14-6 The Equation of Continuity

- Motion of *real fluids* is complicated and poorly understood (e.g., turbulence)
- We discuss motion of an **ideal fluid**

1. Steady flow: Laminar flow, the velocity of the moving fluid at any fixed point does not change with time

2. Incompressible flow: The ideal fluid density has a constant, uniform value

3. Nonviscous flow: Viscosity is, roughly, resistance to flow, fluid analog of friction. No resistive force here

4. Irrotational flow: May flow in a circle, but a dust grain suspended in the fluid will not rotate about com

14-6 The Equation of Continuity

- Visualize fluid flow by adding a *tracer*
- Each bit of tracer (see figure 14-13) follows a *streamline*
- A streamline is the path a tiny element of fluid follows
- Velocity is tangent to streamlines, so they can never intersect (then 1 point would experience 2 velocities)

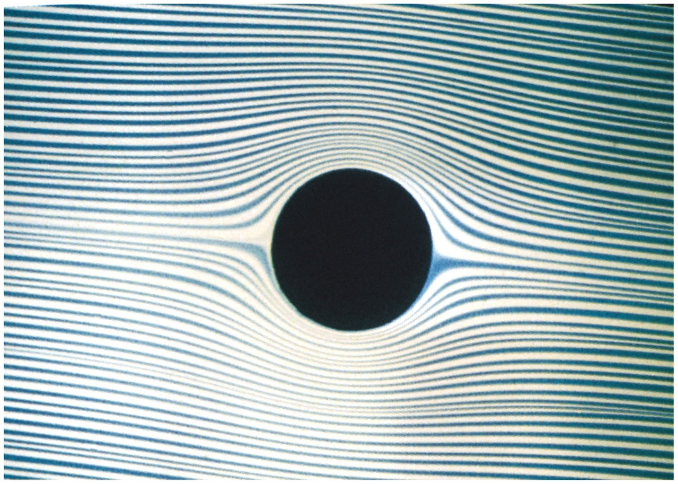


Figure 14-13

Courtesy D. H. Peregrine, University of Bristol

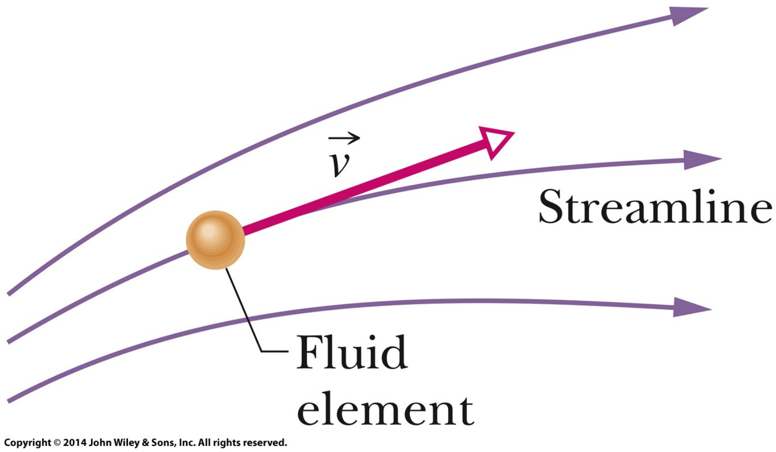


Figure 14-14

14-7 Bernoulli's Equation

- Figure 14-19 represents a tube through which an ideal fluid flows
- Applying the conservation of energy to the equal volumes of input and output fluid:

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2.$$

Eq. (14-28)

- The $\frac{1}{2}\rho v^2$ term is called the fluid's **kinetic energy density**

