19. Table 19-1 gives $M$ = 28.0 g/mol for nitrogen. This value can be used in Eq. 19-22 with $T$ in Kelvins to obtain the results. A variation on this approach is to set up ratios, using the fact that Table 19-1 also gives the rms speed for nitrogen gas at 300 K (the value is 517 m/s). Here we illustrate the latter approach, using $v$ for $v_{rms}$:

$$\frac{v_2}{v_1} = \frac{\sqrt{3RT_2/M}}{\sqrt{3RT_1/M}} = \sqrt{\frac{T_2}{T_1}}.$$

(a) With $T_2 = (20.0 + 273.15)$ K ≈ 293 K, we obtain $v_2 = (517\,\text{m/s})\sqrt{\frac{293\,\text{K}}{300\,\text{K}}} = 511\,\text{m/s}.$

(b) In this case, we set $v_3 = \frac{1}{2}v_2$ and solve $v_3/v_2 = \sqrt{T_3/T_2}$ for $T_3$:

$$T_3 = T_2\left(\frac{v_3}{v_2}\right)^2 = (293\,\text{K})\left(\frac{1}{2}\right)^2 = 73.0\,\text{K}$$

which we write as 73.0 – 273 = – 200°C.

(c) Now we have $v_4 = 2v_2$ and obtain

$$T_4 = T_2\left(\frac{v_4}{v_2}\right)^2 = (293\,\text{K})(4) = 1.17\times10^3\,\text{K}$$

which is equivalent to 899°C.

21. **THINK** According to kinetic theory, the rms speed is (see Eq. 19-34) $v_{\text{rms}} = \sqrt{3RT/M}$, where $T$ is the temperature and $M$ is the molar mass.

**EXPRESS** The rms speed is defined as $v_{\text{rms}} = \sqrt{(v^2)_{\text{avg}}}$, where $(v^2)_{\text{avg}} = \int_0^\infty v^2 P(v)dv$, with the Maxwell's speed distribution function given by

$$P(v) = 4\pi\left(\frac{M}{2\pi RT}\right)^{3/2} v^2 e^{-Mv^2/2RT}.$$

According to Table 19-1, the molar mass of molecular hydrogen is 2.02 g/mol = 2.02 × $10^{-3}$ kg/mol.

**ANALYZE** At $T = 2.7\text{ K}$, we find the rms speed to be

$$v_{\text{rms}} = \sqrt{\frac{3\,(8.31\,\text{J/mol}\cdot\text{K})(2.7\,\text{K})}{2.02\times10^{-3}\,\text{kg/mol}}} = 1.8\times10^2\ \text{m/s}.$$

**LEARN** The corresponding average speed and most probable speed are

$$v_{\text{avg}} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8(8.31\,\text{J/mol}\cdot\text{K})(2.7\,\text{K})}{\pi(2.02\times10^{-3}\,\text{kg/mol})}} = 1.7\times10^2\ \text{m/s}$$

and

$$v_p = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2(8.31\,\text{J/mol}\cdot\text{K})(2.7\,\text{K})}{2.02\times10^{-3}\,\text{kg/mol}}} = 1.5\times10^2\ \text{m/s},$$

respectively.

23. In the reflection process, only the normal component of the momentum changes, so for one molecule the change in momentum is $2mv\cos\theta$, where $m$ is the mass of the molecule, $v$ is its speed, and $\theta$ is the angle between its velocity and the normal to the wall. If $N$ molecules collide with the wall, then the change in their total momentum is $2Nmv\cos\theta$, and if the total time taken for the collisions is $\Delta t$, then the average rate of change of the total momentum is $2(N/\Delta t)mv\cos\theta$. This is the average force exerted by the $N$ molecules on the wall, and the pressure is the average force per unit area:

$$p = \frac{2}{A}\left(\frac{N}{\Delta t}\right) mv\cos\theta = \left(\frac{2}{2.0\times10^{-4}\,\text{m}^2}\right)\left(1.0\times10^{23}\,\text{s}^{-1}\right)\left(3.3\times10^{-27}\,\text{kg}\right)\left(1.0\times10^{3}\,\text{m/s}\right)\cos 55°$$

$$=1.9\times10^{3}\,\text{Pa}.$$

We note that the value given for the mass was converted to kg and the value given for the area was converted to $m^2$.

26. The average translational kinetic energy is given by $K_{avg} = \frac{3}{2}kT$, where $k$ is the Boltzmann constant ($1.38 \times 10^{-23}$ J/K) and $T$ is the temperature on the Kelvin scale. Thus

$$K_{avg} = \frac{3}{2}(1.38 \times 10^{-23}\ \text{J/K})(1600\ \text{K}) = 3.31 \times 10^{-20}\ \text{J}.$$

29. **THINK** Mean free path is the average distance traveled by a molecule between successive collisions.

**EXPRESS** According to Eq. 19-25, the mean free path for molecules in a gas is given by

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 N/V},$$

where $d$ is the diameter of a molecule and $N$ is the number of molecules in volume $V$.

**ANALYZE** (a) Substituting $d = 2.0 \times 10^{-10}$ m and $N/V = 1 \times 10^6$ molecules/m$^3$, we obtain

$$\lambda = \frac{1}{\sqrt{2}\pi(2.0 \times 10^{-10}\text{ m})^2 (1 \times 10^6\text{ m}^{-3})} = 6 \times 10^{12}\text{ m}.$$

(b) At this altitude most of the gas particles are in orbit around Earth and do not suffer randomizing collisions. The mean free path has little physical significance.

