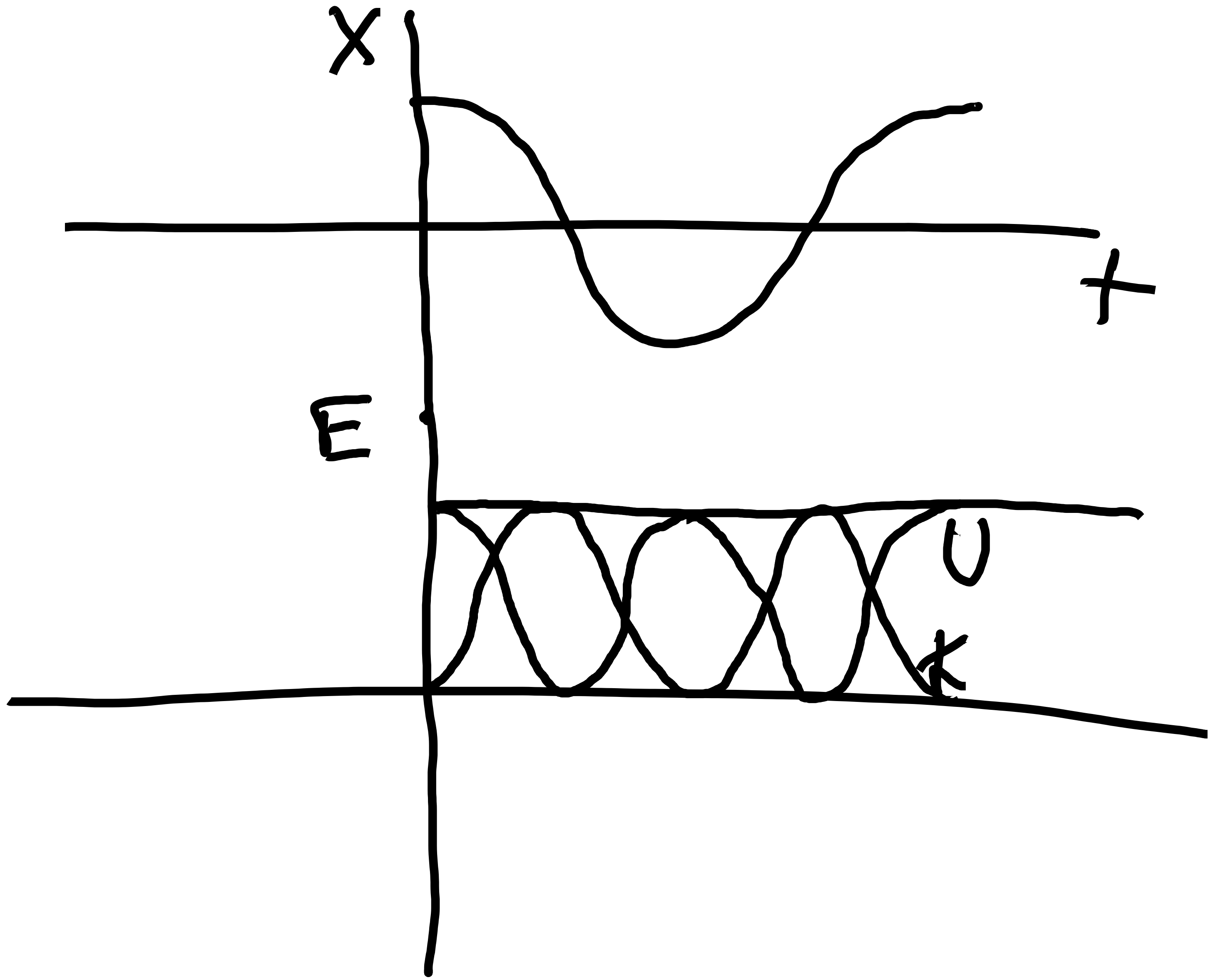

x
t
E
U
K

15-2 Energy in Simple Harmonic Motion

- Write the functions for kinetic and potential energy:

$$U(t) = \tfrac{1}{2}kx^2 = \tfrac{1}{2}kx_m^2 \cos^2(\omega t + \phi).$$

Eq. (15-18)

$$K(t) = \tfrac{1}{2}mv^2 = \tfrac{1}{2}kx_m^2 \sin^2(\omega t + \phi).$$

Eq. (15-20)

