13. The speed of the waves on the cord can be found from Eq. 15-2, $v=\sqrt{F_T/\mu}$. The distance between the children is the wave speed times the elapsed time.

$$\Delta x = v\Delta t = \Delta t\sqrt{\frac{F_T}{m/\Delta x}} \rightarrow \Delta x = (\Delta t)^2\frac{F_T}{m} = (0.50\,\text{s})^2\frac{35\,\text{N}}{0.50\,\text{kg}} = \boxed{18\,\text{m}}$$

19. (*a*) The power transmitted by the wave is assumed to be the same as the output of the oscillator. That power is given by Eq. 15-6. The wave speed is given by Eq. 15-2. Note that the mass per unit length can be expressed as the volume mass density times the cross sectional area.

$$\bar{P} = 2\pi^2\rho Svf^2A^2 = 2\pi^2\rho S\sqrt{\frac{F_T}{\mu}}f^2A^2 = 2\pi^2\rho S\sqrt{\frac{F_T}{\rho S}}f^2A^2 = 2\pi^2 f^2A^2\sqrt{S\rho F_T}$$

$$= 2\pi^2(60.0\,\text{Hz})^2(0.0050\,\text{m})^2\sqrt{\pi\left(5.0\times10^{-3}\,\text{m}\right)^2\left(7800\,\text{kg/m}^3\right)(7.5\,\text{N})} = \boxed{0.38\,\text{W}}$$

(*b*) The frequency and amplitude are both squared in the equation. Thus is the power is constant, and the frequency doubles, the amplitude must be halved, and so be $\boxed{0.25\,\text{cm}}$.

20. Consider a wave traveling through an area *S* with speed *v*, much like Figure 15-11. Start with Eq. 15-7, and use Eq. 15-6.

$$I = \frac{\bar{P}}{S} = \frac{E}{St} = \frac{El}{Sl\,t} = \frac{E}{Sl}\frac{l}{t} = \frac{\text{energy}}{\text{volume}}\times v$$

21. (*a*) We start with Eq. 15-6. The linear mass density is the mass of a given volume of the cord divided by the cross-sectional area of the cord.

$$\bar{P} = 2\pi^2\rho Svf^2A^2 \ ; \ \mu = \frac{m}{l} = \frac{\rho V}{l} = \frac{\rho Sl}{l} = \rho S \rightarrow \bar{P} = 2\pi^2\mu vf^2A^2$$

(*b*) The speed of the wave is found from the given tension and mass density, according to Eq. 15-2.

$$\bar{P} = 2\pi^2\mu vf^2A^2 = 2\pi^2f^2A^2\mu\sqrt{F_T/\mu} = 2\pi^2f^2A^2\sqrt{\mu F_T}$$

$$= 2\pi^2(120\,\text{Hz})^2(0.020\,\text{m})^2\sqrt{(0.10\,\text{kg/m})(135\,\text{N})} = \boxed{420\,\text{W}}$$

22. (a) The only difference is the direction of motion.

$$D(x,t) = 0.015\sin(25x+1200t)$$

(*b*) The speed is found from the wave number and the angular frequency, Eq. 15-12.

$$v = \frac{\omega}{k} = \frac{1200\,\text{rad/s}}{25\,\text{rad/m}} = \boxed{48\,\text{m/s}}$$

24. The traveling wave is given by $D = 0.22\sin(5.6x+34t)$.

(*a*) The wavelength is found from the coefficient of *x*.

$$5.6\,\text{m}^{-1} = \frac{2\pi}{\lambda} \rightarrow \lambda = \frac{2\pi}{5.6\,\text{m}^{-1}} = 1.122\,\text{m} \approx \boxed{1.1\,\text{m}}$$

(*b*) The frequency is found from the coefficient of *t*.

$$34\,\text{s}^{-1} = 2\pi f \rightarrow f = \frac{34\,\text{s}^{-1}}{2\pi} = 5.411\,\text{Hz} \approx \boxed{5.4\,\text{Hz}}$$

(*c*) The velocity is the ratio of the coefficients of *t* and *x*.

$$v = \lambda f = \frac{2\pi}{5.6\,\text{m}^{-1}}\frac{34\,\text{s}^{-1}}{2\pi} = 6.071\,\text{m/s} \approx \boxed{6.1\,\text{m/s}}$$

Because both coefficients are positive, the velocity is in the $\boxed{\text{negative } x \text{ direction}}$.