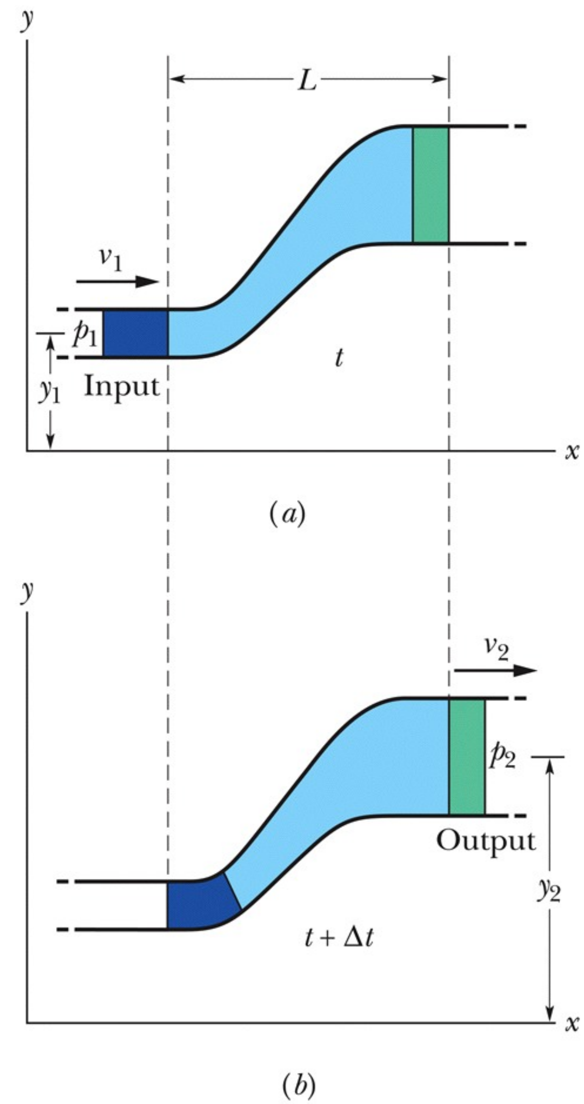


Figure 14-19

$W = \Delta K$ work-energy theorem

$\Delta K = \frac{1}{2} \rho \Delta V (v_2^2 - v_1^2)$

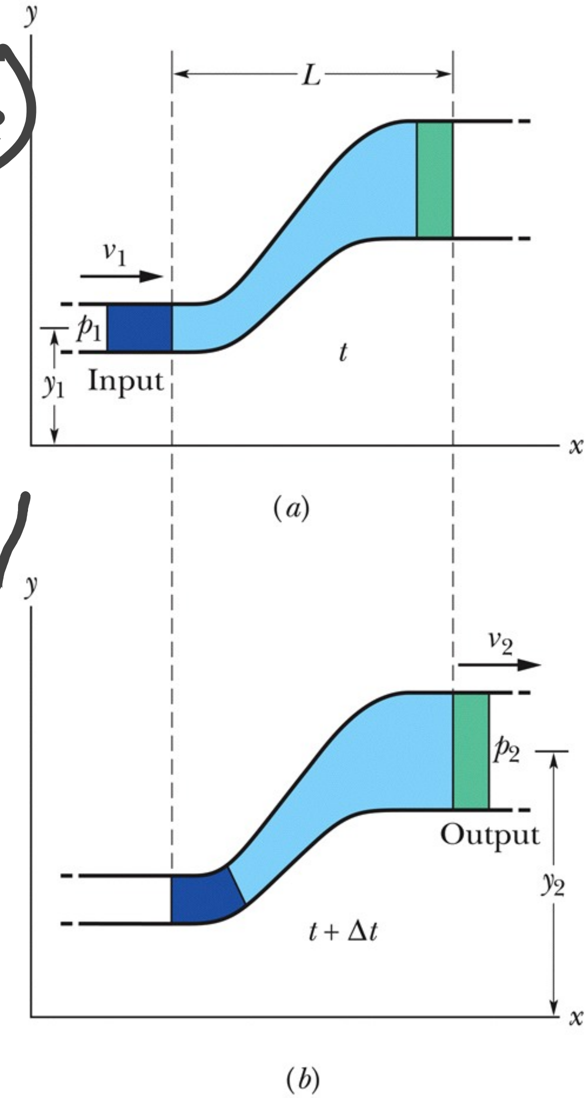
$W = W_g + W_p$ (gravity + pressure)

$W_g = \rho \Delta V g (y_1 - y_2)$

$W_p = F_1 \Delta x_1 - F_2 \Delta x_2 = (p_1 - p_2) \Delta V$

$W_g + W_p = \Delta K \Longrightarrow$

$p_1 + \rho g y_1 + \frac{1}{2} \rho v_1^2 =$ same w/ 2



14-7 Bernoulli's Equation

- Equivalent to Eq. 14-28, we can write:

$$p + \frac{1}{2}\rho v^2 + \rho g y = \text{a constant}$$ **Eq. (14-29)**

- These are both forms of **Bernoulli's Equation**
- Applying this for a fluid at rest we find Eq. 14-7
- Applying this for flow through a horizontal pipe:

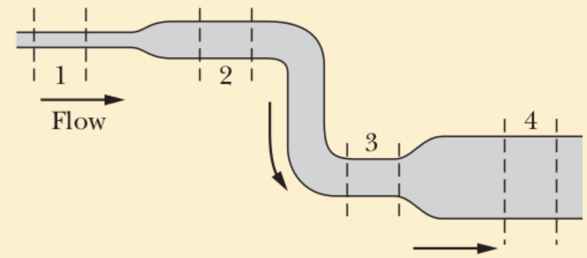
$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2,$$ **Eq. (14-30)**

If the speed of a fluid element increases as the element travels along a horizontal streamline, the pressure of the fluid must decrease, and conversely.