- Their sum is defined by:

$$E = U + K = \tfrac{1}{2}kx_m^2.$$

Eq. (15-21)

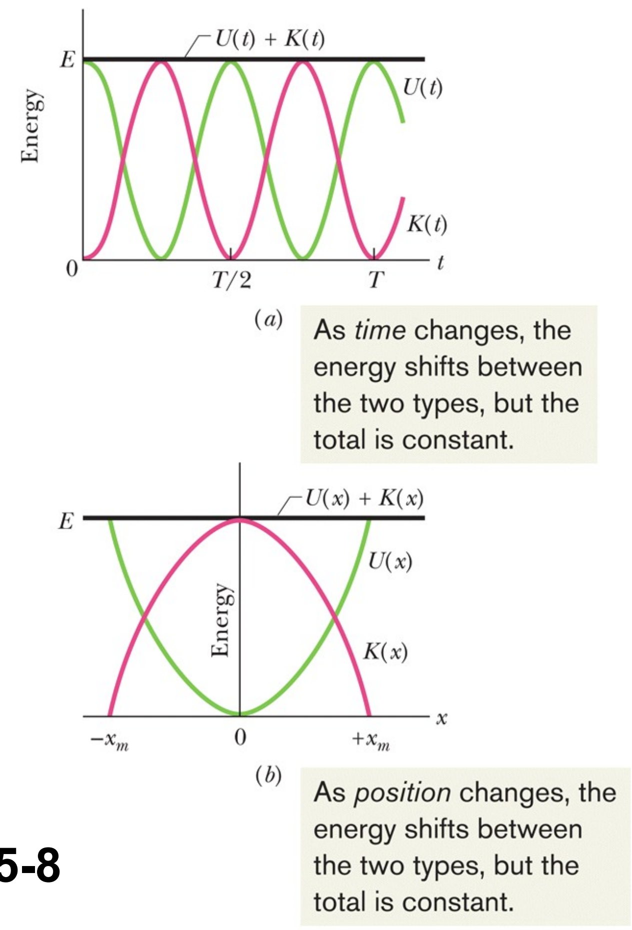


Figure 15-8

15-2 Energy in Simple Harmonic Motion

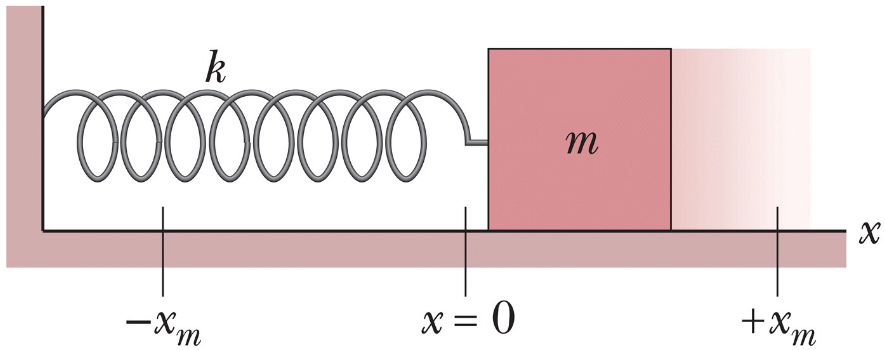


Figure 15-7

Checkpoint 4

In Fig. 15-7, the block has a kinetic energy of 3 J and the spring has an elastic potential energy of 2 J when the block is at $x = +2.0$ cm. (a) What is the kinetic energy when the block is at $x = 0$? What is the elastic potential energy when the block is at (b) $x = -2.0$ cm and (c) $x = -x_m$?

Answer: (a) 5 J (b) 2 J (c) 5 J

15-3 An Angular Simple Harmonic Oscillator

Learning Objectives

15.23 Describe the motion of an angular simple harmonic oscillator.

15.24 For an angular simple harmonic oscillator, apply the relationship between the torque τ and the angular displacement θ (from equilibrium).

15.25 For an angular simple harmonic oscillator, apply the relationship between the period T (or frequency f), the rotational inertia I, and the torsion constant κ.

15.26 For an angular simple harmonic oscillator at any instant, apply the relationship between the angular acceleration α, the angular frequency ω, and the angular displacement θ.

