = 2\pi n$

destructive:

$\phi = \pi, 3\pi, \ldots = 2\pi(n + \frac{1}{2}) = \pi(2n+1)$



$$y_1(x,t) = y_m \sin(kx - \omega t) \quad \longrightarrow$$

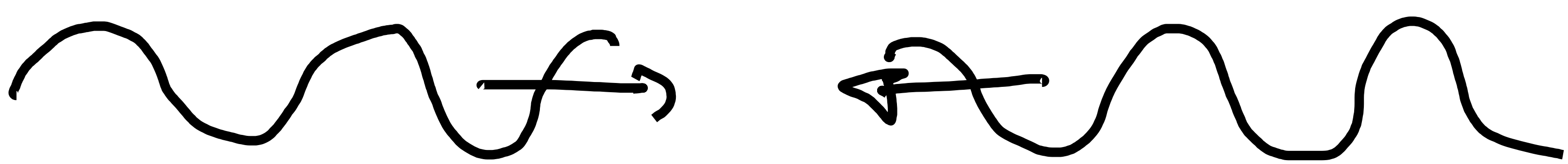
$$y_2(x,t) = y_m \sin(kx + \omega t) \quad \longleftarrow$$

$$y(x,t) = y_1(x,t) + y_2(x,t)$$

$$\sin\alpha + \sin\beta = 2 \sin\frac{\alpha+\beta}{2} \cos\frac{\alpha-\beta}{2}$$

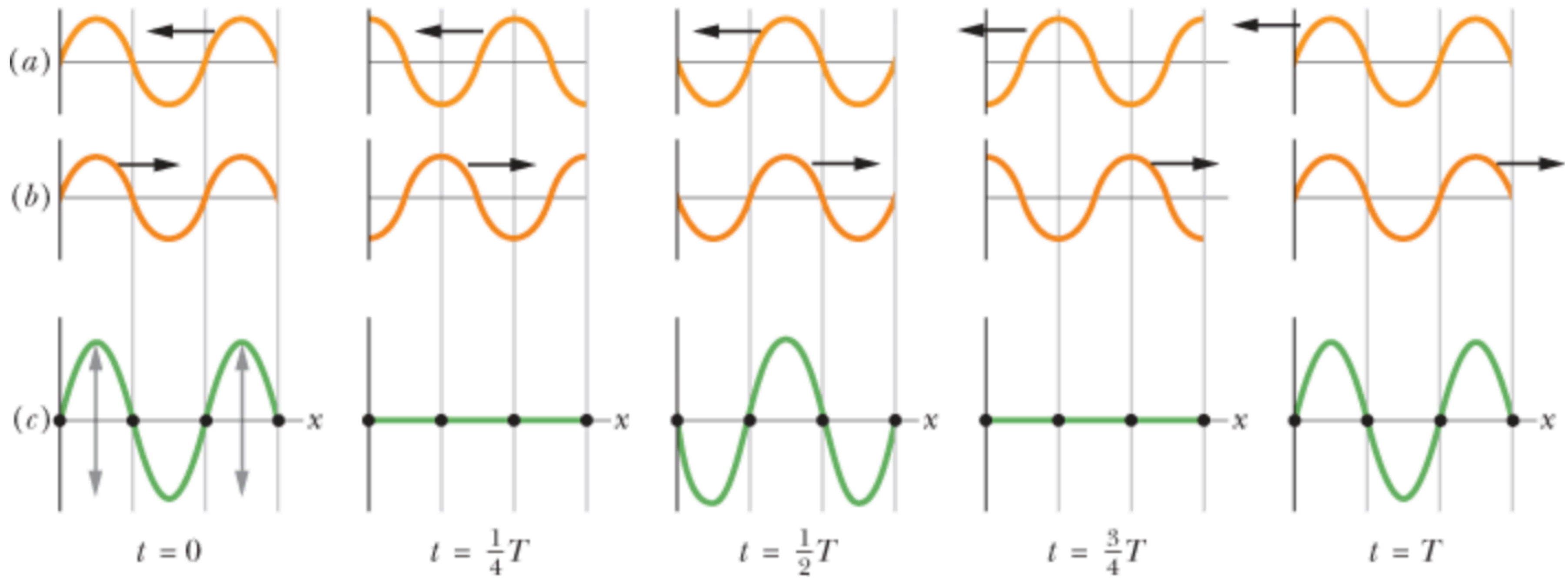
$$\boxed{y(x,t) = 2y_m \sin(kx) \cos(\omega t)}$$

<u>Standing waves</u> = superposition of waves traveling in <u>opposite direction</u>



As the waves move through each other, some points never move and some move the most.