14-7 Bernoulli's Equation

Checkpoint 4

Water flows smoothly through the pipe shown in the figure, descending in the process. Rank the four numbered sections of pipe according to (a) the volume flow rate R_V through them, (b) the flow speed v through them, and (c) the water pressure p within them, greatest first.



Answer: (a) all the same volume flow rate
(b) 1, 2 & 3, 4
(c) 4, 3, 2, 1

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2.$$

Sample Problem 14.07 Bernoulli principle for a leaky water tank

Bernoulli principle for a leaky water tank

In the old West, a desperado fires a bullet into an open water tank (Fig. 14-20), creating a hole a distance h below the water surface. What is the speed v of the water exiting the tank?

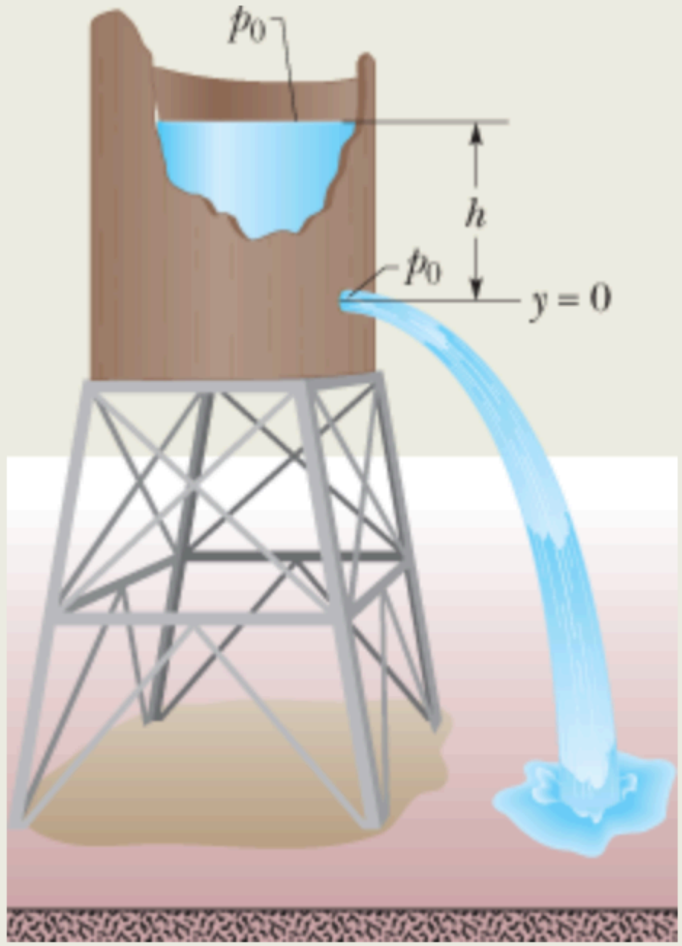


Figure 14-20

Water pours through a hole in a water tank, at a distance h below the water surface. The pressure at the water surface and at the hole is atmospheric pressure p_0.

(1) This situation is essentially that of water moving (downward) with speed v_0 through a wide pipe (the tank) of cross-sectional area A and then moving (horizontally) with speed v through a narrow pipe (the hole) of cross-sectional area a. (2) Because the water flowing through the wide pipe must entirely pass through the narrow pipe, the volume flow rate R_V must be the same in the two "pipes." (3) We can also relate v to v_0 (and to h) through Bernoulli's equation (Eq. 14-28).

