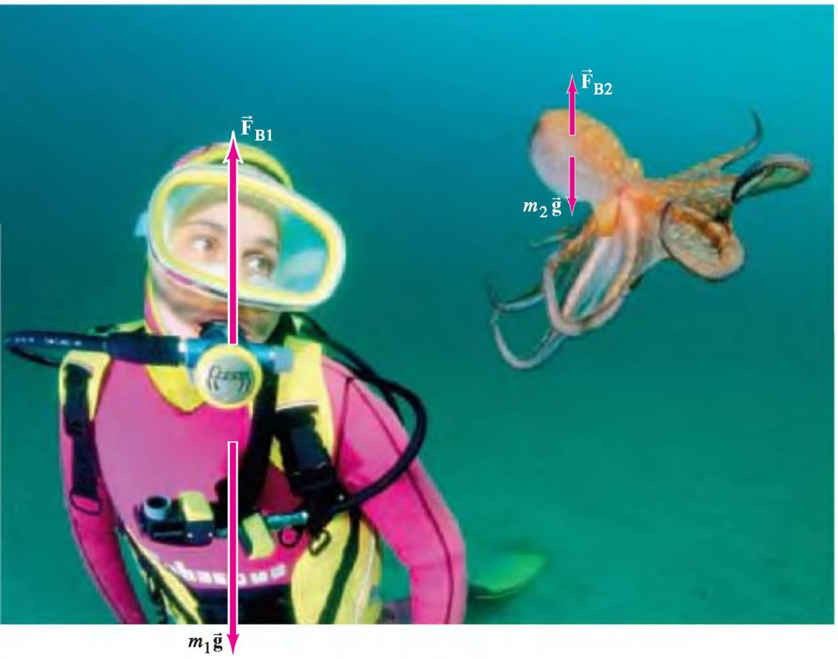


Underwater divers and sea creatures experience a buoyant force $(\vec{F}_B)$ that closely balances their weight $m\vec{g}$. The buoyant force is equal to the weight of the volume of fluid displaced (Archimedes' principle) and arises because the pressure increases with depth in the fluid. Sea creatures have a density very close to that of water, so their weight very nearly equals the buoyant force. Humans have a density slightly less than water, so they can float.

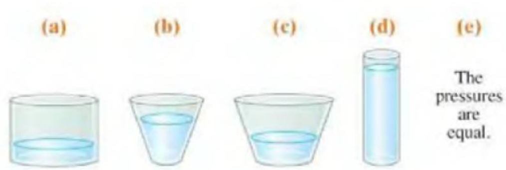
When fluids flow, interesting effects occur because the pressure in the fluid is lower where the fluid velocity is higher (Bernoulli's principle).

CHAPTER 13

Fluids

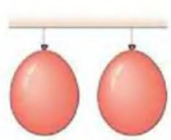
CHAPTER-OPENING QUESTIONS—Guess now!

1. Which container has the largest pressure at the bottom? Assume each container holds the same volume of water.



2. Two balloons are tied and hang with their nearest edges about 3 cm apart. If you blow between the balloons (not *at* the balloons, but at the opening between them), what will happen?

(a) Nothing.
(b) The balloons will move closer together.
(c) The balloons will move farther apart.



CONTENTS

In previous Chapters we considered objects that were solid and assumed to maintain their shape except for a small amount of elastic deformation. We sometimes treated objects as point particles. Now we are going to shift our attention to materials that are very deformable and can flow. Such "fluids" include liquids and gases. We will examine fluids both at rest (fluid statics) and in motion (fluid dynamics).

13–1 Phases of Matter

The three common **phases**, or **states**, of matter are solid, liquid, and gas. We can distinguish these three phases as follows. A **solid** maintains a fixed shape and a fixed size; even if a large force is applied to a solid, it does not readily change in shape or volume. A **liquid** does not maintain a fixed shape—it takes on the shape of its container—but like a solid it is not readily compressible, and its volume can be changed significantly only by a very large force. A **gas** has neither a fixed shape nor a fixed volume—it will expand to fill its container. For example, when air is pumped into an automobile tire, the air does not all run to the bottom of the tire as a liquid would; it spreads out to fill the whole volume of the tire. Since liquids and gases do not maintain a fixed shape, they both have ability to flow; they are thus often referred to collectively as **fluids**.