$k = \frac{2\pi}{\lambda}$




Nodes: sin(kx)=0

Antinodes: sin(kx)=+/-1

$\sin(kx)\cos(\omega t)$

$kx = n\pi = \frac{2\pi}{\lambda}x \Rightarrow x = \frac{n\lambda}{2}$

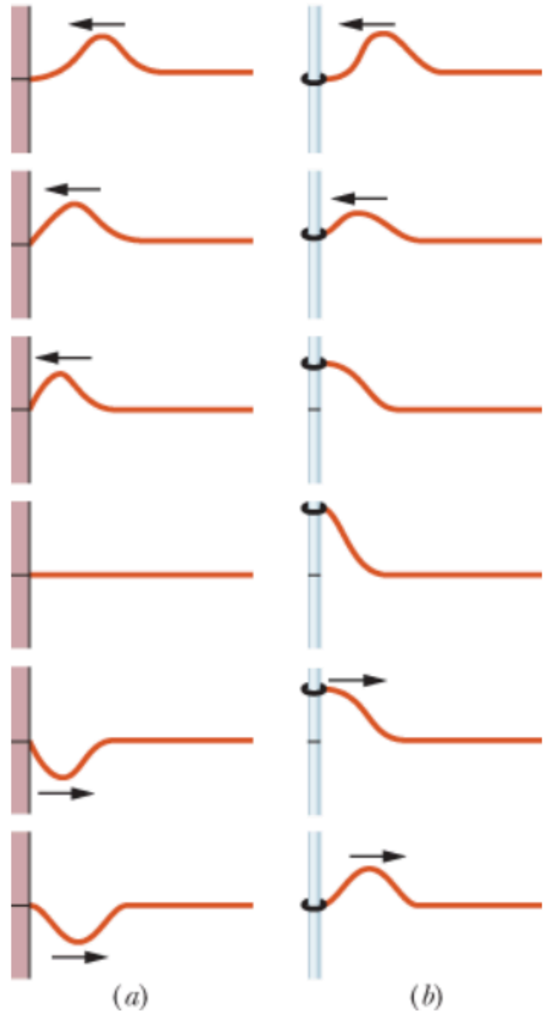
$kx = \frac{\pi}{2}, \frac{3\pi}{2}, (2n+1)\frac{\pi}{2} = \frac{2\pi}{\lambda}x$

$x = (n+\frac{1}{2})\lambda$

# Reflection at a boundary

There are two ways a pulse can reflect from the end of a string.

node




antinode

**Figure 16-19**
(*a*) A pulse incident from the right is reflected at the left end of the string, which is tied to a wall. Note that the reflected pulse is inverted from the incident pulse. (*b*) Here the left end of the string is tied to a ring that can slide without friction up and down the rod. Now the pulse is not inverted by the reflection.

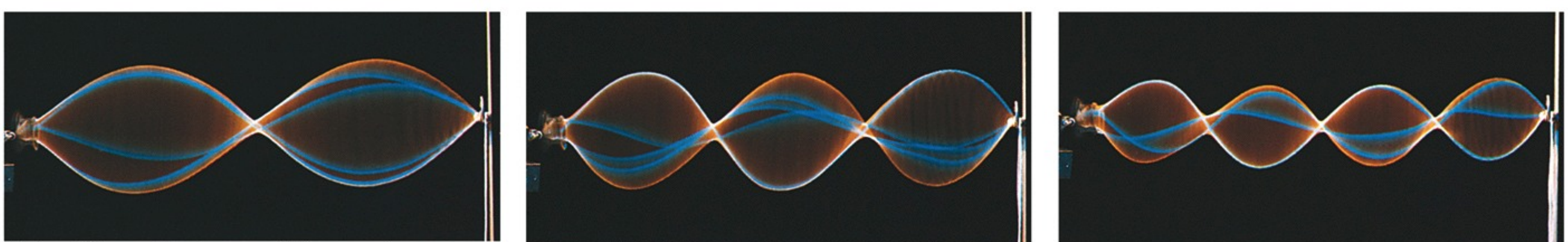
# 16-7 Standing Waves and Resonance

## Standing Waves

•The interference of two identical sinusoidal waves moving in opposite directions produces standing waves. For a string with fixed ends, the standing wave is given by

$$\overbrace{y'(x,t)}^{\text{Displacement}} = \underbrace{[2y_m \sin kx]}_{\substack{\text{Magnitude gives}\\ \text{amplitude}\\ \text{at position } x}}\underbrace{\cos \omega t}_{\substack{\text{Oscillating}\\ \text{term}}}$$





