2. (a) The acceleration amplitude is related to the maximum force by Newton's second law: $F_{max} = ma_m$. The textbook notes (in the discussion immediately after Eq. 15-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where ω is the angular frequency ($\omega = 2\pi f$ since there are 2π radians in one cycle). The frequency is the reciprocal of the period: $f = 1/T = 1/0.20 = 5.0$ Hz, so the angular frequency is $\omega = 10\pi$ (understood to be valid to two significant figures). Therefore,

$$F_{max} = m\omega^2 x_m = (0.12\text{ kg})(10\pi\text{ rad/s})^2(0.085\text{ m}) = 10\text{ N}.$$

(b) Using Eq. 15-12, we obtain

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow k = m\omega^2 = (0.12\text{kg})(10\pi\text{ rad/s})^2 = 1.2\times10^2\text{ N/m}.$$

5. **THINK** The blade of the shaver undergoes simple harmonic motion. We want to find its amplitude, maximum speed and maximum acceleration.

EXPRESS The amplitude x_m is half the range of the displacement D. Once the amplitude is known, the maximum speed v_m is related to the amplitude by $v_m = \omega x_m$, where ω is the angular frequency. Similarly, the maximum acceleration is $a_m = \omega^2 x_m$.

ANALYZE (a) The amplitude is $x_m = D/2 = (2.0\text{ mm})/2 = 1.0$ mm.

(b) The maximum speed v_m is related to the amplitude x_m by $v_m = \omega x_m$, where ω is the angular frequency. Since $\omega = 2\pi f$, where f is the frequency,

$$v_m = 2\pi f x_m = 2\pi(120\text{ Hz})(1.0\times10^{-3}\text{ m}) = 0.75\text{ m/s}.$$

(c) The maximum acceleration is

$$a_m = \omega^2 x_m = (2\pi f)^2 x_m = (2\pi(120\text{ Hz}))^2(1.0\times10^{-3}\text{ m}) = 5.7\times10^2\text{ m/s}^2.$$

LEARN In SHM, acceleration is proportional to the displacement x_m.

9. (a) Making sure our calculator is in radians mode, we find

$$x = 6.0\cos\left[3\pi(2.0) + \frac{\pi}{3}\right] = 3.0 \text{ m}.$$

(b) Differentiating with respect to time and evaluating at $t = 2.0$ s, we find

$$v = \frac{dx}{dt} = -3\pi(6.0)\sin\left[3\pi(2.0) + \frac{\pi}{3}\right] = -49 \text{ m/s}.$$

(c) Differentiating again, we obtain

$$a = \frac{dv}{dt} = -(3\pi)^2(6.0)\cos\left[3\pi(2.0) + \frac{\pi}{3}\right] = -2.7\times 10^2 \text{ m/s}^2.$$

(d) In the second paragraph after Eq. 15-3, the textbook defines the phase of the motion. In this case (with $t = 2.0$ s) the phase is $3\pi(2.0) + \pi/3 \approx 20$ rad.

(e) Comparing with Eq. 15-3, we see that $\omega = 3\pi$ rad/s. Therefore, $f = \omega/2\pi = 1.5$ Hz.

(f) The period is the reciprocal of the frequency: $T = 1/f \approx 0.67$ s.

11. When displaced from equilibrium, the net force exerted by the springs is $-2kx$ acting in a direction so as to return the block to its equilibrium position ($x = 0$). Since the acceleration $a = d^2x/dt^2$, Newton's second law yields

$$m\frac{d^2x}{dt^2} = -2kx.$$

Substituting $x = x_m \cos(\omega t + \phi)$ and simplifying, we find $\omega^2 = 2k/m$, where ω is in radians per unit time. Since there are 2π radians in a cycle, and frequency f measures cycles per second, we obtain

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{2k}{m}} = \frac{1}{2\pi}\sqrt{\frac{2(7580\ \text{N/m})}{0.245\ \text{kg}}} = 39.6\ \text{Hz}.$$

13. **THINK** The mass-spring system undergoes simple harmonic motion. Given the amplitude and the period, we can determine the corresponding frequency, angular frequency, spring constant, maximum speed and maximum force.

