3. **THINK** The wavelength of light in a medium depends on the index of refraction of the medium. The nature of the interference, whether constructive or destructive, depends on the phase difference of the two waves.

EXPRESS We take the phases of both waves to be zero at the front surfaces of the layers. The phase of the first wave at the back surface of the glass is given by $\phi_1 = k_1L - \omega t$, where k_1 $(= 2\pi/\lambda_1)$ is the angular wave number and λ_1 is the wavelength in glass. Similarly, the phase of the second wave at the back surface of the plastic is given by $\phi_2 = k_2L - \omega t$, where k_2 $(= 2\pi/\lambda_2)$ is the angular wave number and λ_2 is the wavelength in plastic. The angular frequencies are the same since the waves have the same wavelength in air and the frequency of a wave does not change when the wave enters another medium. The phase difference is

$$\phi_1 - \phi_2 = (k_1 - k_2)L = 2\pi\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)L.$$

Now, $\lambda_1 = \lambda_{air}/n_1$, where λ_{air} is the wavelength in air and n_1 is the index of refraction of the glass. Similarly, $\lambda_2 = \lambda_{air}/n_2$, where n_2 is the index of refraction of the plastic. This means that the phase difference is

$$\phi_1 - \phi_2 = \frac{2\pi}{\lambda_{air}}(n_1 - n_2)L.$$

ANALYZE (a) The value of L that makes this 5.65 rad is

$$L = \frac{(\phi_1 - \phi_2)\lambda_{air}}{2\pi(n_1 - n_2)} = \frac{5.65(400 \times 10^{-9}\text{ m})}{2\pi(1.60 - 1.50)} = 3.60 \times 10^{-6}\text{ m}.$$

(b) A phase difference of 5.65 rad is less than 2π rad = 6.28 rad, the phase difference for completely constructive interference, but greater than π rad (= 3.14 rad), the phase difference for completely destructive interference. The interference is, therefore, intermediate, neither completely constructive nor completely destructive. It is, however, closer to completely constructive than to completely destructive.

LEARN The phase difference of two light waves can change when they travel through different materials having different indices of refraction.

14. (a) For the maximum adjacent to the central one, we set $m = 1$ in Eq. 35-14 and obtain

$$\theta_1 = \sin^{-1}\left(\frac{m\lambda}{d}\right)\Bigg|_{m=1} = \sin^{-1}\left[\frac{(1)(\lambda)}{100\lambda}\right] = 0.010\,\text{rad}.$$

(b) Since $y_1 = D \tan \theta_1$ (see Fig. 35-10(a)), we obtain

$$y_1 = (500 \text{ mm}) \tan (0.010 \text{ rad}) = 5.0 \text{ mm}.$$

The separation is $\Delta y = y_1 - y_0 = y_1 - 0 = 5.0$ mm.

15. **THINK** The interference at a point depends on the path-length difference of the light rays reaching that point from the two slits.

EXPRESS The angular positions of the maxima of a two-slit interference pattern are given by $\Delta L = d\sin\theta = m\lambda$, where ΔL is the path-length difference, d is the slit separation, λ is the wavelength, and m is an integer. If θ is small, sin θ may be approximated by θ in radians. Then, $\theta = m\lambda/d$ to good approximation. The angular separation of two adjacent maxima is $\Delta\theta = \lambda/d$.

ANALYZE Let λ' be the wavelength for which the angular separation is greater by10.0%. Then, $1.10\lambda/d = \lambda'/d$. or

$$\lambda' = 1.10\lambda = 1.10(589 \text{ nm}) = 648 \text{ nm}.$$

LEARN The angular separation $\Delta\theta$ is proportional to the wavelength of the light. For small θ, we have

$$\Delta\theta' = \left(\frac{\lambda'}{\lambda}\right)\Delta\theta.$$

16. The distance between adjacent maxima is given by $\Delta y = \lambda D/d$ (see Eqs. 35-17 and 35-18). Dividing both sides by D, this becomes $\Delta\theta = \lambda/d$ with θ in radians. In the steps that follow, however, we will end up with an expression where degrees may be directly used. Thus, in the present case,

$$\Delta\theta_n = \frac{\lambda_n}{d} = \frac{\lambda}{nd} = \frac{\Delta\theta}{n} = \frac{0.20°}{1.33} = 0.15°.$$