Richard Megna/Fundamental Photographs

Stroboscopic photographs reveal (imperfect) standing wave patterns on a string being made to oscillate by an oscillator at the left end. The patterns occur at certain frequencies of oscillation.

$$\lambda f = v$$

$$f_n = \frac{v}{\lambda} = n\frac{v}{2L}$$

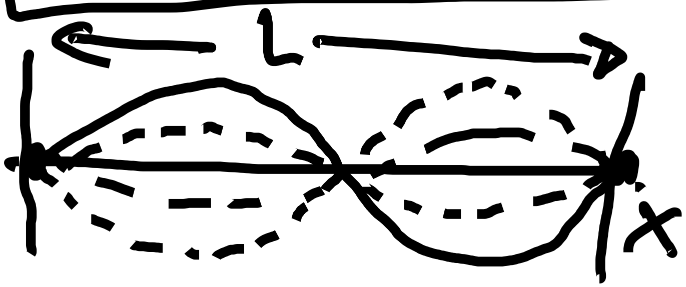
Standing waves

$$v = \sqrt{\frac{\tau}{\mu}}$$

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$y(x,t) = 2y_m \sin(kx)\cos(\omega t)$$

standing wave




$\sin(kx) = 0$ for $x = 0$

$x = L$

$kL = n\pi$ $\quad n = 1, 2, \ldots$

$$n\lambda = 2L$$

$$n \cdot \frac{\lambda}{2} = L$$


$$L k = \frac{2\pi}{\lambda} L = n\pi$$

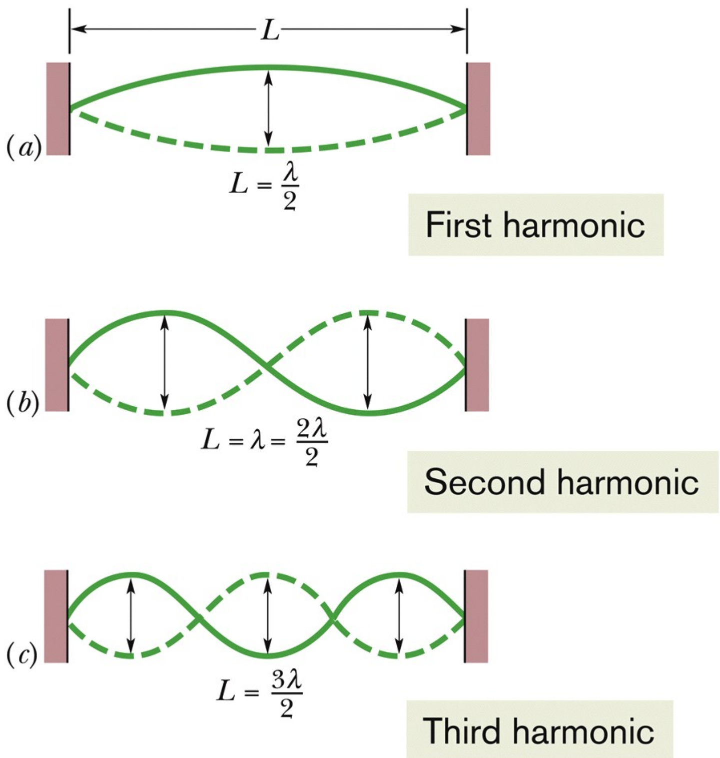
$\lambda = 2L$

$\lambda = 2L/n$

## 16-7 Standing Waves and Resonance

### Harmonics

•Standing waves on a string can be set up by reflection of traveling waves from the ends of the string. If an end is fixed, it must be the position of a node. This limits the frequencies at which standing waves will occur on a given string. Each possible frequency is a **resonant frequency**, and the corresponding standing wave pattern is an oscillation mode. For a stretched string of length L with fixed ends, the resonant frequencies are






$$f = \frac{v}{\lambda} = n\frac{v}{2L}, \quad \text{for } n = 1, 2, 3, \ldots$$

# 16 Summary

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## Waves

- Transverse Waves
- Longitudinal Waves

## Wave Speed

- Angular velocity/ Angular wave number

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f.$$ **Eq. (16-13)**

## Sinusoidal Waves

- Wave moving in positive direction (vector)