EXPRESS The angular frequency ω is given by $\omega = 2\pi f = 2\pi/T$, where f is the frequency and T is the period, with $f = 1/T$. The angular frequency is related to the spring constant k and the mass m by $\omega = \sqrt{k/m}$. The maximum speed v_m is related to the amplitude x_m by $v_m = \omega x_m$.

ANALYZE (a) The motion repeats every 0.500 s so the period must be $T = 0.500$ s.

(b) The frequency is the reciprocal of the period: $f = 1/T = 1/(0.500 \text{ s}) = 2.00$ Hz.

(c) The angular frequency is $\omega = 2\pi f = 2\pi(2.00 \text{ Hz}) = 12.6$ rad/s.

(d) We solve for the spring constant k and obtain

$$k = m\omega^2 = (0.500 \text{ kg})(12.6 \text{ rad/s})^2 = 79.0 \text{ N/m}.$$

(e) The amplitude is x_m=35.0 cm = 0.350 m, so the maximum speed is

$$v_m = \omega x_m = (12.6 \text{ rad/s})(0.350 \text{ m}) = 4.40 \text{ m/s}.$$

(f) The maximum force is exerted when the displacement is a maximum. Thus, we have

$$F_m = kx_m = (79.0 \text{ N/m})(0.350 \text{ m}) = 27.6 \text{ N}.$$

16. They pass each other at time t, at $x_1 = x_2 = \frac{1}{2}x_m$ where

$$x_1 = x_m \cos(\omega t + \phi_1) \quad \text{and} \quad x_2 = x_m \cos(\omega t + \phi_2).$$

From this, we conclude that $\cos(\omega t + \phi_1) = \cos(\omega t + \phi_2) = \frac{1}{2}$, and therefore that the phases (the arguments of the cosines) are either both equal to $\pi/3$ or one is $\pi/3$ while the other is $-\pi/3$. Also at this instant, we have $v_1 = -v_2 \neq 0$ where

$$v_1 = -x_m\omega \sin(\omega t + \phi_1) \quad \text{and} \quad v_2 = -x_m\omega \sin(\omega t + \phi_2).$$

This leads to $\sin(\omega t + \phi_1) = -\sin(\omega t + \phi_2)$. This leads us to conclude that the phases have opposite sign. Thus, one phase is $\pi/3$ and the other phase is $-\pi/3$; the wt term cancels if we take the phase difference, which is seen to be $\pi/3 - (-\pi/3) = 2\pi/3$.

19. Both parts of this problem deal with the critical case when the maximum acceleration becomes equal to that of free fall. The textbook notes (in the discussion immediately after Eq. 15-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where ω is the angular frequency; this is the expression we set equal to $g = 9.8\ \text{m/s}^2$.

(a) Using Eq. 15-5 and $T = 1.0$ s, we have

$$\left(\frac{2\pi}{T}\right)^2 x_m = g \Rightarrow x_m = \frac{gT^2}{4\pi^2} = 0.25\ \text{m}.$$

(b) Since $\omega = 2\pi f$, and $x_m = 0.050$ m is given, we find

$$(2\pi f)^2 x_m = g \quad \Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\frac{g}{x_m}} = 2.2\ \text{Hz}.$$

23. **THINK** The maximum force that can be exerted by the surface must be less than the static frictional force or else the block will not follow the surface in its motion.

EXPRESS The static frictional force is given by $f_s = \mu_s F_N$, where μ_s is the coefficient of static friction and F_N is the normal force exerted by the surface on the block. Since the block does not accelerate vertically, we know that $F_N = mg$, where m is the mass of the block. If the block follows the table and moves in simple harmonic motion, the magnitude of the maximum force exerted on it is given by

$$F = ma_m = m\omega^2 x_m = m(2\pi f)^2 x_m,$$

where a_m is the magnitude of the maximum acceleration, ω is the angular frequency, and f is the frequency. The relationship $\omega = 2\pi f$ was used to obtain the last form.