Calculations:

From Eq. 14-24,

$$R_V = av = Av_0$$

and thus

$$v_0 = \frac{a}{A}v .$$

Because $a \ll A$, we see that $v_0 \ll v$. To apply Bernoulli's equation, we take the level of the hole as our reference level for measuring elevations (and thus gravitational potential energy). Noting that the pressure at the top of the tank and at the bullet hole is the atmospheric pressure p_0 (because both places are exposed to the atmosphere), we write Eq. 14-28 as

$$p_0 + \frac{1}{2}\rho v_0^2 + \rho gh = p_0 + \frac{1}{2}\rho v^2 + \rho g(0) . \qquad (14\text{-}39)$$

(Here the top of the tank is represented by the left side of the equation and the hole by the right side. The zero on the right indicates that the hole is at our reference level.) Before we solve Eq. 14-39 for v, we can use our result that $v_0 \ll v$ to simplify it: We assume that v_0^2, and thus the term $\frac{1}{2}\rho v_0^2$ in Eq. 14-39, is negligible relative to the other terms, and we drop it. Solving the remaining equation for v then yields

$$v = \sqrt{2gh} . \qquad \text{(Answer)}$$

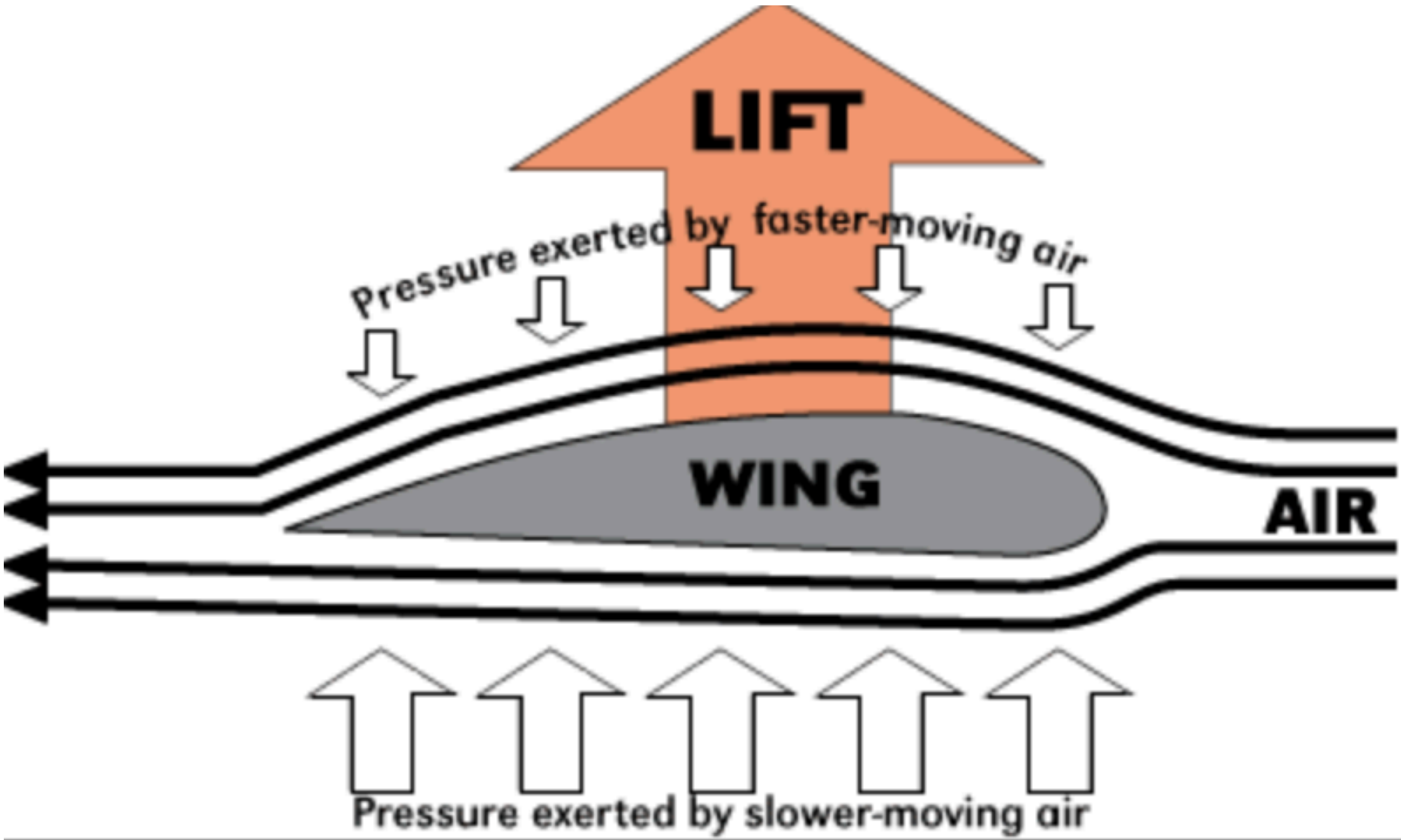
This is the same speed that an object would have when falling a height h from rest.

