

Problem 1 (10 pts)

An ice cube of mass 20g is floating in water, a fly of mass 1g is standing on it. The fly flies away and the cube starts oscillating up and down.

Density of water = 1g/cm^3 ; density of ice = 0.917g/cm^3 .

- Find the amplitude of oscillation, in cm.
- Find the frequency of oscillation, in Hz.
- Find the energy of this oscillator, in erg.

Problem 2 (10 pts)

A hot air balloon has volume 2800m^3 when filled with hot air. It is made of nylon that has thermal conductivity $0.25\text{W}/(\text{m K})$. The weight of the nylon, basket, burners and fuel is 400 kg. The burners heat the air in the balloon to 100°C . The ambient temperature is 20°C , and the density of air at sea level at 20°C is 1.225kg/m^3 and decreases with height h according to the equation $\rho(h) = \rho_0 e^{-h/h_0}$, with $h_0=8000\text{m}$. Assume the pressure of the hot air inside the balloon is the same as the pressure outside.

- How many people of weight 65kg each can this balloon lift?
- The burners supply 2MW power. How thick is the nylon?
- If the weight of the people is just so the balloon can rise, how high will it go if the temperature is raised to 110°C ?

Problem 3 (10 pts)

Two identical bodies are at initial absolute temperatures T_0 and $3T_0$ respectively. Their heat capacity is independent of temperature. When put into thermal contact, heat Q_0 flows from one to the other until they reach thermal equilibrium.

- Find by how much the entropy of the universe changes in this process. Give the answer in terms of Q_0 and T_0 .
- If instead of putting the bodies into thermal contact you operate a heat engine between them: what is the maximum amount of work you can extract in the process of the two bodies reaching thermal equilibrium, starting from initial temperatures T_0 and $3T_0$? Give your answer in terms of Q_0 .
- What is the final temperature of the bodies at the end of this process? Give your answer in terms of T_0 .
- What is the total change in entropy of the environment in this process?

Problem 4 (10 pts)

A cylinder with a movable piston has 0.1 moles of water vapor occupying initially a volume of 20L ($1\text{L}=10^{-3}\text{m}^3$). There are no other gases in the container. The container is in thermal contact with a heat reservoir at temperature 60°C .

Assume water vapor behaves as an ideal gas until it condenses. The saturated vapor pressure of water at 60°C is $2 \times 10^4\text{Pa}$. Gas constant is $R=8.314\text{J/mol K}$. Heat of vaporization of water at this temperature is $L_v=130\text{J/mol}$.

The piston is slowly pushed down to reduce the volume to a final volume of 5L.

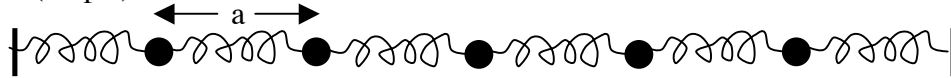
- How much liquid water and how much vapor is there at the end (in moles), to at least 1% accuracy?
- What was the total work done on the system in the process? Give your answer in J.
- What was the total amount of heat absorbed by the heat reservoir in the process, in J?

Problem 5 (10 pts+5 pts extra credit)

10^{20} atoms of a monatomic gas of atomic weight 20 are in a cubic box that is thermally insulated from its environment. Initially, $1/6$ of the atoms are moving in the $+x$ direction, $1/6$ in the $-x$ direction, and similarly for y and z , all with speed 350m/s . After a while, the system reaches equilibrium. $k=1.38\times 10^{-23}\text{J/K}$, $u=1.6605\times 10^{-27}\text{kg}$.

- What is the temperature when the system reaches equilibrium? Give answer in K.
- What is the most probable value of the speed of an atom in equilibrium? Give your answer in m/s.
- Assume that very slowly the walls of this container are moved inward till the volume is reduced to $1/2$ its initial value. What is the most probable value for the speed now?
For extra credit: assume now this is a diatomic gas instead, with molecular weight 20, initially the molecules have translational motion as given above and no rotational motion. Assume vibrational motion never plays a role.
- Repeat (a) and (b) for this case.
- Repeat (c) for this case.

Problem 6 (10 pts)



Consider the system of 5 masses connected by springs shown in the picture. All masses and springs are identical and can move in the horizontal direction, the ends are fixed. The period of oscillation for the lowest frequency normal mode is 10 s. The distance between the masses in equilibrium is a .

Consider the possible normal modes where the mass in the center doesn't move.

- For one such mode where the two masses to the left of the center mass move in the same direction, what is the period, in seconds?
- For another such mode where the two masses to the left of the center mass move in opposite direction, what is the period, in seconds?
- Find the wavelengths of those two modes, in terms of a .
- Find the speed of traveling waves with the wavelengths calculated in (c) in this chain, expressed as (number) $\times(a/s)$. Explain why they are the same, similar or very different.

Problem 7 (10 pts)

A guitar string is under 800N tension and emits sound of fundamental frequency 500Hz . If you put your finger on the string to reduce its length by 2cm , the frequency increases to 513Hz .

- Find the mass of the string, in g.
- Find the length of the string, in cm.
- Find the speed of traveling waves in this string, in m/s.

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Problem 8 (10 pts+3 pts extra credit)

A sound wave of wavelength 10cm is traveling in a monatomic gas at speed 500 m/s. The density of the gas is 700 kg/m^3 . The amplitude of motion of the gas molecules when the wave passes by is 10^{-8}m .

- (a) Find the angular frequency ω of this wave, in rad/s.
- (b) What is the maximum speed that a gas molecule acquires when the wave passes by? (in mm/s).
- (c) What is the maximum pressure variation when the wave passes by, in Pa ($=\text{N/m}^2$)?
- (d) What is the intensity of this sound wave, in W/m^2 ?
- (e) If the gas is monatomic, what is its pressure? (in the absence of the sound wave).