ANALYZE We substitute $F = m(2\pi f)^2 x_m$ and $F_N = mg$ into $F < \mu_s F_N$ to obtain $m(2\pi f)^2 x_m < \mu_s mg$. The largest amplitude for which the block does not slip is

$$x_m = \frac{\mu_s g}{(2\pi f)^2} = \frac{(0.50)(9.8\ \text{m/s}^2)}{(2\pi \times 2.0\ \text{Hz})^2} = 0.031\ \text{m}.$$

LEARN A larger amplitude would require a larger force at the end points of the motion. The block slips if the surface cannot supply a larger force.

25. (a) We interpret the problem as asking for the equilibrium position; that is, the block is gently lowered until forces balance (as opposed to being suddenly released and allowed to oscillate). If the amount the spring is stretched is x, then we examine force-components along the incline surface and find

$$kx = mg\sin\theta \;\Rightarrow\; x = \frac{mg\sin\theta}{k} = \frac{(14.0\text{ N})\sin 40.0°}{120\text{ N/m}} = 0.0750\text{ m}$$

at equilibrium. The calculator is in degrees mode in the above calculation. The distance from the top of the incline is therefore (0.450 + 0.75) m = 0.525 m.

(b) Just as with a vertical spring, the effect of gravity (or one of its components) is simply to shift the equilibrium position; it does not change the characteristics (such as the period) of simple harmonic motion. Thus, Eq. 15-13 applies, and we obtain

$$T = 2\pi\sqrt{\frac{14.0\text{ N}/9.80\text{ m/s}^2}{120\text{ N/m}}} = 0.686\text{ s.}$$

27. **THINK** This problem explores the relationship between energies, both kinetic and potential, with amplitude in SHM.

EXPRESS In simple harmonic motion, let the displacement be

$$x(t) = x_m \cos(\omega t + \phi).$$

The corresponding velocity is

$$v(t) = dx/dt = -\omega x_m \sin(\omega t + \phi).$$

Using the expressions for $x(t)$ and $v(t)$, we find the potential and kinetic energies to be

$$U(t) = \frac{1}{2}kx^2(t) = \frac{1}{2}kx_m^2 \cos^2(\omega t + \phi)$$
$$K(t) = \frac{1}{2}mv^2(t) = \frac{1}{2}m\omega^2 x_m^2 \sin^2(\omega t + \phi) = \frac{1}{2}kx_m^2 \sin^2(\omega t + \phi)$$

where $k = m\omega^2$ is the spring constant and x_m is the amplitude. The total energy is

$$E = U(t) + K(t) = \frac{1}{2}kx_m^2\left[\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)\right] = \frac{1}{2}kx_m^2.$$

ANALYZE (a) The condition $x(t) = x_m / 2$ implies that $\cos(\omega t + \phi) = 1/2$, or $\sin(\omega t + \phi) = \sqrt{3}/2$. Thus, the fraction of energy that is kinetic is

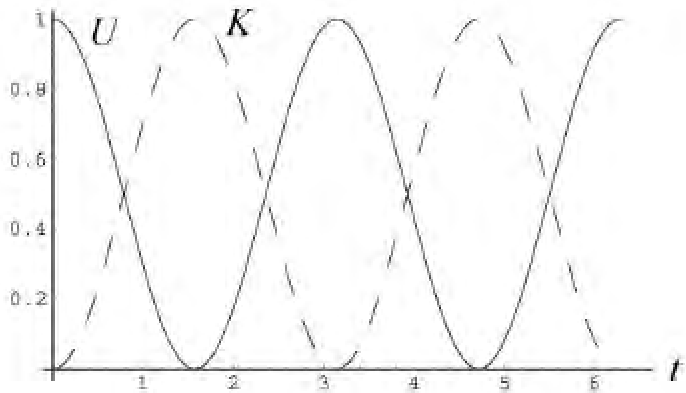
$$\frac{K}{E} = \sin^2(\omega t + \phi) = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}.$$

(b) Similarly, we have $\frac{U}{E} = \cos^2(\omega t + \phi) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$.