15-3 An Angular Simple Harmonic Oscillator

- A **torsion pendulum**: elasticity from a twisting wire
- Moves in **angular simple harmonic motion**

$$\tau = -\kappa\theta. \quad \text{Eq. (15-22)}$$

- κ is called the torsion constant
- Angular form of Hooke's law
- Replace linear variables with their angular analogs and
- we find:

$$T = 2\pi\sqrt{\frac{I}{\kappa}} \quad \text{Eq. (15-23)}$$

$$\omega = \sqrt{\frac{\kappa}{I}}$$

$$T = \frac{2\pi}{\omega}$$

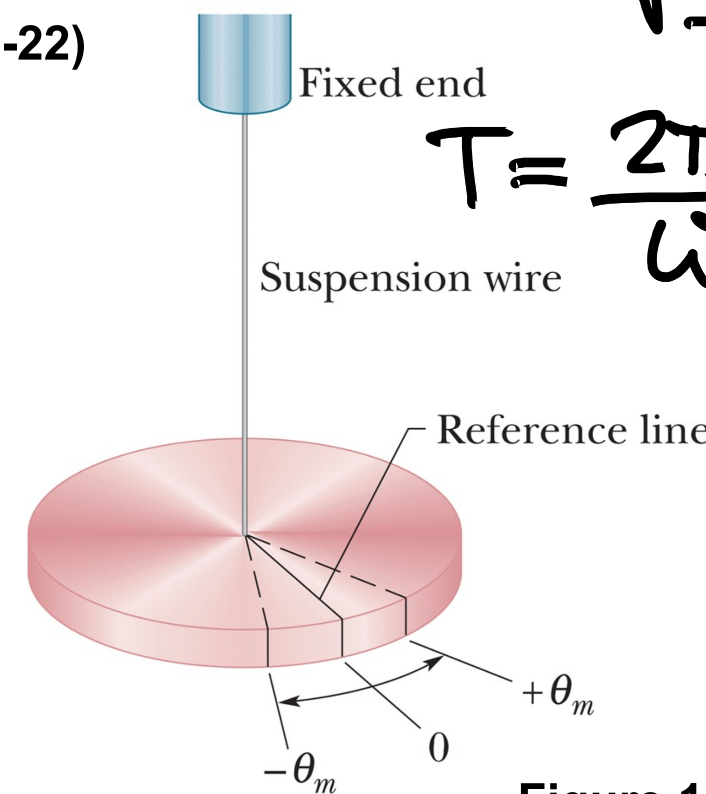


Figure 15-9

15-4 Pendulums, Circular Motion

Learning Objectives

15.27 Describe the motion of an oscillating simple pendulum.

15.28 Draw a free-body diagram.

15.29-31 Distinguish between a simple and physical pendulum, and relate their variables.

15.32 Find angular frequency from torque and angular displacement or acceleration and displacement.

15.33 Distinguish angular frequency from $d\theta/dt$.

15.34 Determine phase and amplitude.

15.35 Describe how free-fall acceleration can be measured with a pendulum.

15.36 For a physical pendulum, find the center of the oscillation.

15.37 Relate SHM to uniform circular motion.

15-4 Pendulums, Circular Motion

- A **simple pendulum**: a *bob* of mass *m* suspended from an unstretchable, massless string
- Bob feels a restoring torque:

$$\tau = -L(F_g \sin\theta),$$ **Eq. (15-24)**

- Relating this to moment of inertia:

$$\alpha = -\frac{mgL}{I}\theta.$$ **Eq. (15-26)**

- Angular acceleration proportional to position but opposite in sign

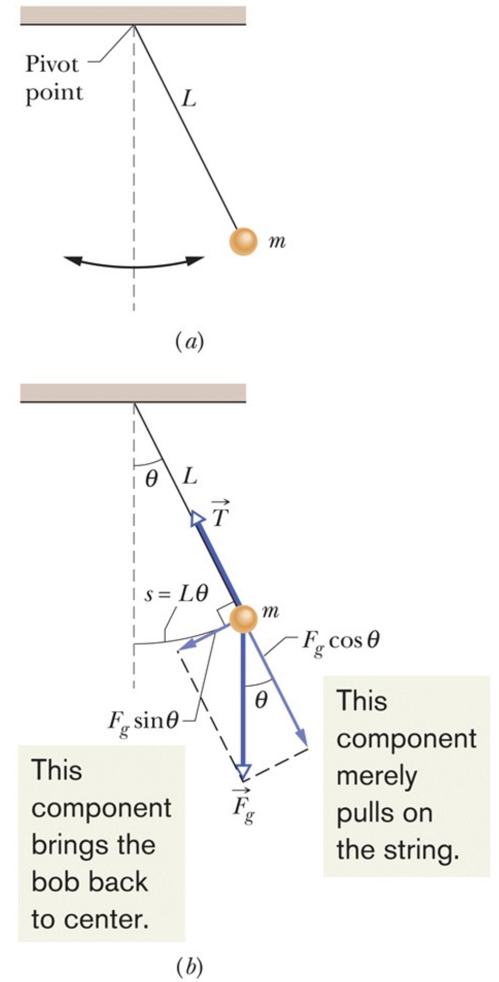
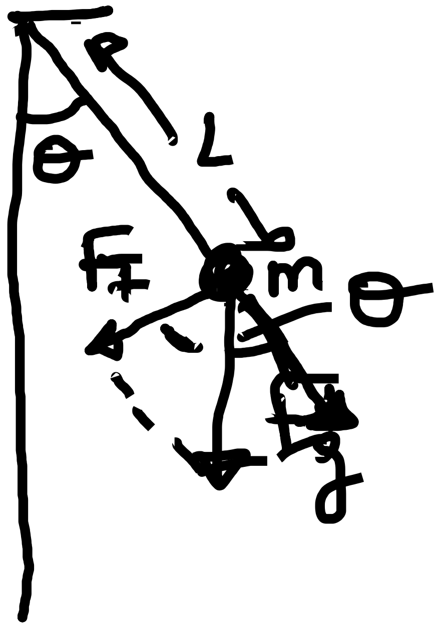


Figure 15-11



$$F_g = mg$$

$$F_t = F_g \sin\theta$$

$$m\frac{d^2s}{dt^2} = -mg\sin\theta$$

$$\boxed{ds = L\,d\theta}$$

$$\cancel{m}L\frac{d^2\theta}{dt^2} = -\cancel{m}g\sin\theta$$

$$\ddot{\theta} = -\frac{g}{L}\sin\theta \approx -\frac{g}{L}\theta$$

$$\omega = \sqrt{\frac{g}{L}}$$

$$ds = r\,d\theta$$

torque

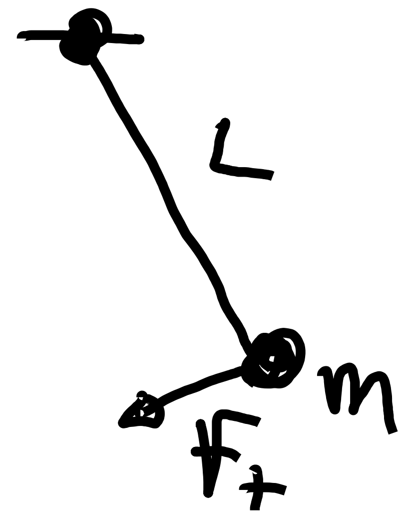
$$\tau = -L F_t = -L F_g \sin\theta$$

$$\tau = I \ddot{\theta}$$

$$I \ddot{\theta} = -Lmg\sin\theta \approx -Lmg\theta$$

$$\ddot{\theta} = -\frac{Lmg}{I}\theta \Rightarrow \omega = \sqrt{\frac{L \cdot m \cdot g}{I}}$$

$$I = \int r^2 dm = mL^2 \Rightarrow \omega = \sqrt{\frac{g}{L}}$$



15-4 Pendulums, Circular Motion

- **Angular amplitude** θ_m of the motion must be small
- The angular frequency is:

$$\omega = \sqrt{\frac{mgL}{I}}.$$

- The period is (for simple pendulum, $I = mL^2$):

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Eq. (15-28)

