1. **THINK** If the expansion of the gas is reversible and isothermal, then there's no change in internal energy. However, if the process is reversible and adiabatic, then there would be no change in entropy.

EXPRESS Since the gas is ideal, its pressure p is given in terms of the number of moles n, the volume V, and the temperature T by $p = nRT/V$. If the expansion is isothermal, the work done by the gas is

$$W=\int_{V_1}^{V_2} p\,dV = nRT\int_{V_1}^{V_2}\frac{dV}{V} = nRT\ln\frac{V_2}{V_1},$$

and the corresponding change in entropy is $\Delta S=\int(1/T)\,dQ=Q/T$, where Q is the heat absorbed (see Eq. 20-2).

ANALYZE (a) With $V_2 = 2.00V_1$ and $T = 400$ K, we obtain

$$W = nRT\ln 2.00 = (4.00\text{ mol})(8.31\text{ J/mol}\cdot\text{K})(400\text{ K})\ln 2.00 = 9.22\times10^3\text{ J}.$$

(b) According to the first law of thermodynamics, $\Delta E_{\text{int}} = Q - W$. Now the internal energy of an ideal gas depends only on the temperature and not on the pressure and volume. Since the expansion is isothermal, $\Delta E_{\text{int}} = 0$ and $Q = W$. Thus,

$$\Delta S = \frac{W}{T} = \frac{9.22\times10^3\text{ J}}{400\text{ K}} = 23.1\text{ J/K}.$$

(c) The change in entropy ΔS is zero for all reversible adiabatic processes.

LEARN The general expression for ΔS for reversible processes is given by Eq. 20-4:

$$\Delta S = S_f - S_i = nR\ln\left(\frac{V_f}{V_i}\right) + nC_V\ln\left(\frac{T_f}{T_i}\right).$$

Note that ΔS does not depend on how the gas changes from its initial state i to the final state f.

5. We use the following relation derived in Sample Problem 20.01 — "Entropy change of two blocks coming to equilibrium:"

$$\Delta S = mc\ln\left(T_f / T_i\right).$$

(a) The energy absorbed as heat is given by Eq. 19-14. Using Table 19-3, we find

$$Q = cm\Delta T = \left(386\,\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(2.00\text{ kg})(75\text{ K}) = 5.79\times10^{4}\text{ J}$$

where we have used the fact that a change in Kelvin temperature is equivalent to a change in Celsius degrees.

(b) With $T_f = 373.15$ K and $T_i = 298.15$ K, we obtain

$$\Delta S = (2.00\text{ kg})\left(386\,\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)\ln\left(\frac{373.15}{298.15}\right) = 173\text{ J/K.}$$

6. (a) This may be considered a reversible process (as well as isothermal), so we use $\Delta S = Q/T$ where $Q = Lm$ with $L = 333$ J/g from Table 19-4. Consequently,

$$\Delta S = \frac{\text{a}333\text{ J / gfa}12.0\text{ gf}}{273\text{ K}} = 14.6\text{ J / K.}$$

(b) The situation is similar to that described in part (a), except with $L = 2256$ J/g, $m = 5.00$ g, and $T = 373$ K. We therefore find $\Delta S = 30.2$ J/K.

7. (a) We refer to the copper block as block 1 and the lead block as block 2. The equilibrium temperature T_f satisfies $m_1c_1(T_f - T_{i,1}) + m_2c_2(T_f - T_{i2}) = 0$, which we solve for T_f:

$$T_f = \frac{m_1c_1T_{i,1} + m_2c_2T_{i,2}}{m_1c_1 + m_2c_2} = \frac{(50.0\text{ g})(386\text{ J/kg}\cdot\text{K})(400\text{ K}) + (100\text{ g})(128\text{ J/kg}\cdot\text{K})(200\text{ K})}{(50.0\text{ g})(386\text{ J/kg}\cdot\text{K}) + (100\text{ g})(128\text{ J/kg}\cdot\text{K})}$$

$$= 320\text{ K}.$$

(b) Since the two-block system in thermally insulated from the environment, the change in internal energy of the system is zero.