(c) Since $E = \frac{1}{2}kx_m^2$ and $U = \frac{1}{2}kx(t)^2$, $U/E = x^2/x_m^2$. Solving $x^2/x_m^2 = 1/2$ for x, we get $x = x_m / \sqrt{2}$.

LEARN The figure to the right depicts the potential energy (solid line) and kinetic energy (dashed line) as a function of time, assuming $x(0) = x_m$. The curves intersect when $K = U = E/2$, or equivalently,

$$\cos^2 \omega t = \sin^2 \omega t = 1/2.$$



28. The total mechanical energy is equal to the (maximum) kinetic energy as it passes through the equilibrium position ($x = 0$):

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}(2.0\text{ kg})(0.85\text{ m/s})^2 = 0.72\text{ J}.$$

Looking at the graph in the problem, we see that $U(x = 10) = 0.5$ J. Since the potential function has the form $U(x) = bx^2$, the constant is $b = 5.0\times10^{-3}\,\text{J/cm}^2$. Thus, $U(x) = 0.72$ J when $x = 12$ cm.

(a) Thus, the mass does turn back before reaching $x = 15$ cm.

(b) It turns back at $x = 12$ cm.

33. The problem consists of two distinct parts: the completely inelastic collision (which is assumed to occur instantaneously, the bullet embedding itself in the block before the block moves through significant distance) followed by simple harmonic motion (of mass $m + M$ attached to a spring of spring constant k).

(a) Momentum conservation readily yields $v' = mv/(m + M)$. With $m = 9.5$ g, $M = 5.4$ kg, and $v = 630$ m/s, we obtain $v' = 1.1$ m/s.

(b) Since v' occurs at the equilibrium position, then $v' = v_m$ for the simple harmonic motion. The relation $v_m = \omega x_m$ can be used to solve for x_m, or we can pursue the alternate (though related) approach of energy conservation. Here we choose the latter:

$$\frac{1}{2}(m+M)v'^2 = \frac{1}{2}kx_m^2 \quad \Rightarrow \quad \frac{1}{2}(m+M)\frac{m^2v^2}{(m+M)^2} = \frac{1}{2}kx_m^2$$

which simplifies to

$$x_m = \frac{mv}{\sqrt{k(m+M)}} = \frac{(9.5\times10^{-3}\text{kg})(630\text{ m/s})}{\sqrt{(6000\text{ N/m})(9.5\times10^{-3}\text{kg} + 5.4\text{kg})}} = 3.3\times10^{-2}\text{ m.}$$

39. **THINK** The balance wheel in the watch undergoes angular simple harmonic oscillation. From the amplitude and period, we can calculate the corresponding angular velocity and angular acceleration.

EXPRESS We take the angular displacement of the wheel to be $\theta(t) = \theta_m \cos(2\pi t/T)$, where θ_m is the amplitude and T is the period. We differentiate with respect to time to find the angular velocity:

$$\Omega = d\theta/dt = -(2\pi/T)\theta_m \sin(2\pi t/T).$$

The symbol Ω is used for the angular velocity of the wheel so it is not confused with the angular frequency.

ANALYZE (a) The maximum angular velocity is

$$\Omega_m = \frac{2\pi\theta_m}{T} = \frac{(2\pi)(\pi \text{ rad})}{0.500 \text{ s}} = 39.5 \text{ rad/s}.$$

(b) When $\theta = \pi/2$, then $\theta/\theta_m = 1/2$, $\cos(2\pi t/T) = 1/2$, and

$$\sin(2\pi t/T) = \sqrt{1-\cos^2(2\pi t/T)} = \sqrt{1-(1/2)^2} = \sqrt{3}/2$$

where the trigonometric identity $\cos^2\theta + \sin^2\theta = 1$ is used. Thus,

$$\Omega = -\frac{2\pi}{T}\theta_m \sin\left(\frac{2\pi t}{T}\right) = -\left(\frac{2\pi}{0.500 \text{ s}}\right)(\pi \text{ rad})\left(\frac{\sqrt{3}}{2}\right) = -34.2 \text{ rad / s}.$$

During another portion of the cycle its angular speed is +34.2 rad/s when its angular displacement is $\pi/2$ rad.