19. **THINK** The condition for a maximum in the two-slit interference pattern is $d \sin\theta = m\lambda$, where d is the slit separation, λ is the wavelength, m is an integer, and θ is the angle made by the interfering rays with the forward direction.

EXPRESS If θ is small, $\sin\theta$ may be approximated by θ in radians. Then, $\theta = m\lambda/d$, and the angular separation of adjacent maxima, one associated with the integer m and the other associated with the integer $m + 1$, is given by $\Delta\theta = \lambda/d$. The separation on a screen a distance D away is given by

$$\Delta y = D\,\Delta\theta = \lambda D/d.$$

ANALYZE Thus,

$$\Delta y = \frac{(500\times10^{-9}\,\text{m})(5.40\,\text{m})}{1.20\times10^{-3}\,\text{m}} = 2.25\times10^{-3}\,\text{m} = 2.25\,\text{mm}.$$

LEARN For small θ, the spacing is nearly uniform. However, away from the center of the pattern, θ increases and the spacing gets larger.

21. The maxima of a two-slit interference pattern are at angles θ given by $d \sin \theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer. If θ is small, $\sin \theta$ may be replaced by θ in radians. Then, $d\theta = m\lambda$. The angular separation of two maxima associated with different wavelengths but the same value of m is

$$\Delta\theta = (m/d)(\lambda_2 - \lambda_1),$$

and their separation on a screen a distance D away is

$$\Delta y = D \tan \Delta\theta \approx D\,\Delta\theta = \left(\frac{mD}{d}\right)(\lambda_2 - \lambda_1)$$

$$= \left(\frac{3(1.0\text{ m})}{5.0 \times 10^{-3}\text{ m}}\right)\left[600 \times 10^{-9}\text{ m} - 480 \times 10^{-9}\text{ m}\right] = 7.2 \times 10^{-5}\text{ m}.$$

The small angle approximation $\tan \Delta\theta \approx \Delta\theta$ (in radians) is made.

23. Initially, source A leads source B by 90°, which is equivalent to 1/4 wavelength. However, source A also lags behind source B since r_A is longer than r_B by 100 m, which is $100\,\text{m}/400\,\text{m} = 1/4$ wavelength. So the net phase difference between A and B at the detector is zero.

25. Let the distance in question be x. The path difference (between rays originating from S_1 and S_2 and arriving at points on the $x > 0$ axis) is

$$\sqrt{d^2 + x^2} - x = \left(m + \frac{1}{2}\right)\lambda,$$

where we are requiring destructive interference (half-integer wavelength phase differences) and $m = 0, 1, 2, \ldots$. After some algebraic steps, we solve for the distance in terms of m:

$$x = \frac{d^2}{(2m+1)\lambda} - \frac{(2m+1)\lambda}{4}.$$

To obtain the largest value of x, we set $m = 0$:

$$x_0 = \frac{d^2}{\lambda} - \frac{\lambda}{4} = \frac{(3.00\lambda)^2}{\lambda} - \frac{\lambda}{4} = 8.75\lambda = 8.75(900 \text{ nm}) = 7.88\times10^3 \text{ nm} = 7.88\,\mu\text{m}.$$

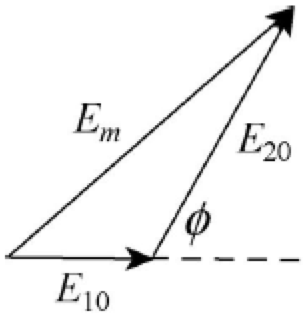
29. **THINK** The intensity is proportional to the square of the resultant field amplitude.