(c) The change in entropy is

$$\Delta S = \Delta S_1 + \Delta S_2 = m_1c_1 \ln\left(\frac{T_f}{T_{i,1}}\right) + m_2c_2 \ln\left(\frac{T_f}{T_{i,2}}\right)$$

$$= (50.0\text{ g})(386\text{ J/kg}\cdot\text{K})\ln\left(\frac{320\text{ K}}{400\text{ K}}\right) + (100\text{ g})(128\text{ J/kg}\cdot\text{K})\ln\left(\frac{320\text{ K}}{200\text{ K}}\right)$$

$$= +1.72\text{ J/K}.$$

9. The ice warms to 0°C, then melts, and the resulting water warms to the temperature of the lake water, which is 15°C. As the ice warms, the energy it receives as heat when the temperature changes by dT is $dQ = mc_I\,dT$, where m is the mass of the ice and c_I is the specific heat of ice. If T_i (= 263 K) is the initial temperature and T_f (= 273 K) is the final temperature, then the change in its entropy is

$$\Delta S = \int \frac{dQ}{T} = mc_I \int_{T_i}^{T_f} \frac{dT}{T} = mc_I \ln\frac{T_f}{T_i} = (0.010\text{ kg})(2220\text{ J/kg}\cdot\text{K})\ln\left(\frac{273\text{ K}}{263\text{ K}}\right) = 0.828\text{ J/K}.$$

Melting is an isothermal process. The energy leaving the ice as heat is mL_F, where L_F is the heat of fusion for ice. Thus,

$$\Delta S = Q/T = mL_F/T = (0.010\text{ kg})(333\times10^3\text{ J/kg})/(273\text{ K}) = 12.20\text{ J/K}.$$

For the warming of the water from the melted ice, the change in entropy is

$$\Delta S = mc_w \ln\frac{T_f}{T_i},$$

where c_w is the specific heat of water (4190 J/kg · K). Thus,

$$\Delta S = (0.010\text{ kg})(4190\text{ J/kg}\cdot\text{K})\ln\left(\frac{288\text{ K}}{273\text{ K}}\right) = 2.24\text{ J/K}.$$

The total change in entropy for the ice and the water it becomes is

$$\Delta S = 0.828\text{ J/K} + 12.20\text{ J/K} + 2.24\text{ J/K} = 15.27\text{ J/K}.$$

Since the temperature of the lake does not change significantly when the ice melts, the change in its entropy is $\Delta S = Q/T$, where Q is the energy it receives as heat (the negative of the energy it supplies the ice) and T is its temperature. When the ice warms to 0°C,

$$Q = -mc_I\left(T_f - T_i\right) = -(0.010\text{ kg})(2220\text{ J/kg}\cdot\text{K})(10\text{ K}) = -222\text{ J}.$$

When the ice melts,

$$Q = -mL_F = -(0.010\text{ kg})(333\times10^3\text{ J/kg}) = -3.33\times10^3\text{ J}.$$

When the water from the ice warms,

$$Q = -mc_w(T_f - T_i) = -(0.010\text{ kg})(4190\text{ J/kg}\cdot\text{K})(15\text{ K}) = -629\text{ J}.$$

The total energy leaving the lake water is

$$Q = -222\text{ J} - 3.33\times10^3\text{ J} - 6.29\times10^2\text{ J} = -4.18\times10^3\text{ J}.$$

The change in entropy is

$$\Delta S = -\frac{4.18\times10^3\text{ J}}{288\text{ K}} = -14.51\text{ J/K}.$$

The change in the entropy of the ice–lake system is $\Delta S = (15.27 - 14.51)\text{ J/K} = 0.76\text{ J/K}$.

15. (a) The final mass of ice is (1773 g + 227 g)/2 = 1000 g. This means 773 g of water froze. Energy in the form of heat left the system in the amount mL_F, where m is the mass of the water that froze and L_F is the heat of fusion of water. The process is isothermal, so the change in entropy is

$$\Delta S = Q/T = -mL_F/T = -(0.773\ \text{kg})(333 \times 10^3\ \text{J/kg})/(273\ \text{K}) = -943\ \text{J/K}.$$

(b) Now, 773 g of ice is melted. The change in entropy is

$$\Delta S = \frac{Q}{T} = \frac{mL_F}{T} = +943\ \text{J/K}.$$

(c) Yes, they are consistent with the second law of thermodynamics. Over the entire cycle, the change in entropy of the water–ice system is zero even though part of the cycle is irreversible. However, the system is not closed. To consider a closed system, we must include whatever exchanges energy with the ice and water. Suppose it is a constant-temperature heat reservoir during the freezing portion of the cycle and a Bunsen burner during the melting portion. During freezing the entropy of the reservoir increases by 943 J/K. As far as the reservoir–water–ice system is concerned, the process is adiabatic and reversible, so its total entropy does not change. The melting process is irreversible, so the total entropy of the burner–water–ice system increases. The entropy of the burner either increases or else decreases by less than 943 J/K.

