Homework-9 (French !!)

7.2 $y = 0.3 \sin \pi(0.5x - 50t)$.

(a) Amplitude $= 0.3\,cm = 0.003\,m$.

wavelength $= 2\pi/k = 2\pi/0.5\pi = 4\,c.m.$

wavenumber $(k) = 0.5\pi\ c.m^{-1}$

frequency $(f) = \omega/2\pi = \frac{50\pi}{2\pi} = 25\,Hz$.

period $(T) = 1/f = 0.04$ sec.

velocity $= \lambda f = (4 \times 25)\ c.m/s = 100\ c.m/s = 1\,m/s$.

(b) $\frac{\partial y}{\partial t} = (0.3)(-50\pi) \cos \pi(0.5x - 50t)$

$\therefore \left.\frac{\partial y}{\partial t}\right|_{MAX} = (15\pi)$. c.m/s

7.3. $y = A \sin(kx - \omega t)$

$A = 0.003\,m$.

$\omega = 2\pi f = (10\pi)\,s^{-1}$ $\because f = 5\,s^{-1}$

$k = 2\pi/\lambda = 2\pi/(v/f)$ $(\because \lambda = v/f)$

$= 2\pi f/v = \omega/v = \left(\frac{10\pi}{3000}\right) = \pi/300$

$\therefore\ y(x,t) = 0.003 \sin\left(\frac{\pi}{300} x + 10\pi t\right)$

because it's moving negative x direction

7.5.

(a) $v = \sqrt{T/\mu}$

$= \sqrt{\frac{50}{0.1}}\ ms^{-1}$

$= 10\sqrt{5}\ ms^{-1}$.

(b) $v = \lambda f$.

$\therefore\ \lambda = v/f = vT$

$= 10\sqrt{5} \times 0.1\ m$.

$= \sqrt{5}\ m$.

(c) amplitude $= 0.02\,m$.

@ $t = 0$, $\quad y(x, t=0) = 0.02 \sin(kx + \varphi)$.

$0.01 = y(0,0) = 0.02 \sin\varphi$

$\therefore\ \sin\varphi = 1/2 \Rightarrow \varphi = \pi/6$.

Now, $k = 2\pi/\lambda$, $\omega = 2\pi/T$

$\therefore\ y(x,t) = 0.02 \sin\left(\frac{2\pi}{\sqrt{5}} x - \frac{2\pi}{0.1} t + \pi/6\right)$

negative sign makes sure $\frac{\partial y}{\partial t}$ negative !!

had it been the case $\varphi = 5\pi/6$, $\sin\varphi$ would still be $1/2$ but to keep $\frac{\partial y}{\partial t}(0,0)$ negative, we would have to choose left going wave, which is not the case given in question

Hence, $y(x,t) = 0.02 \sin \pi \left(\frac{2}{\sqrt{5}} x - 20t + \frac{1}{3}\right)$ (Answer)

7.6. time taken for the wave to travel from one end to the other

(a) $t = \left(\frac{l}{v}\right)$ where l is length of the string
v is the velocity.

And $v = \sqrt{T/\mu} = \sqrt{\frac{Tl}{m}}$

$= \sqrt{g}\sqrt{\frac{Tl}{mg}}$

it's given $\left(\frac{T}{mg}\right) = 100$

$v = \sqrt{gl}\sqrt{100} = 10\sqrt{gl}$

$\therefore t = l/v = \frac{1}{10}\sqrt{l/g}$

$\therefore \sqrt{l/g} = 10t = 10 \times 0.1\ s = 1\,s.$

$\therefore l = 9.8\ m.$ (Answer)

(b) 3rd normal mode, $\Rightarrow 3\lambda/2 = 9.8 \Rightarrow \lambda = 6.53\ m.$

$\therefore k = 2\pi/\lambda = 0.96\ m^{-1}$

And $\omega = kv = 0.96 \times l/t = 0.96 \times 9.8/0.1\ s^{-1}$

$= 94.08\ s^{-1}$

3rd normal mode (standing wave)

$\therefore y(x,t) = A \sin(0.96x)\cos(94.08t)$ ✓

7.7. $y(x,t) = 0.02 \sin \pi (x - v t)$

same T & same μ

$\Downarrow$

same v

$\therefore v = 98 \ ms^{-1}$

$\therefore y(x,t) = 0.02 \sin(\pi x - 98\pi t)$

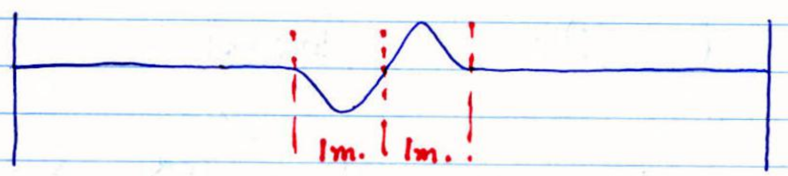
$\frac{\partial y}{\partial t}(x,t) = -1.96\pi \cos \pi (x - 98t)$.

$\therefore y(5, 0.1) = 0.02 \sin \pi (5 - 9.8) = -0.012 \ m$

$\frac{\partial y}{\partial t}(5, 0.1) = -1.96\pi \cos \pi (5 - 9.8) = 4.98 \ ms^{-1}$.

7.12.

(a)



velocity profile

(b) the pulse moving forward by distance 1 m. means the particle on the string moves up by 0.1 m.

approximate velocity would be 0.1 m divided by the time it takes for the pulse to move ahead by 1 m.

the time $= \frac{1 m.}{40 m/s} = 0.025$ s.

$\therefore$ average velocity of the particle

$= 0.1/0.025 \ ms^{-1} = 4 \ ms^{-1}$.

c). $v = \sqrt{T/\mu} \Rightarrow T = \mu v^2$

$= \left(2/100\right) v^2$ (2kg in 100 m. $\therefore \mu = 2/100$ kg/m)

$= \frac{2}{100} \times 1600$ ~~N~~

$= 32$ N (Answer)

d) velocity remains same $\Rightarrow v = 40 ms^{-1}$

$\lambda = 5$ m. $\quad k = 2\pi/5 = 0.4\pi$

$f = v/\lambda = 8 \ s^{-1}$, $\quad \omega = 2\pi f = 16\pi \ s^{-1}$

$y(x,t) = 0.2 \ Sin \ \pi(0.4x + 16t)$.

7.17. (a) $y_1 + y_2 = A\left[\sin(5x - 10t) + \sin(4x - 9t)\right]$

$$= 2A \sin\left(\frac{9x - 19t}{2}\right) \cos\left(\frac{x - t}{2}\right).$$

(b) The envelope has a lower frequency
$\omega = 1/2$, $k = 1/2$ [coming from Cos term]

$\therefore$ group velocity $= \omega/k = 1\ ms^{-1}$.

(c) distance between points of zero amplitude is determined by the $\sin\left(\frac{9x - 19t}{2}\right)$ part

distance $= \pi/\frac{9}{2} = \left(2\pi/9\right)$.

[If we've $\sin(kx - \omega t)$ form, then $\lambda = 2\pi/k$,

here $k = 9/2$, distance between zero amplitude $\left(\lambda/2\right) = \pi/k$].