$$\frac{1}{2} v_0^2 + gh = \frac{1}{2} v^2$$

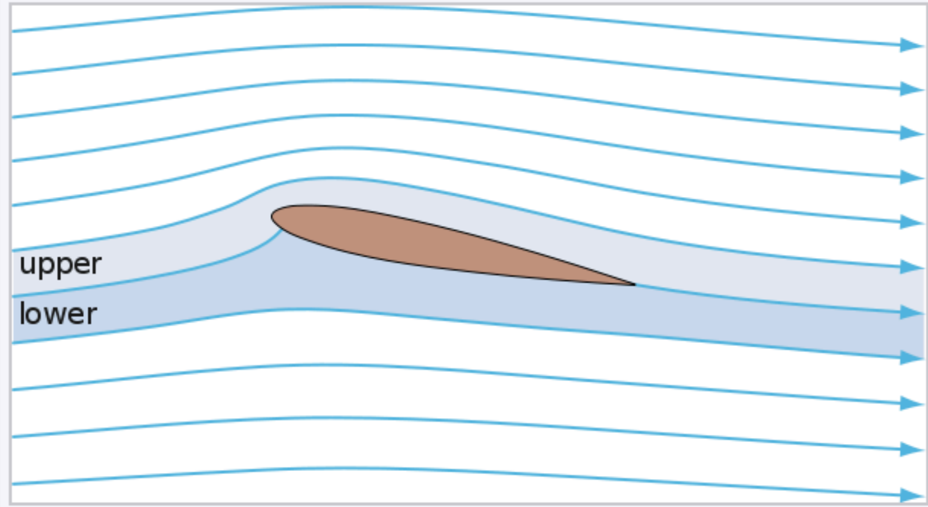
$$v_0 = \frac{A}{A_0} v \Longrightarrow \frac{1}{2} v^2 \left(1 - \frac{A^2}{A_0^2}\right) = gh$$

$$\Longrightarrow v = \left(\frac{2gh}{1 - A^2/A_0^2}\right)^{1/2}$$ for $A << A_0$

$$v \approx \sqrt{2gh}$$


LIFT
Pressure exerted by faster-moving air
WING
AIR
Pressure exerted by slower-moving air

Starting with the flow pattern observed in both theory and experiments, the increased flow speed over the upper surface can be explained in terms of streamtube pinching and conservation of mass.[25]



Streamlines and streamtubes around an airfoil generating lift. Note the narrower streamtubes above and the wider streamtubes below.

Assuming that the air is incompressible, the rate of volume flow (e.g. liters or gallons per minute) must be constant within each streamtube since matter is not created or destroyed. If a streamtube becomes narrower, the flow speed must increase in the narrower region to maintain the constant flow rate. This is an application of the principle of conservation of mass.[26]

The upper stream tubes constrict as they flow up and around the airfoil. Conservation of mass says that the flow speed must increase as the stream tube area decreases.[25] Similarly, the lower stream tubes expand and the flow slows down.

From Bernoulli's principle, the pressure on the upper surface where the flow is moving faster is lower than the pressure on the lower surface where it is moving slower. This pressure difference creates a net aerodynamic force, pointing upward.

14 Summary

Density

$$\rho = \frac{m}{V}$$

Eq. (14-2)

Fluid Pressure

- A substance that can flow
- Can exert a force perpendicular to its surface

$$p = \frac{F}{A}$$

Eq. (14-4)

Pressure Variation with Height and Depth

$$p = p_0 + \rho g h$$

Eq. (14-8)

Pascal's Principle

- A change in pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and to the walls of the containing vessel

14 Summary

Archimedes' Principle

$$F_b = m_f g \qquad \text{(buoyant force)},$$

Eq. (14-16)

$$\text{weight}_{\text{app}} = \text{weight} - F_b$$

Eq. (14-19)

Flow of Ideal Fluids

$$R_V = Av = \text{a constant}$$

Eq. (14-24)

$$R_m = \rho R_V = \rho A v = \text{a constant}$$

Eq. (14-25)

Bernoulli's Equation

$$p + \tfrac{1}{2}\rho v^2 + \rho g y = \text{a constant}$$

Eq. (14-29)

Oscillations

$$x(t+T) = x\left(t+\frac{2\pi}{\omega}\right) = x_m \cos(\omega t + 2\pi + \phi)$$