18. (a) It is possible to motivate, starting from Eq. 20-3, the notion that heat may be found from the integral (or "area under the curve") of a curve in a *TS* diagram, such as this one. Either from calculus, or from geometry (area of a trapezoid), it is straightforward to find the result for a "straight-line" path in the *TS* diagram:

$$Q_{\text{straight}} = \left(\frac{T_i + T_f}{2}\right)\Delta S$$

which could, in fact, be *directly* motivated from Eq. 20-3 (but it is important to bear in mind that this is rigorously true only for a process that forms a straight line in a graph that plots *T* versus *S*). This leads to

$$Q = (300\text{ K})\,(15\text{ J/K}) = 4.5\times10^3\text{ J}$$

for the energy absorbed as heat by the gas.

(b) Using Table 19-3 and Eq. 19-45, we find

$$\Delta E_{\text{int}} = n\left(\frac{3}{2}R\right)\Delta T = (2.0\text{ mol})(8.31\text{ J/mol}\cdot\text{K})(200\text{ K} - 400\text{ K}) = -5.0\times10^3\text{ J.}$$

(c) By the first law of thermodynamics, $W = Q - \Delta E_{\text{int}} = 4.5\text{ kJ} - \text{a}{-5.0}\text{ kJ}\text{f} = 9.5\text{ kJ}$.

23. With $T_L = 290$ k, we find

$$\varepsilon = 1 - \frac{T_L}{T_H} \Rightarrow T_H = \frac{T_L}{1-\varepsilon} = \frac{290\text{ K}}{1-0.40}$$

which yields the (initial) temperature of the high-temperature reservoir: $T_H = 483$ K. If we replace $\varepsilon = 0.40$ in the above calculation with $\varepsilon = 0.50$, we obtain a (final) high temperature equal to $T'_H = 580\text{ K}$. The difference is

$$T'_H - T_H = 580\text{ K} - 483\text{ K} = 97\text{ K}.$$

25. We solve (b) first.

(b) For a Carnot engine, the efficiency is related to the reservoir temperatures by Eq. 20-13. Therefore,

$$T_H = \frac{T_H - T_L}{\varepsilon} = \frac{75\text{ K}}{0.22} = 341\text{ K}$$

which is equivalent to 68°C.

(a) The temperature of the cold reservoir is $T_L = T_H - 75 = 341\text{ K} - 75\text{ K} = 266$ K.

27. **THINK** The thermal efficiency of the Carnot engine depends on the temperatures of the reservoirs.

EXPRESS The efficiency of the Carnot engine is given by

$$\varepsilon_C = \frac{T_{\mathrm{H}} - T_{\mathrm{L}}}{T_{\mathrm{H}}},$$

where T_H is the temperature of the higher-temperature reservoir, and T_L the temperature of the lower-temperature reservoir, in kelvin scale. The work done by the engine is $|W| = \varepsilon |Q_{\mathrm{H}}|$.

ANALYZE (a) The efficiency of the engine is

$$\varepsilon_c = \frac{T_{\mathrm{H}} - T_{\mathrm{L}}}{T_{\mathrm{H}}} = \frac{(235-115)\,\mathrm{K}}{(235+273)\,\mathrm{K}} = 0.236 = 23.6\%\,.$$

We note that a temperature difference has the same value on the Kelvin and Celsius scales. Since the temperatures in the equation must be in Kelvins, the temperature in the denominator is converted to the Kelvin scale.