$$y(x, t) = y_m \sin(kx - \omega t)$$ **Eq. (16-2)**

## Traveling Waves

- A functional form for traveling waves

$$y(x, t) = h(kx \pm \omega t)$$ **Eq. (16-17)**

# 16 Summary

WILEY

## Powers

- Average Power is given by

$$P_{avg} = \tfrac{1}{2}\mu v \omega^2 y_m^2$$ **Eq. (16-33)**

## Interference of Waves

- Two sinusoidal waves on the same string exhibit interference

$$y'(x,t) = [2y_m \cos \tfrac{1}{2}\phi] \sin(kx - \omega t + \tfrac{1}{2}\phi).$$

**Eq. (16-51)**

## Standing Waves

- The interference of two identical sinusoidal waves moving in opposite directions produces standing waves.

$$y'(x,t) = [2y_m \sin kx] \cos \omega t.$$ **Eq. (16-60)**

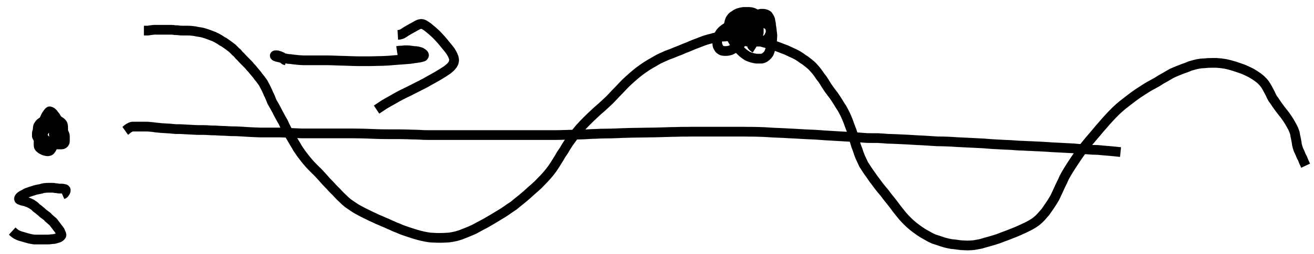
## Resonance

- For a stretched string of length L with fixed ends, the resonant frequencies are

$$f = \frac{v}{\lambda} = n\frac{v}{2L}, \quad \text{for } n = 1, 2, 3, \ldots$$

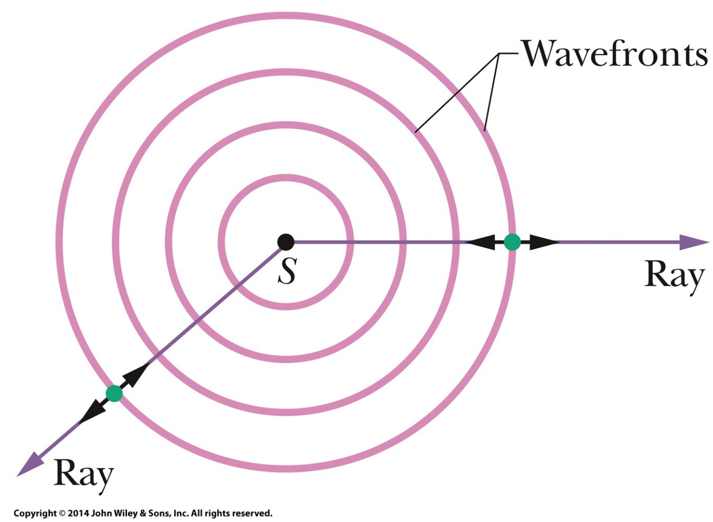
**Eq. (16-66)**

# Waves - II




**Sound waves** are longitudinal mechanical waves that can travel through solids, liquids, or gases.

Point S represents a tiny sound source, called a **point source**, that emits sound waves in all directions. A sound wave travels from a point source S through a three-dimensional medium. The **wavefronts (**surfaces over which the oscillations due to the sound wave have the same value) form spheres centered on S; the **rays** are radial to S. The short, double-headed arrows indicate that elements of the medium oscillate parallel to the rays.