15-1 Simple Harmonic Motion

- The **frequency** of an oscillation is the number of times per second that it completes a full oscillation (cycle)
- Unit of hertz: 1 Hz = 1 oscillation per second
- The time in seconds for one full cycle is the **period**

$$T = \frac{1}{f}. \quad \text{Eq. (15-2)}$$

- Any motion that repeats regularly is called periodic
- **Simple harmonic motion** is periodic motion that is a sinusoidal function of time

$$x(t) = x_m \cos(\omega t + \phi) \quad \text{Eq. (15-3)}$$

15-1 Simple Harmonic Motion

- The value written x_m is how far the particle moves in either direction: the **amplitude**
- The argument of the cosine is the **phase**
- The constant φ is called the **phase angle** or phase constant
- It adjusts for the initial conditions of motion at $t = 0$
- The **angular frequency** is written ω

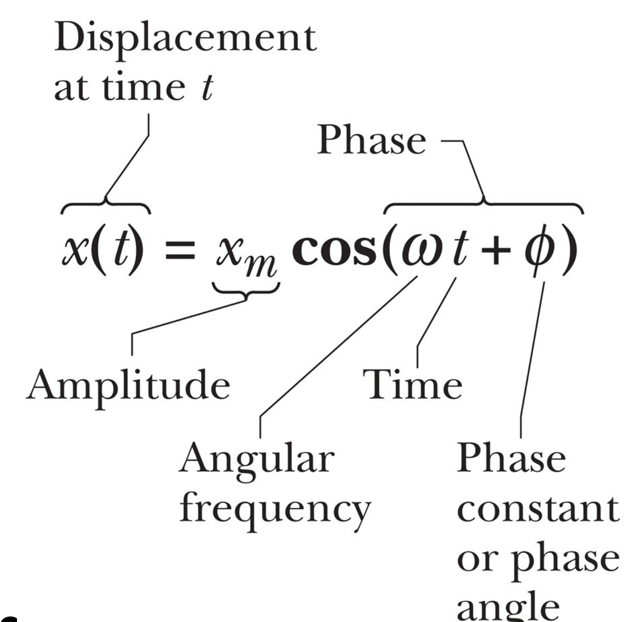


Figure 15-3

15-1 Simple Harmonic Motion

- The angular frequency has the value:

$$\omega = \frac{2\pi}{T} = 2\pi f.$$

Eq. (15-5)