(b) Since the efficiency is given by $\varepsilon = |W|/|Q_{\mathrm{H}}|$, the work done is given by

$$|W| = \varepsilon |Q_{\mathrm{H}}| = 0.236(6.30\times10^4\ \mathrm{J}) = 1.49\times10^4\ \mathrm{J}\,.$$

LEARN Expressing the efficiency as $\varepsilon_c = 1 - T_{\mathrm{L}}/T_{\mathrm{H}}$, we see that ε_c approaches unity (100% efficiency) in the limit $T_L/T_H \to 0$. This is an impossible dream. An alternative version of the second law of thermodynamics is: *there are no perfect engines*. ■

29. (a) The net work done is the rectangular "area" enclosed in the pV diagram:

$$W = (V - V_0)(p - p_0) = (2V_0 - V_0)(2p_0 - p_0) = V_0 p_0 .$$

Inserting the values stated in the problem, we obtain $W = 2.27$ kJ.

(b) We compute the energy added as heat during the "heat-intake" portions of the cycle using Eq. 19-39, Eq. 19-43, and Eq. 19-46:

$$\begin{aligned} Q_{abc} &= nC_V(T_b - T_a) + nC_p(T_c - T_b) = n\left(\frac{3}{2}R\right)T_a\left(\frac{T_b}{T_a} - 1\right) + n\left(\frac{5}{2}R\right)T_a\left(\frac{T_c}{T_a} - \frac{T_b}{T_a}\right) \\ &= nRT_a\left(\frac{3}{2}\left(\frac{T_b}{T_a} - 1\right) + \frac{5}{2}\left(\frac{T_c}{T_a} - \frac{T_b}{T_a}\right)\right) = p_0V_0\left(\frac{3}{2}(2-1) + \frac{5}{2}(4-2)\right) \\ &= \frac{13}{2}p_0V_0 \end{aligned}$$

where, to obtain the last line, the gas law in ratio form has been used. Therefore, since $W = p_0V_0$, we have $Q_{abc} = 13W/2 = 14.8$ kJ.

(c) The efficiency is given by Eq. 20-11:

$$\varepsilon = \frac{W}{|Q_H|} = \frac{2}{13} = 0.154 = 15.4\%.$$

(d) A Carnot engine operating between T_c and T_a has efficiency equal to

$$\varepsilon = 1 - \frac{T_a}{T_c} = 1 - \frac{1}{4} = 0.750 = 75.0\%$$

where the gas law in ratio form has been used.

(e) This is greater than our result in part (c), as expected from the second law of thermodynamics.

33. **THINK** Our engine cycle consists of three steps: isochoric heating (*a* to *b*), adiabatic expansion (*b* to *c*), and isobaric compression (*c* to *a*).

EXPRESS Energy is added as heat during the portion of the process from *a* to *b*. This portion occurs at constant volume (V_b), so $Q_H = nC_V\,\Delta T$. The gas is a monatomic ideal gas, so $C_V = 3R/2$ and the ideal gas law gives

$$\Delta T = (1/nR)(p_b V_b - p_a V_a) = (1/nR)(p_b - p_a)V_b.$$

Thus, $Q_H = \frac{3}{2}(p_b - p_a)V_b$. On the other hand, energy leaves the gas as heat during the portion of the process from *c* to *a*. This is a constant pressure process, so

$$Q_L = nC_p\Delta T = nC_p(T_a - T_c) = nC_p\left(\frac{p_aV_a}{nR} - \frac{p_cV_c}{nR}\right) = \frac{C_p}{R}p_a(V_a - V_c)\ .$$

where C_p is the molar specific heat for constant-pressure process.

ANALYZE (a) V_b and p_b are given. We need to find p_a. Now p_a is the same as p_c and points *c* and *b* are connected by an adiabatic process. With $p_cV_c^\gamma = p_bV_b^\gamma$ for the adiabat, we have ($\gamma = 5/3$ for monatomic gas)

$$p_a = p_c = \left(\frac{V_b}{V_c}\right)^\gamma p_b = \left(\frac{1}{8.00}\right)^{5/3}\left(1.013\times10^6\text{ Pa}\right) = 3.167\times10^4\text{ Pa}.$$

Thus, the energy added as heat is

$$Q_H = \frac{3}{2}(p_b - p_a)V_b = \frac{3}{2}\left(1.013\times10^6\text{ Pa} - 3.167\times10^4\text{ Pa}\right)\left(1.00\times10^{-3}\text{ m}^3\right) = 1.47\times10^3\text{ J}.$$

(b) The energy leaving the gas as heat going from *c* to *a* is

$$Q_L = \frac{5}{2}p_a(V_a - V_c) = \frac{5}{2}\left(3.167\times10^4\text{ Pa}\right)(-7.00)\left(1.00\times10^{-3}\text{ m}^3\right) = -5.54\times10^2\text{ J}\ ,$$

or $|Q_L| = 5.54\times10^2$ J. The substitutions $V_a - V_c = V_a - 8.00\,V_a = -7.00\,V_a$ and $C_p = \frac{5}{2}R$ were made.