B = bulk modulus : $B = -V\frac{dP}{dV}$ ; $B^{-1} = K = -\frac{1}{V}\frac{dV}{dP}$ ← compressibility

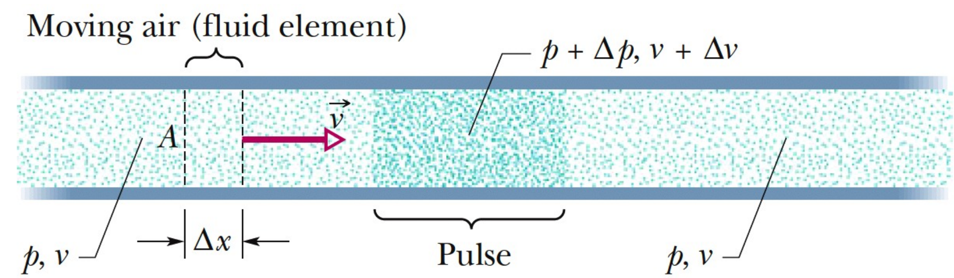
$P = \frac{nRT}{V}$ $\quad B = \frac{nRT}{V} = P$

The speed *v* of a sound wave in a medium having bulk modulus *B* and density *ρ* is

$$v = \sqrt{\frac{B}{\rho}} \qquad \text{(speed of sound)}$$

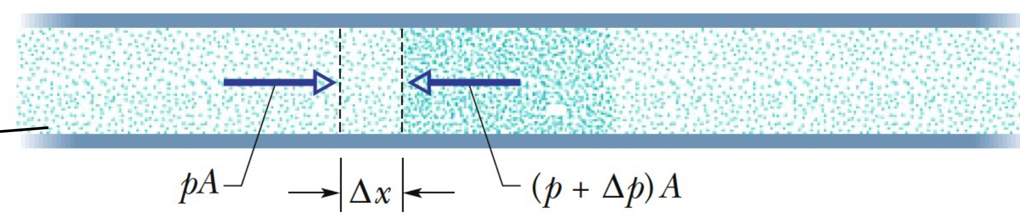
$\rho = \frac{\text{mass}}{\text{volume}}$

Through direct application of Newton's Second law.



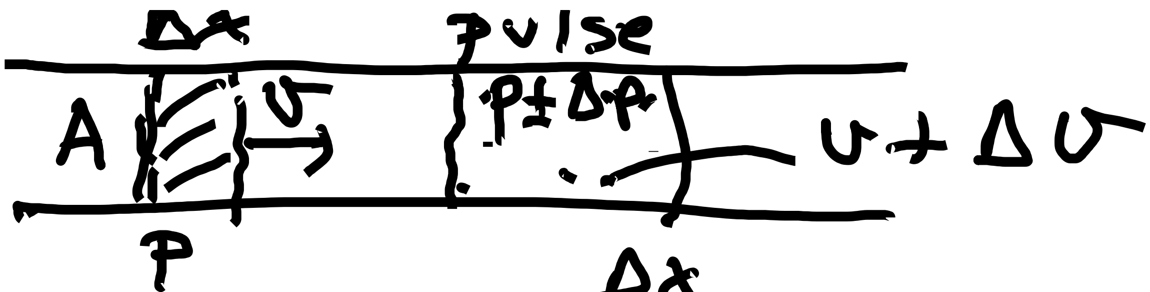



An element of air of width *Δx* moves toward the pulse with speed *v*.






The leading face of the element enters the pulse. The forces acting on the leading and trailing faces (due to air pressure) are shown.