(c) The angular acceleration is

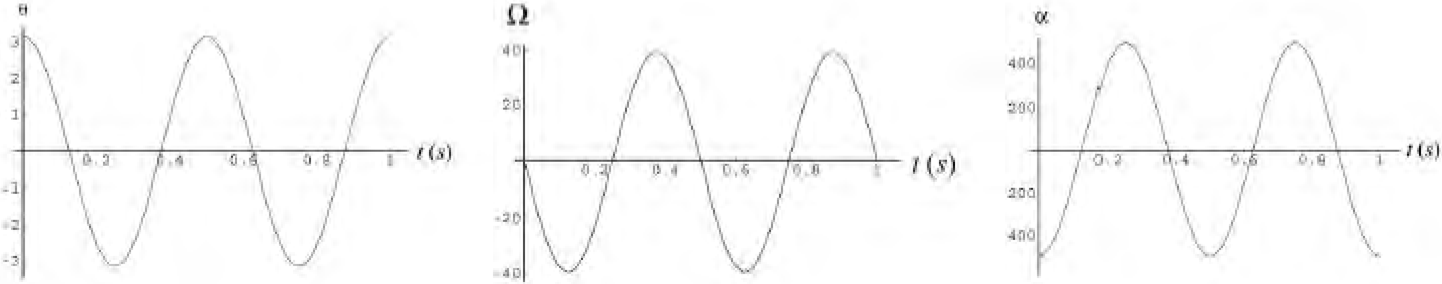
$$\alpha = \frac{d^2\theta}{dt^2} = -\left(\frac{2\pi}{T}\right)^2 \theta_m \cos(2\pi t/T) = -\left(\frac{2\pi}{T}\right)^2 \theta.$$

When $\theta = \pi/4$,

$$\alpha = -\left(\frac{2\pi}{0.500 \text{ s}}\right)^2 \left(\frac{\pi}{4}\right) = -124 \text{ rad/s}^2,$$

or $|\alpha| = 124 \text{ rad/s}^2$.

LEARN The angular displacement, angular velocity and angular acceleration as a function of time are plotted next.



40. We use Eq. 15-29 and the parallel-axis theorem $I = I_{cm} + mh^2$ where $h = d$, the unknown. For a meter stick of mass m, the rotational inertia about its center of mass is $I_{cm} = mL^2/12$ where $L = 1.0$ m. Thus, for $T = 2.5$ s, we obtain

$$T = 2\pi\sqrt{\frac{mL^2/12 + md^2}{mgd}} = 2\pi\sqrt{\frac{L^2}{12gd} + \frac{d}{g}}.$$

Squaring both sides and solving for d leads to the quadratic formula:

$$d = \frac{g(T/2\pi)^2 \pm \sqrt{d^2(T/2\pi)^4 - L^2/3}}{2}.$$

Choosing the plus sign leads to an impossible value for d ($d = 1.5 > L$). If we choose the minus sign, we obtain a physically meaningful result: $d = 0.056$ m.