The division of matter into three phases is not always simple. How, for example, should butter be classified? Furthermore, a fourth phase of matter can be distinguished, the **plasma** phase, which occurs only at very high temperatures and consists of ionized atoms (electrons separated from the nuclei). Some scientists believe that so-called colloids (suspensions of tiny particles in a liquid) should also be considered a separate phase of matter. **Liquid crystals**, which are used in TV and computer screens, calculators, digital watches, and so on, can be considered a phase of matter intermediate between solids and liquids. However, for our present purposes we will mainly be interested in the three ordinary phases of matter.

TABLE 13–1
Densities of Substances†

Substance	Density, ρ (kg/m^3)
Solids	
Aluminum	2.70×10^3
Iron and steel	7.8×10^3
Copper	8.9×10^3
Lead	11.3×10^3
Gold	19.3×10^3
Concrete	2.3×10^3
Granite	2.7×10^3
Wood (typical)	$0.3-0.9 \times 10^3$
Glass, common	$2.4-2.8 \times 10^3$
Ice (H_2O)	0.917×10^3
Bone	$1.7-2.0 \times 10^3$
Liquids	
Water (4°C)	1.00×10^3
Blood, plasma	1.03×10^3
Blood, whole	1.05×10^3
Sea water	1.025×10^3
Mercury	13.6×10^3
Alcohol, ethyl	0.79×10^3
Gasoline	0.68×10^3
Gases	
Air	1.29
Helium	0.179
Carbon dioxide	1.98
Steam (water, 100°C)	0.598

†Densities are given at 0°C and 1 atm pressure unless otherwise specified.

13–2 Density and Specific Gravity

It is sometimes said that iron is "heavier" than wood. This cannot really be true since a large log clearly weighs more than an iron nail. What we should say is that iron is more *dense* than wood.

The **density**, ρ, of a substance (ρ is the lowercase Greek letter rho) is defined as its mass per unit volume:

$$\rho = \frac{m}{V}, \tag{13–1}$$

where m is the mass of a sample of the substance and V is its volume. Density is a characteristic property of any pure substance. Objects made of a particular pure substance, such as pure gold, can have any size or mass, but the density will be the same for each.

We will sometimes use the concept of density, Eq. 13–1, to write the mass of an object as

$$m = \rho V,$$

and the weight of an object as

$$mg = \rho V g.$$

The SI unit for density is kg/m^3. Sometimes densities are given in g/cm^3. Note that since $1\,\text{kg/m}^3 = 1000\,\text{g/(100 cm)}^3 = 10^3\,\text{g}/10^6\,\text{cm}^3 = 10^{-3}\,\text{g/cm}^3$, then a density given in g/cm^3 must be multiplied by 1000 to give the result in kg/m^3. Thus the density of aluminum is $\rho = 2.70\,\text{g/cm}^3$, which is equal to $2700\,\text{kg/m}^3$. The densities of a variety of substances are given in Table 13–1. The Table specifies temperature and atmospheric pressure because they affect the density of substances (although the effect is slight for liquids and solids). Note that air is roughly 1000 times less dense than water.

EXAMPLE 13–1 **Mass, given volume and density.** What is the mass of a solid iron wrecking ball of radius 18 cm?

APPROACH First we use the standard formula $V = \frac{4}{3}\pi r^3$ (see inside rear cover) to obtain the volume of the sphere. Then Eq. 13–1 and Table 13–1 give us the mass m.

SOLUTION The volume of the sphere is

$$V = \tfrac{4}{3}\pi r^3 = \tfrac{4}{3}(3.14)(0.18\,\mathrm{m})^3 = 0.024\,\mathrm{m}^3.$$

From Table 13–1, the density of iron is $\rho = 7800\,\mathrm{kg/m^3}$, so Eq. 13–1 gives

$$m = \rho V = \left(7800\,\mathrm{kg/m^3}\right)\left(0.024\,\mathrm{m^3}\right) = 190\,\mathrm{kg}.$$

The **specific gravity** of a substance is defined as the ratio of the density of that substance to the density of water at 4.0°C. Because specific gravity (abbreviated SG) is a ratio, it is a simple number without dimensions or units. The density of water is $1.00\,\mathrm{g/cm^3} = 1.00 \times 10^3\,\mathrm{kg/m^3}$, so the specific gravity of any substance will be equal numerically to its density specified in $\mathrm{g/cm^3}$, or 10^{-3} times its density specified in $\mathrm{kg/m^3}$. For example (see Table 13–1), the specific gravity of lead is 11.3, and that of alcohol is 0.79.