- A **physical pendulum** has a complicated mass distribution

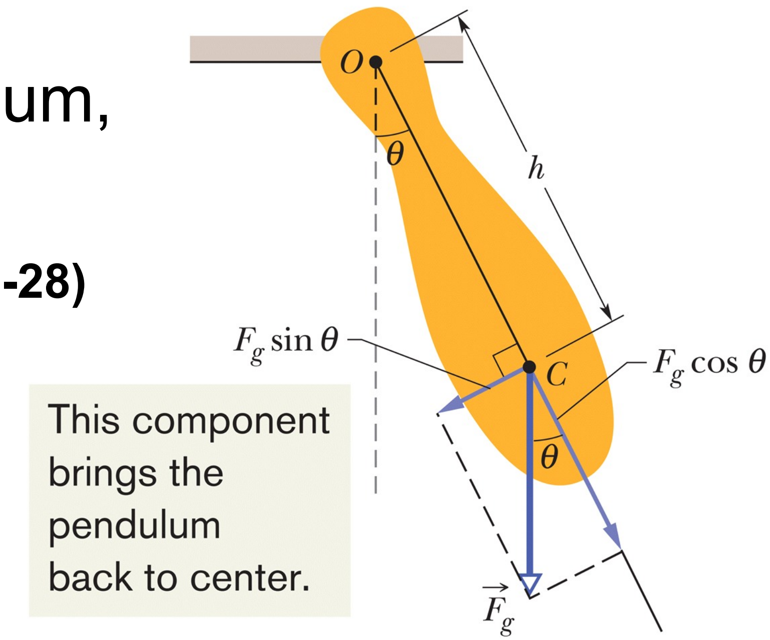


Figure 15-12

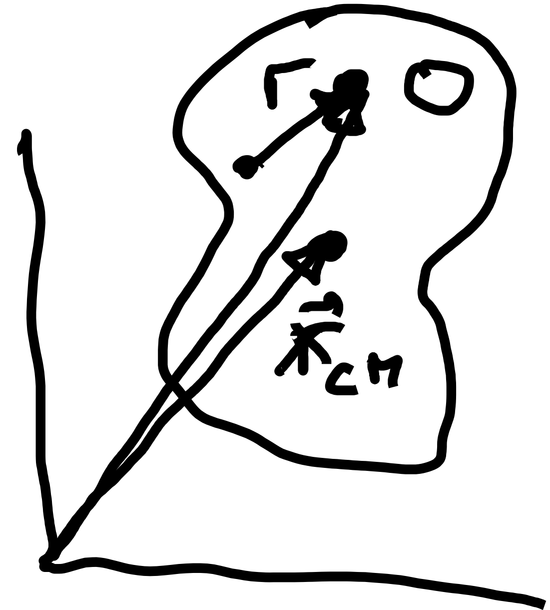
15-4 Pendulums, Circular Motion

- An analysis is the same except rather than length L we have distance h to the com, and I will be particular to the mass distribution
- The period is:

$$T = 2\pi\sqrt{\frac{I}{mgh}} \qquad \textbf{Eq. (15-29)}$$

- A physical pendulum will not show SHM if pivoted about its com
- The *center of oscillation* of a physical pendulum is the length L_0 of a simple pendulum with the same period