EXPRESS Let the electric field components of the two waves be written as

$$E_1 = E_{10}\sin\omega t$$
$$E_2 = E_{20}\sin(\omega t + \phi),$$

where E_{10} = 1.00, E_{20} = 2.00, and ϕ = 60°. The resultant field is $E = E_1 + E_2$. We use phasor diagram to calculate the amplitude of E.

ANALYZE The phasor diagram is shown next.



The resultant amplitude E_m is given by the trigonometric law of cosines:

$$E_m^2 = E_{10}^2 + E_{20}^2 - 2E_{10}E_{20}\cos(180° - \phi).$$

Thus,

$$E_m = \sqrt{(1.00)^2 + (2.00)^2 - 2(1.00)(2.00)\cos 120°} = 2.65.$$

LEARN Summing over the horizontal components of the two fields gives

$$\sum E_h = E_{10}\cos 0 + E_{20}\cos 60° = 1.00 + (2.00)\cos 60° = 2.00$$

Similarly, the sum over the vertical components is

$$\sum E_v = E_{10}\sin 0 + E_{20}\sin 60° = 1.00\sin 0° + (2.00)\sin 60° = 1.732.$$

The resultant amplitude is

$$E_m = \sqrt{(2.00)^2 + (1.732)^2} = 2.65,$$

which agrees with what we found above. The phase angle relative to the phasor representing E_1 is

$$\beta = \tan^{-1}\left(\frac{1.732}{2.00}\right) = 40.9°$$

Thus, the resultant field can be written as $E = (2.65)\sin(\omega t + 40.9°)$.

30. In adding these with the phasor method (as opposed to, say, trig identities), we may set $t = 0$ and add them as vectors:

$$y_h = 10\cos 0° + 8.0\cos 30° = 16.9$$

$$y_v = 10\sin 0° + 8.0\sin 30° = 4.0$$

so that

$$y_R = \sqrt{y_h^2 + y_v^2} = 17.4$$

$$\beta = \tan^{-1}\left(\frac{y_v}{y_h}\right) = 13.3° .$$

Thus,

$$y = y_1 + y_2 = y_R \sin(\omega t + \beta) = 17.4\sin(\omega t + 13.3°) .$$

Quoting the answer to two significant figures, we have $y \approx 17\sin(\omega t + 13°)$.

32. (a) We can use phasor techniques or use trig identities. Here we show the latter approach. Since

$$\sin a + \sin(a+b) = 2\cos(b/2)\sin(a+b/2),$$

we find

$$E_1 + E_2 = 2E_0\cos(\phi/2)\sin(\omega t + \phi/2)$$

where E_0 = 2.00 μV/m, ω = 1.26 × 10^{15} rad/s, and ϕ = 39.6 rad. This shows that the electric field amplitude of the resultant wave is

$$E = 2E_0\cos(\phi/2) = 2(2.00\ \mu\text{V/m})\cos(19.2\ \text{rad}) = 2.33\ \mu\text{V/m}.$$

(b) Equation 35-22 leads to

$$I = 4I_0\cos^2(\phi/2) = 1.35\,I_0$$

at point P, and

$$I_{\text{center}} = 4I_0\cos^2(0) = 4\,I_0$$

at the center. Thus, $I/I_{\text{center}} = 1.35/4 = 0.338$.

(c) The phase difference ϕ (in wavelengths) is gotten from ϕ in radians by dividing by 2π. Thus, $\phi = 39.6/2\pi = 6.3$ wavelengths. Thus, point P is between the sixth side maximum (at which ϕ = 6 wavelengths) and the seventh minimum (at which $\phi = 6\frac{1}{2}$ wavelengths).

(d) The rate is given by ω = 1.26 × 10^{15} rad/s.

(e) The angle between the phasors is ϕ = 39.6 rad = 2270° (which would look like about 110° when drawn in the usual way).