The concepts of density and specific gravity are especially helpful in the study of fluids because we are not always dealing with a fixed volume or mass.

13–3 Pressure in Fluids

Pressure and force are related, but they are not the same thing. **Pressure** is defined as force per unit area, where the force F is understood to be the magnitude of the force acting perpendicular to the surface area A:

$$\text{pressure} = P = \frac{F}{A}. \tag{13–2}$$

Although force is a vector, pressure is a scalar. Pressure has magnitude only. The SI unit of pressure is $\mathrm{N/m^2}$. This unit has the official name **pascal** (Pa), in honor of Blaise Pascal (see Section 13–5); that is, $1\,\mathrm{Pa} = 1\,\mathrm{N/m^2}$. However, for simplicity, we will often use $\mathrm{N/m^2}$. Other units sometimes used are $\mathrm{dynes/cm^2}$, and $\mathrm{lb/in.^2}$ (abbreviated "psi"). Several other units for pressure are discussed, along with conversions between them, in Section 13–6 (see also the Table inside the front cover).

CAUTION
Pressure is a scalar, not a vector

EXAMPLE 13–2 **Calculating pressure.** The two feet of a 60-kg person cover an area of $500\,\mathrm{cm^2}$. (*a*) Determine the pressure exerted by the two feet on the ground. (*b*) If the person stands on one foot, what will the pressure be under that foot?

APPROACH Assume the person is at rest. Then the ground pushes up on her with a force equal to her weight mg, and she exerts a force mg on the ground where her feet (or foot) contact it. Because $1\,\mathrm{cm^2} = \left(10^{-2}\,\mathrm{m}\right)^2 = 10^{-4}\,\mathrm{m^2}$, then $500\,\mathrm{cm^2} = 0.050\,\mathrm{m^2}$.

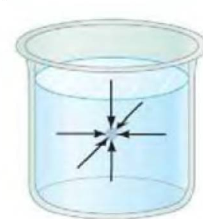
SOLUTION (*a*) The pressure on the ground exerted by the two feet is

$$P = \frac{F}{A} = \frac{mg}{A} = \frac{(60\,\mathrm{kg})\left(9.8\,\mathrm{m/s^2}\right)}{\left(0.050\,\mathrm{m^2}\right)} = 12 \times 10^3\,\mathrm{N/m^2}.$$

(*b*) If the person stands on one foot, the force is still equal to the person's weight, but the area will be half as much, so the pressure will be twice as much: $24 \times 10^3\,\mathrm{N/m^2}$.

Pressure is particularly useful for dealing with fluids. It is an experimental observation that *a fluid exerts pressure in any direction*. This is well known to swimmers and divers who feel the water pressure on all parts of their bodies. At any depth in a fluid at rest, the pressure is the same in all directions at a given depth. To see why, consider a tiny cube of the fluid (Fig. 13–1) which is so small that we can consider it a point and can ignore the force of gravity on it. The pressure on one side of it must equal the pressure on the opposite side. If this weren't true, there would be a net force on the cube and it would start moving. If the fluid is not flowing, then the pressures must be equal.

FIGURE 13–1 Pressure is the same in every direction in a nonmoving fluid at a given depth. If this weren't true, the fluid would be in motion.



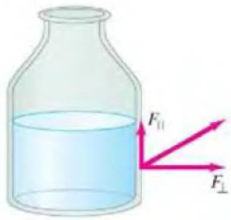
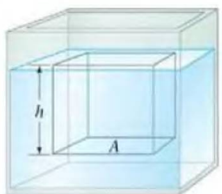


FIGURE 13–2 If there were a component of force parallel to the solid surface of the container, the liquid would move in response to it. For a liquid at rest, $F_\parallel = 0$.

For a fluid at rest, the force due to fluid pressure always acts *perpendicular* to any solid surface it touches. If there were a component of the force parallel to the surface, as shown in Fig. 13–2, then according to Newton's third law the solid surface would exert a force back on the fluid that also would have a component parallel to the surface. Such a component would cause the fluid to flow, in contradiction to our assumption that the fluid is at rest. Thus the force due to the pressure in a fluid at rest is always perpendicular to the surface.

FIGURE 13–3 Calculating the pressure at a depth h in a liquid.