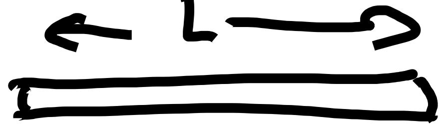
$$I = \int d^3r \, \rho(r) \, r^2$$



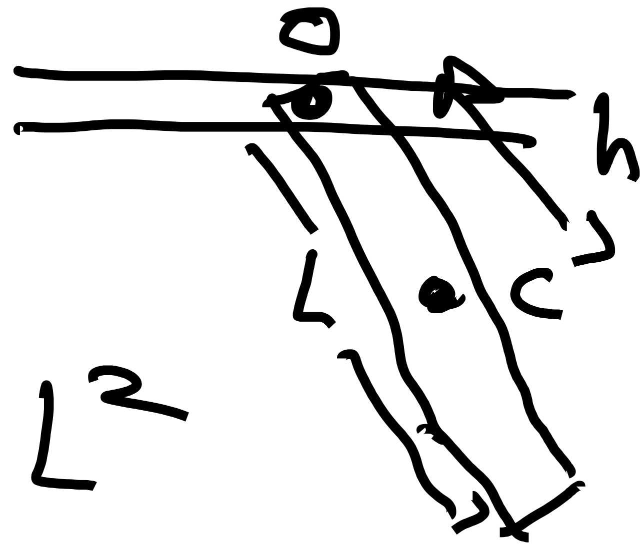
$$I = I_{cm} + m h^2$$

Measure g

Uniform rod



$$h = \frac{1}{2} L$$



$$I_{cm} = \frac{1}{12} m L^2$$

$$I = \frac{1}{12} m L^2 + m \frac{L^2}{4} = \frac{1}{3} m L^2$$

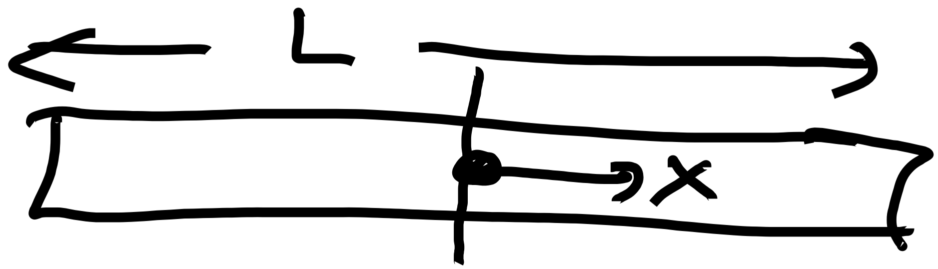
$$I = \frac{1}{3} m L^2$$

$$\omega = \sqrt{\frac{mgh}{I}} \Rightarrow T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{I}{mgh}}$$

$$\frac{2\pi}{T} = \sqrt{\frac{mgh}{I}} \Rightarrow g = \frac{I}{mh}\left(\frac{2\pi}{T}\right)^2$$

$$g = \frac{1}{3} \frac{\cancel{m} L^2}{\cancel{m} h} \left(\frac{2\pi}{T}\right)^2 = \frac{8\pi^2 L}{3T^2}$$

$h = L/2$



$$\rho = \frac{m}{L}$$

$$I_{cm} = \int dx\, x^2 \cdot \rho = \frac{m}{L} \int_{-\frac{L}{2}}^{L/2} dx\, x^2 =$$

$$= \frac{m}{L} \left. \frac{x^3}{3} \right]_{-\frac{L}{2}}^{L/2} = \frac{m}{L} \cdot \frac{L^3}{3 \cdot 8} \cdot 2 = \frac{mL^2}{12}$$

15-4 Pendulums, Circular Motion

- A physical pendulum can be used to determine free-fall acceleration *g*
- Assuming the pendulum is a uniform rod of length L:

$$I = I_{\text{com}} + mh^2 = \tfrac{1}{12}mL^2 + m(\tfrac{1}{2}L)^2 = \tfrac{1}{3}mL^2.$$

Eq. (15-30)

- Then solve Eq. 15-29 for *g*:

$$g = \frac{8\pi^2 L}{3T^2}.$$

Eq. (15-31)

Checkpoint 5

Three physical pendulums, of masses m_0, $2m_0$, and $3m_0$, have the same shape and size and are suspended at the same point. Rank the masses according to the periods of the pendulums, greatest first.

Answer: All the same: mass does not affect the period of a pendulum

15-4 Pendulums, Circular Motion

- Simple harmonic motion is circular motion viewed edge-on

Simple harmonic motion is the projection of uniform circular motion on a diameter of the circle in which the circular motion occurs.

- Figure 15-15 shows a reference particle moving in uniform circular motion
- Its angular position at any time is $\omega t + \varphi$