**LEARN** Mean free path is inversely proportional to the number density, $N/V$. The typical value of $N/V$ at room temperature and atmospheric pressure for ideal gas is

$$\frac{N}{V} = \frac{p}{kT} = \frac{1.01\times10^5\text{ Pa}}{(1.38\times10^{-23}\text{ J/K})(298\text{K})} = 2.46\times10^{25}\text{ molecules/m}^3 = 2.46\times10^{19}\text{ molecules/cm}^3.$$

This is much higher than that in the outer space.

30. We solve Eq. 19-25 for $d$:

$$d = \sqrt{\frac{1}{\lambda\pi\sqrt{2}\,(N/V)}} = \sqrt{\frac{1}{(0.80 \times 10^5\text{ cm})\ \pi\sqrt{2}\,(2.7 \times 10^{19}/\text{cm}^3)}}$$

which yields $d = 3.2 \times 10^{-8}$ cm, or 0.32 nm.

35. (a) The average speed is

$$v_{\text{avg}} = \frac{1}{N}\sum_{i=1}^{N} v_i = \frac{1}{10}[4(200\text{ m/s}) + 2(500\text{ m/s}) + 4(600\text{ m/s})] = 420\text{ m/s}.$$

(b) The rms speed is

$$v_{\text{rms}} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} v_i^2} = \sqrt{\frac{1}{10}[4(200\text{ m/s})^2 + 2(500\text{ m/s})^2 + 4(600\text{ m/s})^2]} = 458\text{ m/s}$$

(c) Yes, $v_{\text{rms}} > v_{\text{avg}}$.

37. **THINK** From the distribution function $P(v)$, we can calculate the average and rms speeds.

**EXPRESS** The distribution function gives the fraction of particles with speeds between $v$ and $v + dv$, so its integral over all speeds is unity: $\int P(v)\,dv = 1$. The average speed is defined as $v_{avg} = \int_0^\infty vP(v)dv$. Similarly, the rms speed is given by $v_{rms} = \sqrt{(v^2)_{avg}}$, where $(v^2)_{avg} = \int_0^\infty v^2 P(v)dv$.

**ANALYZE** (a) Evaluate the integral by calculating the area under the curve in Fig. 19-23. The area of the triangular portion is half the product of the base and altitude, or $\frac{1}{2}av_0$. The area of the rectangular portion is the product of the sides, or $av_0$. Thus,

$$\int P(v)dv = \frac{1}{2}av_0 + av_0 = \frac{3}{2}av_0 ,$$

so $\frac{3}{2}av_0 = 1$ and $av_0 = 2/3 = 0.67$.

(b) For the triangular portion of the distribution $P(v) = av/v_0$, and the contribution of this portion is

$$\frac{a}{v_0}\int_0^{v_0} v^2 dv = \frac{a}{3v_0}v_0^3 = \frac{av_0^2}{3} = \frac{2}{9}v_0,$$

where $2/3v_0$ was substituted for $a$. $P(v) = a$ in the rectangular portion, and the contribution of this portion is

$$a\int_{v_0}^{2v_0} v\,dv = \frac{a}{2}\left(4v_0^2 - v_0^2\right) = \frac{3a}{2}v_0^2 = v_0.$$

Therefore, we have

$$v_{avg} = \frac{2}{9}v_0 + v_0 = 1.22v_0 \quad \Rightarrow \quad \frac{v_{avg}}{v_0} = 1.22 .$$

(c) In calculating $v_{avg}^2 = \int v^2 P(v)\,dv$, we note that the contribution of the triangular section is

$$\frac{a}{v_0}\int_0^{v_0} v^3\,dv = \frac{a}{4v_0}v_0^4 = \frac{1}{6}v_0^2.$$

The contribution of the rectangular portion is

$$a\int_{v_0}^{2v_0} v^2\,dv = \frac{a}{3}\left(8v_0^3 - v_0^3\right) = \frac{7a}{3}v_0^3 = \frac{14}{9}v_0^2.$$

Thus,

$$v_{rms} = \sqrt{\frac{1}{6}v_0^2 + \frac{14}{9}v_0^2} = 1.31v_0 \;\Rightarrow\; \frac{v_{rms}}{v_0} = 1.31\,.$$

(d) The number of particles with speeds between $1.5v_0$ and $2v_0$ is given by $N\int_{1.5v_0}^{2v_0} P(v)dv$. The integral is easy to evaluate since $P(v) = a$ throughout the range of integration. Thus the number of particles with speeds in the given range is

$$Na(2.0v_0 - 1.5v_0) = 0.5N\,av_0 = N/3,$$

where $2/3v_0$ was substituted for $a$. In other words, the fraction of particles in this range is 1/3 or 0.33.

40. We divide Eq. 19-31 by Eq. 19-22:

$$\frac{v_{\text{avg2}}}{v_{\text{rms1}}} = \frac{\sqrt{8RT/\pi M_2}}{\sqrt{3RT/M_1}} = \sqrt{\frac{8M_1}{3\pi M_2}}$$

which, for $v_{\text{avg2}} = 2v_{\text{rms1}}$, leads to

$$\frac{m_1}{m_2} = \frac{M_1}{M_2} = \frac{3\pi}{8}\left(\frac{v_{\text{avg2}}}{v_{\text{rms1}}}\right)^2 = \frac{3\pi}{2} = 4.7\,.$$