(d) The amplitude is the coefficient of the sine function, and so is $\boxed{0.22\,\text{m}}$.

(e) The particles on the cord move in simple harmonic motion with the same frequency as the wave. From Chapter 14, $v_{\text{max}} = D\omega = 2\pi fD$.

$$v_{\text{max}} = 2\pi fD = 2\pi\left(\frac{34\,\text{s}^{-1}}{2\pi}\right)(0.22\,\text{m}) = \boxed{7.5\,\text{m/s}}$$

The minimum speed is when a particle is at a turning point of its motion, at which time the speed is 0.

$$v_{\text{min}} = \boxed{0}$$

26. The displacement of a point on the cord is given by the wave, $D(x,t) = 0.12\sin(3.0x - 15.0t)$. The velocity of a point on the cord is given by $\frac{\partial D}{\partial t}$.

$$D(0.60\,\text{m}, 0.20\,\text{s}) = (0.12\,\text{m})\sin\left[(3.0\,\text{m}^{-1})(0.60\,\text{m}) - (15.0\,\text{s}^{-1})(0.20\,\text{s})\right] = \boxed{-0.11\,\text{m}}$$

$$\frac{\partial D}{\partial t} = (0.12\,\text{m})(-15.0\,\text{s}^{-1})\cos(3.0x - 15.0t)$$

$$\frac{\partial D}{\partial t}(0.60\,\text{m}, 0.20\,\text{s}) = (0.12\,\text{m})(-15.0\,\text{s}^{-1})\cos\left[(3.0\,\text{m}^{-1})(0.60\,\text{m}) - (15.0\,\text{s}^{-1})(0.20\,\text{s})\right] = \boxed{-0.65\,\text{m/s}}$$

34. Find the various derivatives for the linear combination.

$$D(x,t) = C_1D_1 + C_2D_2 = C_1f_1(x,t) + C_2f_2(x,t)$$

$$\frac{\partial D}{\partial x} = C_1\frac{\partial f_1}{\partial x} + C_2\frac{\partial f_2}{\partial x} \quad ; \quad \frac{\partial^2 D}{\partial x^2} = C_1\frac{\partial^2 f_1}{\partial x^2} + C_2\frac{\partial^2 f_2}{\partial x^2}$$

$$\frac{\partial D}{\partial t} = C_1\frac{\partial f_1}{\partial t} + C_2\frac{\partial f_2}{\partial t} \quad ; \quad \frac{\partial^2 D}{\partial t^2} = C_1\frac{\partial^2 f_1}{\partial t^2} + C_2\frac{\partial^2 f_2}{\partial t^2}$$

To satisfy the wave equation, we must have $\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$. Use the fact that both f_1 and f_2 satisfy the wave equation.

$$\frac{\partial^2 D}{\partial x^2} = C_1\frac{\partial^2 f_1}{\partial x^2} + C_2\frac{\partial^2 f_2}{\partial x^2} = C_1\left[\frac{1}{v^2}\frac{\partial^2 f_1}{\partial t^2}\right] + C_2\left[\frac{1}{v^2}\frac{\partial^2 f_2}{\partial t^2}\right] = \frac{1}{v^2}\left[C_1\frac{\partial^2 f_1}{\partial t^2} + C_2\frac{\partial^2 f_2}{\partial t^2}\right] = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$$

Thus we see that $\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$, and so D satisfies the wave equation.