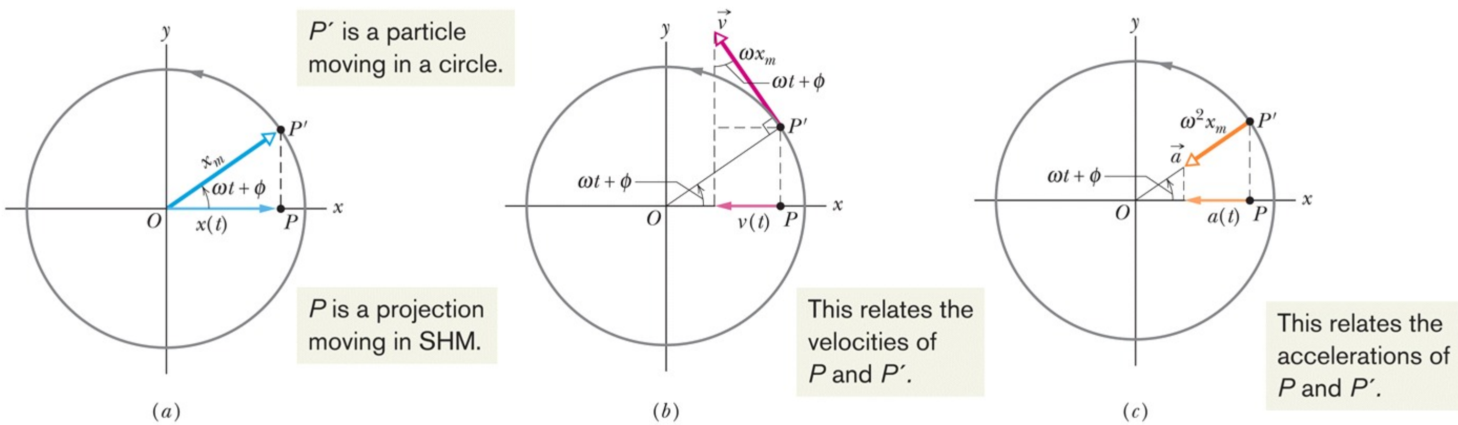
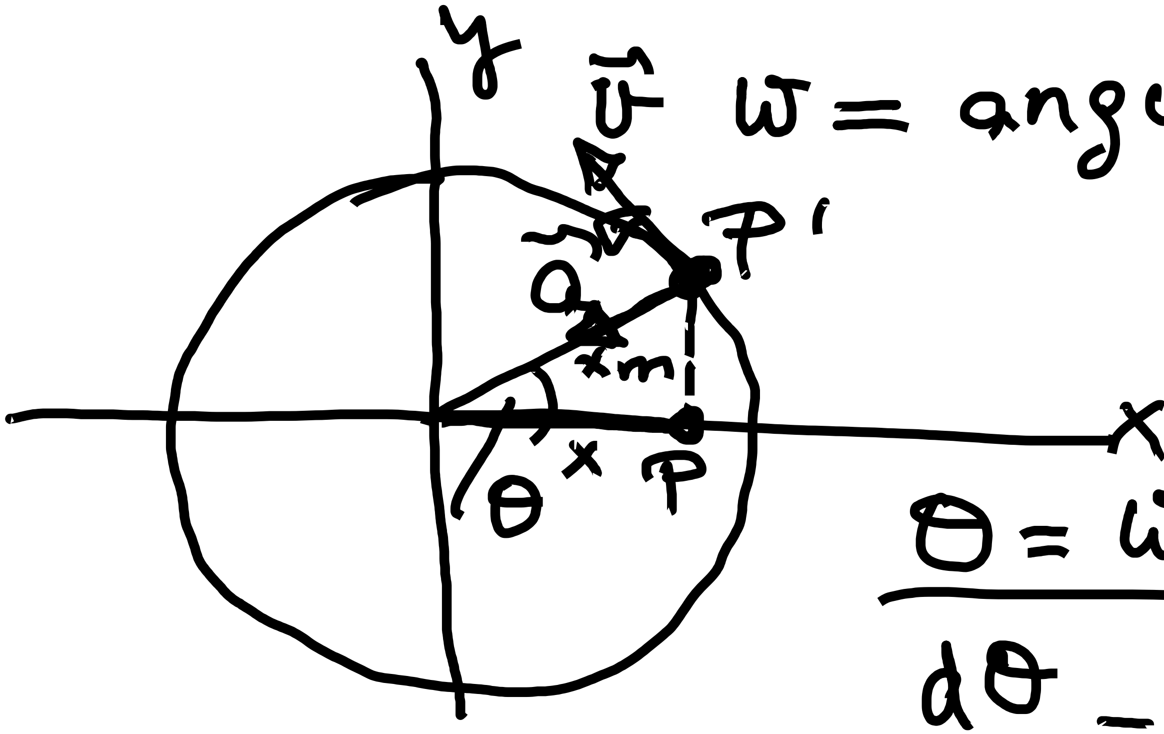


Figure 15-15

$x = x_m \cos(\omega t + \phi)$



ω = angular velocity

$R = x_m$

$a = v^2 / x_m$

$\theta = \omega t + \phi$

$\frac{d\theta}{dt} = \omega$ = angular velocity

ω = const.

$x = x_m \cos\theta$

$x(t) = x_m \cos(\omega t + \phi)$

$v(t) = -\omega x_m \sin(\omega t + \phi)$

$a(t) = -\omega^2 x_m \cos(\omega t + \phi)$.

15-4 Pendulums, Circular Motion

- Projecting its position onto *x*:

$$x(t) = x_m \cos(\omega t + \phi), \qquad \text{Eq. (15-36)}$$

- Similarly with velocity and acceleration:

$$v(t) = -\omega x_m \sin(\omega t + \phi), \qquad \text{Eq. (15-37)}$$

$$a(t) = -\omega^2 x_m \cos(\omega t + \phi), \qquad \text{Eq. (15-38)}$$

- We indeed find this projection is simple harmonic motion