44. To use Eq. 15-29 we need to locate the center of mass and we need to compute the rotational inertia about A. The center of mass of the stick shown horizontal in the figure is at A, and the center of mass of the other stick is 0.50 m below A. The two sticks are of equal mass, so the center of mass of the system is $h = \frac{1}{2}(0.50\text{ m}) = 0.25\text{m}$ below A, as shown in the figure. Now, the rotational inertia of the system is the sum of the rotational inertia I_1 of the stick shown horizontal in the figure and the rotational inertia I_2 of the stick shown vertical. Thus, we have

$$I = I_1 + I_2 = \frac{1}{12}ML^2 + \frac{1}{3}ML^2 = \frac{5}{12}ML^2$$

where L = 1.00 m and M is the mass of a meter stick (which cancels in the next step). Now, with $m = 2M$ (the total mass), Eq. 15-29 yields

$$T = 2\pi\sqrt{\frac{\frac{5}{12}ML^2}{2Mgh}} = 2\pi\sqrt{\frac{5L}{6g}}$$

where $h = L/4$ was used. Thus, T = 1.83 s.

50. (a) The rotational inertia of a uniform rod with pivot point at its end is $I = mL^2/12 + mL^2 = 1/3ML^2$. Therefore, Eq. 15-29 leads to

$$T = 2\pi\sqrt{\frac{\frac{1}{3}ML^2}{Mg(L/2)}} \;\Rightarrow\; L = \frac{3gT^2}{8\pi^2} = \frac{3(9.8\text{ m/s}^2)(1.5\text{ s})^2}{8\pi^2} = 0.84\text{ m}.$$

(b) By energy conservation

$$E_{\text{bottom of swing}} = E_{\text{end of swing}} \;\Rightarrow\; K_m = U_m$$

where $U = Mg\ell(1-\cos\theta)$ with ℓ being the distance from the axis of rotation to the center of mass. If we use the small-angle approximation ($\cos\theta \approx 1 - \frac{1}{2}\theta^2$ with θ in radians (Appendix E)), we obtain

$$U_m = (0.5\text{ kg})(9.8\text{ m/s}^2)\left(\frac{L}{2}\right)\left(\frac{1}{2}\theta_m^2\right)$$

where θ_m = 0.17 rad. Thus, $K_m = U_m$ = 0.031 J. If we calculate $(1 - \cos\theta)$ directly (without using the small angle approximation) then we obtain within 0.3% of the same answer.

51. This is similar to the situation treated in Sample Problem 15.5 — "Physical pendulum, period and length," except that O is no longer at the end of the stick. Referring to the center of mass as C (assumed to be the geometric center of the stick), we see that the distance between O and C is $h = x$. The parallel axis theorem (see Eq. 15-30) leads to

$$I = \frac{1}{12}mL^2 + mh^2 = m\left(\frac{L^2}{12} + x^2\right).$$

Equation 15-29 gives

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{\left(\frac{L^2}{12} + x^2\right)}{gx}} = 2\pi\sqrt{\frac{\left(L^2 + 12x^2\right)}{12gx}}.$$

(a) Minimizing T by graphing (or special calculator functions) is straightforward, but the standard calculus method (setting the derivative equal to zero and solving) is somewhat awkward. We pursue the calculus method but choose to work with $12gT^2/2\pi$ instead of T (it should be clear that $12gT^2/2\pi$ is a minimum whenever T is a minimum). The result is

$$\frac{d\left(\frac{12gT^2}{2\pi}\right)}{dx} = 0 = \frac{d\left(\frac{L^2}{x} + 12x\right)}{dx} = -\frac{L^2}{x^2} + 12$$

which yields $x = L/\sqrt{12} = (1.85\ \text{m})/\sqrt{12} = 0.53\ \text{m}$ as the value of x that should produce the smallest possible value of T.

(b) With $L = 1.85$ m and $x = 0.53$ m, we obtain $T = 2.1$ s from the expression derived in part (a).

59. **THINK** In the presence of a damping force, the amplitude of oscillation of the mass-spring system decreases with time.