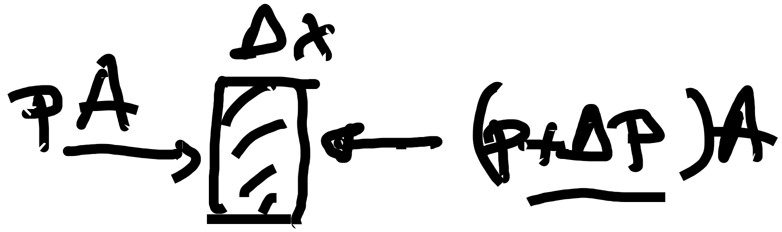

$$B = -V\frac{dp}{dV}$$

$\Delta v < 0$

$V = A v \Delta t$

$\Delta V = A \Delta v \Delta t$

$$\Delta t = \frac{\Delta x}{v}$$




$F = pA - (p+\Delta p)A = -\Delta p \cdot A$ to left

$\Delta m = \rho \Delta V = \rho \Delta x \cdot A = \rho A v \Delta t$

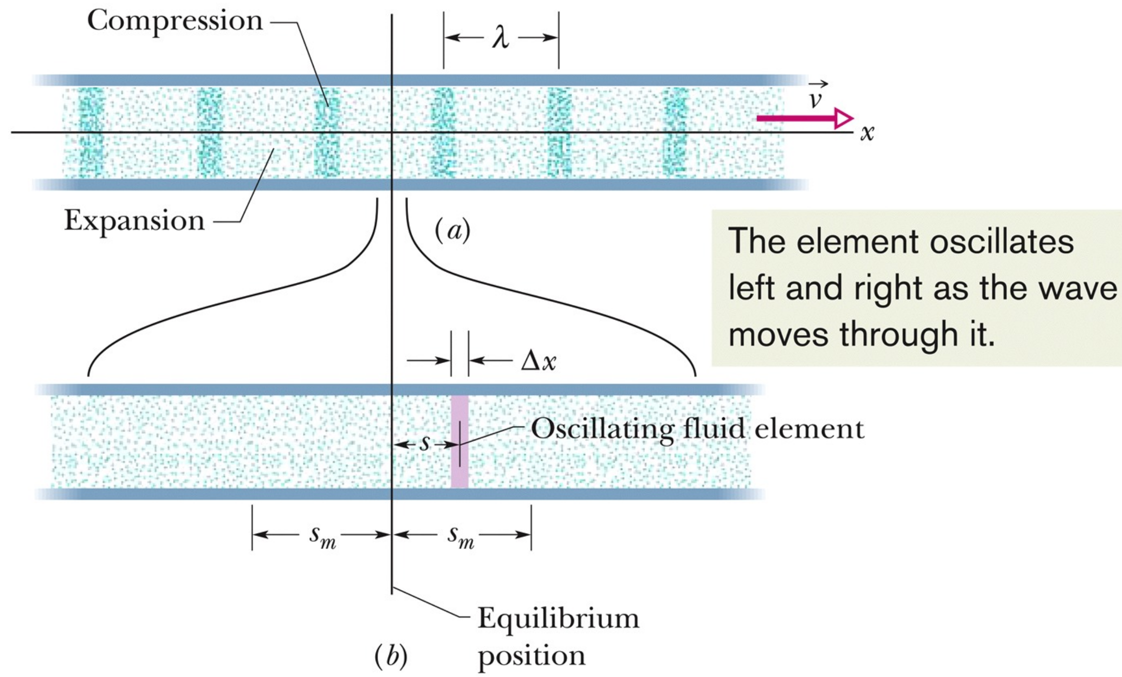
acceleration: $a = \dfrac{\Delta v}{\Delta t}$ $(F = ma)$ $\rho = \dfrac{\text{mass}}{\text{volume}}$

$$-\Delta p A = \rho A v \Delta t \cdot \frac{\Delta v}{\Delta t}$$

$$\boxed{\frac{\Delta v}{v} = \frac{\Delta V}{V}}$$

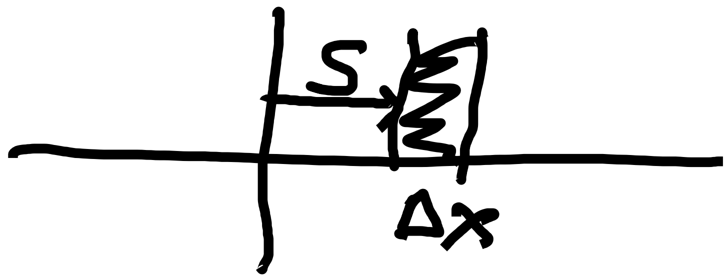
$$\rho v^2 = -\frac{\Delta p}{\Delta v / v} = -\frac{\Delta p}{\frac{\Delta V}{V}} = B \Rightarrow v = \sqrt{\frac{B}{\rho}}$$

(a) A sound wave, traveling through a long air-filled tube with speed *v*, consists of a moving, periodic pattern of expansions and compressions of the air. The wave is shown at an arbitrary instant.






(b) A horizontally expanded view of a short piece of the tube. As the wave passes, an air element of thickness $\Delta x$ oscillates left and right in simple harmonic motion about its equilibrium position. At the instant shown in (b), the element happens to be displaced a distance $s$ to the right of its equilibrium position. Its maximum displacement, either right or left, is $s_m$.