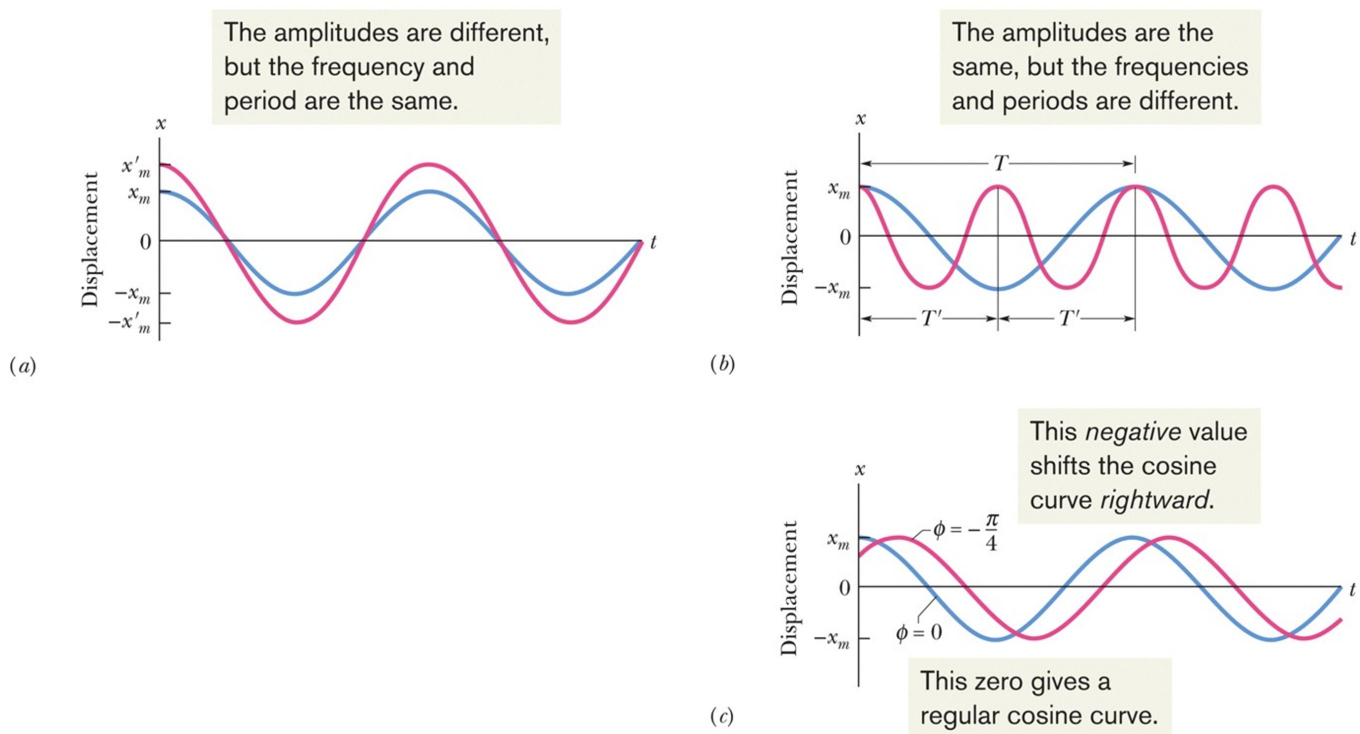


Figure 15-5

$$x(t) = x_m \cos(\omega t + \phi)$$

$$v(t) = \frac{dx}{dt} = -\omega x_m \sin(\omega t + \phi)$$

$$a(t) = \frac{d^2x}{dt^2} = -\omega^2 x_m \cos(\omega t + \phi)$$

$$\Rightarrow a(t) = -\omega^2 x(t)$$

~~F(t)~~ $m\,a(t) = -m\omega^2 x(t)$

$$F = -m\omega^2 x$$

$$\boxed{F = -kx}$$

$$k = m\omega^2 \qquad \omega = \sqrt{\frac{k}{m}}$$

15-1 Simple Harmonic Motion

- We can apply Newton's second law

$$F = ma = m(-\omega^2 x) = -(m\omega^2)x. \qquad \textbf{Eq. (15-9)}$$

- Relating this to Hooke's law we see the similarity

> Simple harmonic motion is the motion of a particle when the force acting on it is proportional to the particle's displacement but in the opposite direction.

- **Linear simple harmonic oscillation** (*F* is proportional to *x* to the first power) gives:

$$\omega = \sqrt{\frac{k}{m}} \quad \text{(angular frequency).} \qquad \textbf{Eq. (15-12)}$$

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \text{(period).} \qquad \textbf{Eq. (15-13)}$$

15-2 Energy in Simple Harmonic Motion

Learning Objectives

15.19 For a spring-block oscillator, calculate the kinetic energy and elastic potential energy at any given time.

15.20 Apply the conservation of energy to relate the total energy of a spring-block oscillator at one instant to the total energy at another instant.

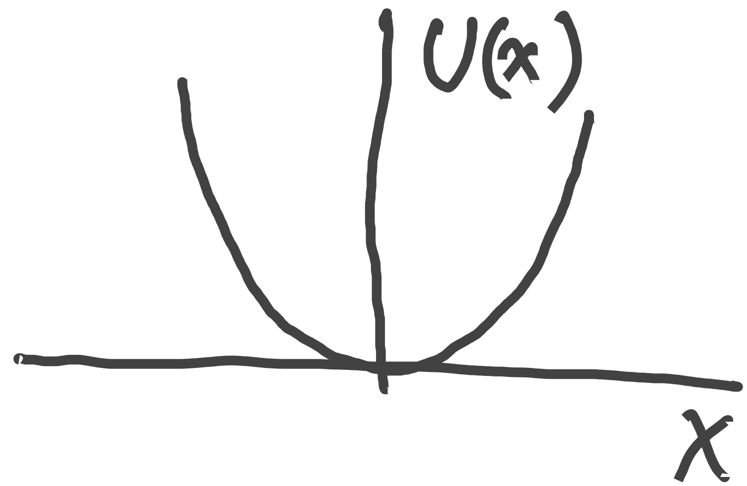
15.21 Sketch a graph of the kinetic energy, potential energy, and total energy of a spring-block oscillator, first as a function of time and then as a function of the oscillator's position.

15.22 For a spring-block oscillator, determine the block's position when the total energy is entirely kinetic energy and when it is entirely potential energy.