(c) For a complete cycle, the change in the internal energy is zero and

$$W = Q = Q_H - Q_L = 1.47 \times 10^3 \text{ J} - 5.54 \times 10^2 \text{ J} = 9.18 \times 10^2 \text{ J}.$$

(d) The efficiency is

$$\varepsilon = W/Q_H = (9.18 \times 10^2 \text{ J})/(1.47 \times 10^3 \text{ J}) = 0.624 = 62.4\%.$$

LEARN To summarize, the heat engine in this problem intakes energy as heat (from, say, consuming fuel) equal to $|Q_H| = 1.47$ kJ and exhausts energy as heat equal to $|Q_L| = 554$ J; its efficiency and net work are $\varepsilon = 1 - |Q_L|/|Q_H|$ and $W = |Q_H| - |Q_L|$. The less the exhaust heat $|Q_L|$, the more efficient is the engine.

34. (a) Using Eq. 19-54 for process $D \to A$ gives

$$p_D V_D^{\gamma} = p_A V_A^{\gamma} \quad \Rightarrow \quad \frac{p_0}{32}(8V_0)^{\gamma} = p_0 V_0^{\gamma}$$

which leads to $8^{\gamma} = 32 \Rightarrow \gamma = 5/3$. The result (see Sections 19-9 and 19-11) implies the gas is monatomic.

(b) The input heat is that absorbed during process $A \to B$:

$$Q_{\rm H} = nC_p\Delta T = n\left(\frac{5}{2}R\right)T_A\left(\frac{T_B}{T_A}-1\right) = nRT_A\left(\frac{5}{2}\right)(2-1) = p_0V_0\left(\frac{5}{2}\right)$$

and the exhaust heat is that liberated during process $C \to D$:

$$Q_{\rm L} = nC_p\Delta T = n\left(\frac{5}{2}R\right)T_D\left(1-\frac{T_{\rm L}}{T_D}\right) = nRT_D\left(\frac{5}{2}\right)(1-2) = -\frac{1}{4}p_0V_0\left(\frac{5}{2}\right)$$

where in the last step we have used the fact that $T_D = \frac{1}{4}T_A$ (from the gas law in ratio form). Therefore, Eq. 20-12 leads to

$$\varepsilon = 1 - \left|\frac{Q_{\rm L}}{Q_{\rm H}}\right| = 1 - \frac{1}{4} = 0.75 = 75\%.$$

37. **THINK** The performance of the refrigerator is related to its rate of doing work.

EXPRESS The coefficient of performance for a refrigerator is given by

$$K = \frac{\text{what we want}}{\text{what we pay for}} = \frac{|Q_L|}{|W|},$$

where Q_L is the energy absorbed from the cold reservoir as heat and W is the work done during the refrigeration cycle, a negative value. The first law of thermodynamics yields

$Q_H + Q_L - W = 0$ for an integer number of cycles. Here Q_H is the energy ejected to the hot reservoir as heat. Thus, $Q_L = W - Q_H$. Q_H is negative and greater in magnitude than W, so $|Q_L| = |Q_H| - |W|$. Thus,

$$K = \frac{|Q_H| - |W|}{|W|}.$$

The solution for $|W|$ is $|W| = |Q_H|/(K + 1)$.

ANALYZE In one hour, $|Q_H| = 7.54\ \text{MJ}$. With $K = 3.8$, the work done is

$$|W| = \frac{7.54\ \text{MJ}}{3.8+1} = 1.57\ \text{MJ}.$$

The rate at which work is done is $P = |W|/\Delta t = (1.57 \times 10^6\ \text{J})/(3600\ \text{s}) = 440\ \text{W}$.

LEARN The greater the value of K, the less the amount of work $|W|$ required to transfer the heat.