EXPRESS As discussed in 15-8, when a damping force is present, we have

$$x(t) = x_m e^{-bt/2m}\cos(\omega' t + \phi)$$

where b is the damping constant and the angular frequency is given by

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}\,.$$

ANALYZE (a) We want to solve $e^{-bt/2m} = 1/3$ for t. We take the natural logarithm of both sides to obtain $-bt/2m = \ln(1/3)$. Therefore,

$$t = -(2m/b)\ln(1/3) = (2m/b)\ln 3.$$

Thus,

$$t = \frac{2(1.50\text{ kg})}{0.230\text{ kg/s}}\ln 3 = 14.3\text{ s}.$$

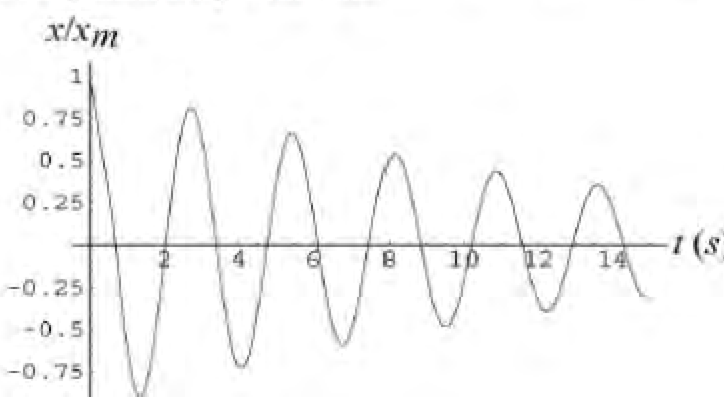
(b) The angular frequency is

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} = \sqrt{\frac{8.00\text{ N/m}}{1.50\text{ kg}} - \frac{(0.230\text{ kg/s})^2}{4(1.50\text{ kg})^2}} = 2.31\text{ rad/s}.$$

The period is $T = 2\pi/\omega' = (2\pi)/(2.31\text{ rad/s}) = 2.72$ s and the number of oscillations is

$$t/T = (14.3\text{ s})/(2.72\text{ s}) = 5.27.$$

LEARN The displacement $x(t)$ as a function of time is shown below. The amplitude, $x_m e^{-bt/2m}$, decreases exponentially with time.



60. (a) From Hooke's law, we have

$$k = \frac{(500\text{ kg})(9.8\text{ m/s}^2)}{10\text{cm}} = 4.9\times10^2\text{ N/cm}.$$

(b) The amplitude decreasing by 50% during one period of the motion implies

$$e^{-bT/2m} = \frac{1}{2} \quad \text{where} \quad T = \frac{2\pi}{\omega'}.$$

Since the problem asks us to estimate, we let $\omega' \approx \omega = \sqrt{k/m}$. That is, we let

$$\omega' \approx \sqrt{\frac{49000\text{ N/m}}{500\text{ kg}}} \approx 9.9\text{ rad/s},$$

so that $T \approx 0.63$ s. Taking the (natural) log of both sides of the above equation, and rearranging, we find

$$b = \frac{2m}{T}\ln 2 \approx \frac{2(500\text{ kg})}{0.63\text{ s}}(0.69) = 1.1\times10^3\text{ kg/s}.$$

Note: if one worries about the $\omega' \approx \omega$ approximation, it is quite possible (though messy) to use Eq. 15-43 in its full form and solve for b. The result would be (quoting more figures than are significant)

$$b = \frac{2\ln 2\sqrt{mk}}{\sqrt{(\ln 2)^2 + 4\pi^2}} = 1086\text{ kg/s}$$

which is in good agreement with the value gotten "the easy way" above.

61. (a) We set $\omega = \omega_d$ and find that the given expression reduces to $x_m = F_m/b\omega$ at resonance.

(b) In the discussion immediately after Eq. 15-6, the book introduces the velocity amplitude $v_m = \omega x_m$. Thus, at resonance, we have $v_m = \omega F_m/b\omega = F_m/b$.

62. With $\omega = 2\pi/T$ then Eq. 15-28 can be used to calculate the angular frequencies for the given pendulums. For the given range of $2.00 < \omega < 4.00$ (in rad/s), we find only two of the given pendulums have appropriate values of ω: pendulum (d) with length of 0.80 m (for which ω = 3.5 rad/s) and pendulum (e) with length of 1.2 m (for which ω = 2.86 rad/s).