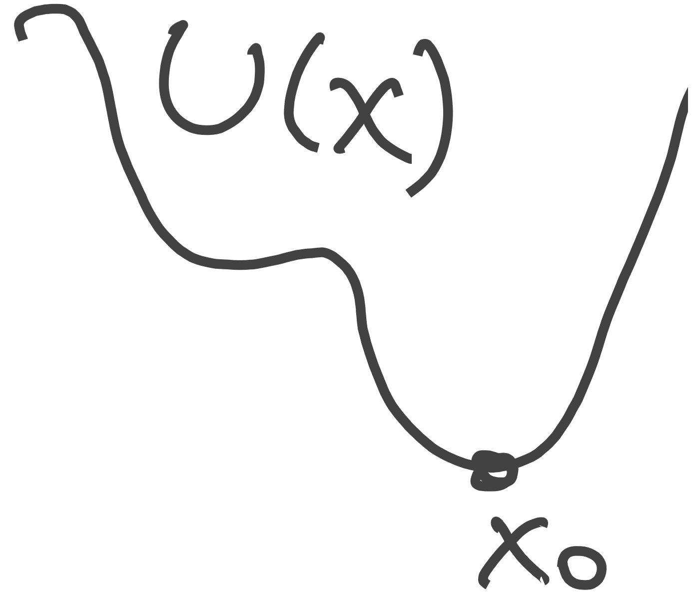
$$F = -kx$$

$$U(x) = \frac{1}{2}kx^2$$

$$F = -\frac{dU}{dx}$$



$$U(x) = U(x_0) + U'(x_0)(x - x_0) + \frac{1}{2}U''(x_0)(x - x_0)^2 + \frac{1}{6}U'''(x_0)(x - x_0)^3 + \ldots (x - x_0)^4 \ldots$$



$$U(x) = \frac{1}{2}kx^2$$

$$U(x) = \frac{1}{2}kx^2 = \frac{1}{2}k\,x_m^2\cos^2(\omega t + \phi)$$

$$K = \frac{1}{2}mv^2 \;;\; v = -\omega x_m \sin(\omega t + \phi)$$

$$K = \frac{1}{2}m\omega^2 x_m^2 \sin^2(\omega t + \phi)$$

~~$U + K =$~~

$$K + U = \frac{1}{2}k\,x_m^2 \underbrace{\left(\cos^2 + \sin^2\right)}_{1} = E$$

$$E = \frac{1}{2}k\,x_m^2 = U(x_m) = K(x = 0)$$

15-2 Energy in Simple Harmonic Motion

- Write the functions for kinetic and potential energy:

$$U(t) = \tfrac{1}{2}kx^2 = \tfrac{1}{2}kx_m^2 \cos^2(\omega t + \phi).$$

Eq. (15-18)

$$K(t) = \tfrac{1}{2}mv^2 = \tfrac{1}{2}kx_m^2 \sin^2(\omega t + \phi).$$

Eq. (15-20)

- Their sum is defined by:

$$E = U + K = \tfrac{1}{2}kx_m^2.$$

Eq. (15-21)

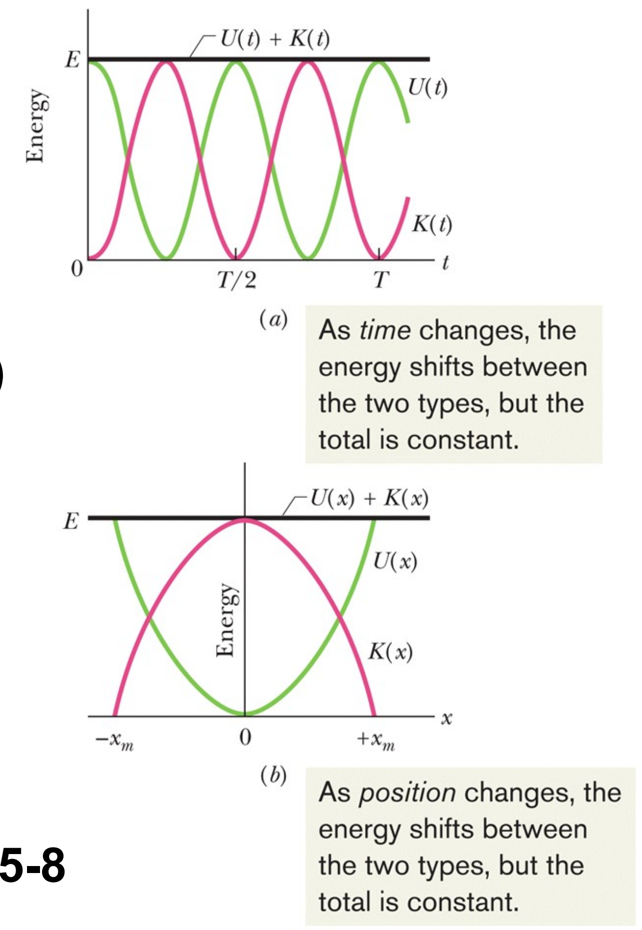


Figure 15-8