**Displacement**: A sound wave causes a longitudinal displacement s of a mass element in a medium as given by

$$s(x, t) = s_m \cos(kx - \omega t).$$

where $s_m$ is the displacement amplitude (maximum displacement) from equilibrium, $k = 2\pi/\lambda$, and $\omega = 2\pi f$, $\lambda$ and $f$ being the wavelength and frequency, respectively, of the sound wave.

**Pressure**: The sound wave also causes a pressure change $\Delta p$ of the medium from the equilibrium pressure:

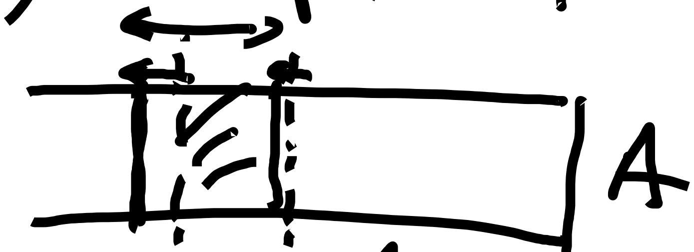
$$\Delta p(x, t) = \Delta p_m \sin(kx - \omega t).$$

where the pressure amplitude is

$$\Delta p_m = (v\rho\omega)s_m.$$

$$s(x,t) = s_m \cos(kx - \omega t)$$

$$\Delta p(x,t) = \Delta p_m \sin(kx - \omega t) \qquad \Delta p_m \ll P$$




$$\Delta P = -B\frac{\Delta V}{V} = -B\frac{ds}{dx}$$

$$\Delta V = A\,\Delta s$$

$$\frac{\Delta s}{\Delta x} = \frac{ds}{dx} = -s_m k \sin(kx - \omega t)$$

$$V = A \cdot \Delta x$$

$$\frac{\Delta V}{V} = \frac{A\,\Delta s}{A\,\Delta x}$$

$$\Delta p = \underbrace{B s_m k}_{} \sin(kx - \omega t)$$

$$v \cdot v \cdot \rho\, s_m k \qquad v = \frac{\omega}{k}$$

$$\boxed{\Delta p_m = v \rho \omega s_m}$$

$$\boxed{v = \frac{\omega}{k}}$$

$$v^2 = \frac{B}{\rho}$$

$$B = v^2 \rho$$

$\Delta p_m = 28\,\text{Pa}$

$1\,\text{atm} = 10^5\,\text{Pa}$

$\Delta p_m = v \rho \omega s_m$

$\omega = 2\pi f$

$\rho = 1.21\,\text{kg/m}^3$

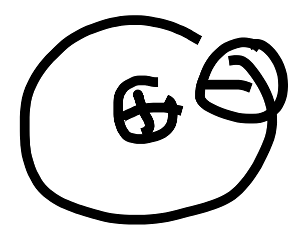
$f = 1000\,\text{Hz}, \quad v = 343\,\text{m/s}$

$$s_m = \frac{\Delta p_m}{v\rho\omega} = \frac{28}{343 \cdot 1.21 \cdot 2\pi \cdot 1000}\,\text{m}$$

$s_m = 1.1 \times 10^{-5}\,\text{m} = 0.01\,\text{mm}$

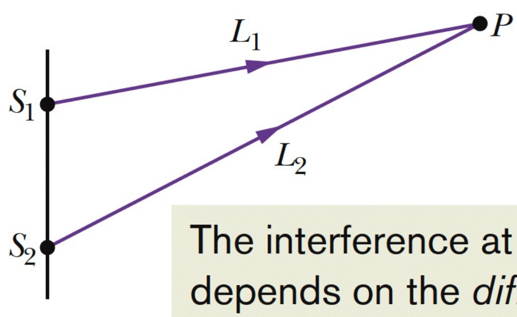
faintest sound: $3 \times 10^{-5}\,\text{Pa} = \Delta p_m$

$s_m = 1.1 \times 10^{-11}\,\text{m}$



H

$\longleftrightarrow 0.5 \times 10^{-10}\,\text{m}$



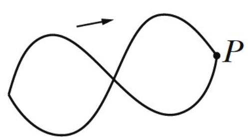

The interference at *P* depends on the *difference* in the path lengths to reach *P*.



Two point sources $S_1$ and $S_2$ emit spherical sound waves in phase. The rays indicate that the waves pass through a common point *P*. The waves (represented with transverse waves) arrive at *P*.