39. **THINK** A large (small) value of coefficient of performance K means that less (more) work would be required to transfer the heat

EXPRESS A Carnot refrigerator working between a hot reservoir at temperature T_H and a cold reservoir at temperature T_L has a coefficient of performance K that is given by

$$K = \frac{T_L}{T_H - T_L},$$

where T_H is the temperature of the higher-temperature reservoir, and T_L the temperature of the lower-temperature reservoir, in Kelvin scale. Equivalently, the coefficient of performance is the energy Q_L drawn from the cold reservoir as heat divided by the work done: $K = |Q_L|/|W|$.

ANALYZE For the refrigerator of this problem, T_H = 96° F = 309 K and T_L = 70° F = 294 K, so

$$K = (294\text{ K})/(309\text{ K} - 294\text{ K}) = 19.6.$$

Thus, with $|W| = 1.0\text{ J}$, the amount of heat removed from the room is

$$|Q_L| = K|W| = (19.6)(1.0\text{ J}) = 20\text{ J}.$$

LEARN The Carnot air conditioner in this problem (with K = 19.6) are much more efficient than that of the typical room air conditioners ($K \approx 2.5$).

42. The work done by the motor in $t = 10.0$ min is $|W| = Pt = (200\text{ W})(10.0\text{ min})(60\text{ s/min}) = 1.20 \times 10^5$ J. The heat extracted is then

$$|Q_L| = K|W| = \frac{T_L|W|}{T_H - T_L} = \frac{(270\,\text{K})\left(1.20\times10^5\,\text{J}\right)}{300\,\text{K} - 270\,\text{K}} = 1.08\times10^6\,\text{J}.$$

45. We need nine labels:

Label	Number of molecules on side 1	Number of molecules on side 2
I	8	0
II	7	1
III	6	2
IV	5	3
V	4	4
VI	3	5
VII	2	6
VIII	1	7
IX	0	8

The multiplicity W is computing using Eq. 20-20. For example, the multiplicity for label IV is

$$W = \frac{8!}{(5!)(3!)} = \frac{40320}{(120)(6)} = 56$$

and the corresponding entropy is (using Eq. 20-21)

$$S = k \ln W = \left(1.38\times10^{-23}\text{ J/K}\right)\ln(56) = 5.6\times10^{-23}\text{ J/K}.$$

In this way, we generate the following table:

Label	W	S
I	1	0
II	8	2.9×10^{-23} J/K
III	28	4.6×10^{-23} J/K
IV	56	5.6×10^{-23} J/K
V	70	5.9×10^{-23} J/K
VI	56	5.6×10^{-23} J/K
VII	28	4.6×10^{-23} J/K
VIII	8	2.9×10^{-23} J/K
IX	1	0

46. (a) We denote the configuration with n heads out of N trials as $(n; N)$. We use Eq. 20-20:

$$W(25;50) = \frac{50!}{(25!)(50-25)!} = 1.26\times 10^{14}.$$

(b) There are 2 possible choices for each molecule: it can either be in side 1 or in side 2 of the box. If there are a total of N independent molecules, the total number of available states of the N-particle system is

$$N_{\text{total}} = 2\times 2\times 2\times\cdots\times 2 = 2^N.$$

With $N = 50$, we obtain $N_{\text{total}} = 2^{50} = 1.13 \times 10^{15}$.

(c) The percentage of time in question is equal to the probability for the system to be in the central configuration:

$$p(25;50) = \frac{W(25;50)}{2^{50}} = \frac{1.26\times 10^{14}}{1.13\times 10^{15}} = 11.1\%.$$

With $N = 100$, we obtain

(d) $W(N/2, N) = N!/[(N/2)!]^2 = 1.01 \times 10^{29}$,

(e) $N_{\text{total}} = 2^N = 1.27 \times 10^{30}$,

(f) and $p(N/2; N) = W(N/2, N)/ N_{\text{total}} = 8.0\%$.

Similarly, for $N = 200$, we obtain

(g) $W(N/2, N) = 9.25 \times 10^{58}$,

(h) $N_{\text{total}} = 1.61 \times 10^{60}$,

(i) and $p(N/2; N) = 5.7\%$.

(j) As N increases, the number of available microscopic states increases as 2^N, so there are more states to be occupied, leaving the probability less for the system to remain in its central configuration. Thus, the time spent there decreases with an increase in N.