## Fully Destructive Interference

(exactly out of phase)

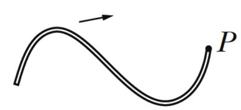



If the difference is equal to, say, 2.5$\lambda$, then the waves arrive exactly out of phase. This is how transverse waves would look.



## Fully Constructive Interference

(exactly in phase)




If the difference is equal to, say, 2.0$\lambda$, then the waves arrive exactly in phase. This is how transverse waves would look.

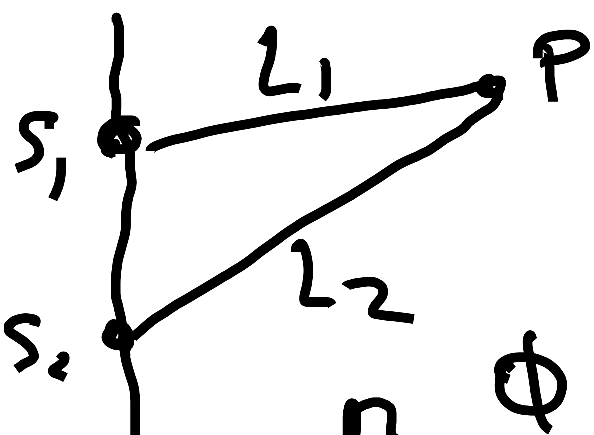
Interference:

$$s_1(x,t) = s_m \cos(kx - \omega t)$$

$$s_2(x,t) = s_m \cos(kx - \omega t + \phi)$$

$$s_{tot} = s_1 + s_2 = 2 s_m \cos\left(\frac{\phi}{2}\right) \cos\left(kx - \omega t + \frac{\phi}{2}\right)$$

Amplitude $|s_{tot}| = \left|2 s_m \cos\frac{\phi}{2}\right|$




$$\Delta L = |L_2 - L_1|$$

$$\phi = k\,\Delta L = \frac{2\pi}{\lambda}\Delta L$$

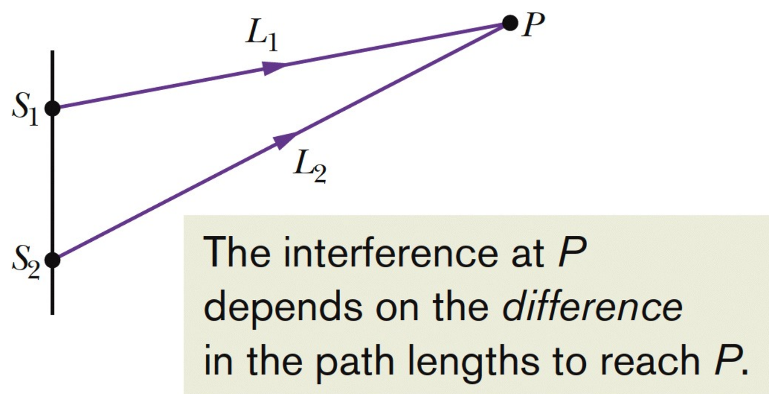
$$n \quad \frac{\phi}{2\pi} = \frac{\Delta L}{\lambda}$$

constructive $\phi = 2n\pi$

$$\frac{2\pi}{\lambda}\Delta L = 2n\pi$$

$$\Delta L = n\lambda$$

# Path Length Difference






- The interference of two sound waves with identical wavelengths passing through a common point depends on their phase difference there $\phi$. If the sound waves were emitted in phase and are traveling in approximately the same direction, $\phi$ is given by

$$\phi = \frac{\Delta L}{\lambda} 2\pi.$$

where $\Delta L$ is their **path length difference**.

- **Fully constructive interference** occurs when $\phi$ is an integer and multiple of $2\pi$,

$$\phi = m(2\pi), \qquad \text{for } m = 0, 1, 2, \ldots,$$

and, equivalently, when $\Delta L$ is related to wavelength $\lambda$ by

$$\frac{\Delta L}{\lambda} = 0, 1, 2, \ldots \quad \text{(fully constructive interference).}$$

- **Fully destructive interference** occurs when $\phi$ is an odd multiple of $\pi$,

$$\phi = (2m + 1)\pi, \qquad \text{for } m = 0, 1, 2, \ldots,$$

and, equivalently, when $\Delta L$ is related to wavelength $\lambda$ by

$$\frac{\Delta L}{\lambda} = 0.5, 1.5, 2.5, \ldots \quad \text{(fully destructive interference).}$$