Let us now calculate quantitatively how the pressure in a liquid of uniform density varies with depth. Consider a point at a depth h below the surface of the liquid, as shown in Fig. 13–3 (that is, the surface is a height h above this point). The pressure due to the liquid at this depth h is due to the weight of the column of liquid above it. Thus the force due to the weight of liquid acting on the area A is $F = mg = (\rho V)g = \rho Ahg$, where Ah is the volume of the column of liquid, ρ is the density of the liquid (assumed to be constant), and g is the acceleration of gravity. The pressure P due to the weight of liquid is then

$$P = \frac{F}{A} = \frac{\rho Ahg}{A}$$

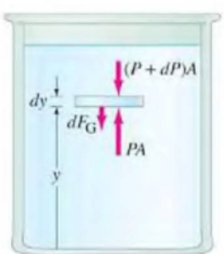
$$P = \rho g h. \qquad \text{[liquid]} \quad \textbf{(13–3)}$$

Note that the area A doesn't affect the pressure at a given depth. The fluid pressure is directly proportional to the density of the liquid and to the depth within the liquid. In general, the pressure at equal depths within a uniform liquid is the same.

EXERCISE A Return to Chapter-Opening Question 1, page 339, and answer it again now. Try to explain why you may have answered differently the first time.

Equation 13–3 tells us what the pressure is at a depth h in the liquid, due to the liquid itself. But what if there is additional pressure exerted at the surface of the liquid, such as the pressure of the atmosphere or a piston pushing down? And what if the density of the fluid is not constant? Gases are quite compressible and hence their density can vary significantly with depth. Liquids, too, can be compressed, although we can often ignore the variation in density. (One exception is in the depths of the ocean where the great weight of water above significantly compresses the water and increases its density.) To cover these, and other cases, we now treat the general case of determining how the pressure in a fluid varies with depth.

FIGURE 13–4 Forces on a flat, slablike volume of fluid for determining the pressure P at a height y in the fluid.



As shown in Fig. 13–4, let us determine the pressure at any height y above some reference point† (such as the ocean floor or the bottom of a tank or swimming pool). Within this fluid, at the height y, we consider a tiny, flat, slablike volume of the fluid whose area is A and whose (infinitesimal) thickness is dy, as shown. Let the pressure acting upward on its lower surface (at height y) be P. The pressure acting downward on the top surface of our tiny slab (at height $y + dy$) is designated $P + dP$. The fluid pressure acting on our slab thus exerts a force equal to PA upward on our slab and a force equal to $(P + dP)A$ downward on it. The only other force acting vertically on the slab is the (infinitesimal) force of gravity dF_G, which on our slab of mass dm is

$$dF_G = (dm)g = \rho g\, dV = \rho g A\, dy,$$

where ρ is the density of the fluid at the height y. Since the fluid is assumed to be at rest, our slab is in equilibrium so the net force on it must be zero. Therefore we have

$$PA - (P + dP)A - \rho g A\, dy = 0,$$

which when simplified becomes

$$\frac{dP}{dy} = -\rho g. \qquad \textbf{(13–4)}$$

This relation tells us how the pressure within the fluid varies with height above any reference point. The minus sign indicates that the pressure decreases with an increase in height; or that the pressure increases with depth (reduced height).

†Now we are measuring y positive upwards, the reverse of what we did to get Eq. 13–3 where we measured the depth (i.e. downward as positive).

If the pressure at a height y_1 in the fluid is P_1, and at height y_2 it is P_2, then we can integrate Eq. 13–4 to obtain

$$\int_{P_1}^{P_2} dP = -\int_{y_1}^{y_2} \rho g\, dy$$

$$P_2 - P_1 = -\int_{y_1}^{y_2} \rho g\, dy, \qquad \textbf{(13–5)}$$

where we assume ρ is a function of height y: $\rho = \rho(y)$. This is a general relation, and we apply it now to two special cases: (1) pressure in liquids of uniform density and (2) pressure variations in the Earth's atmosphere.

For liquids in which any variation in density can be ignored, $\rho =$ constant and Eq. 13–5 is readily integrated:

$$P_2 - P_1 = -\rho g (y_2 - y_1). \qquad \textbf{(13–6a)}$$

For the everyday situation of a liquid in an open container—such as water in a glass, a swimming pool, a lake, or the ocean—there is a free surface at the top exposed to the atmosphere. It is convenient to measure distances from this top surface. That is, we let h be the *depth* in the liquid where $h = y_2 - y_1$ as shown in Fig. 13–5. If we let y_2 be the position of the top surface, then P_2 represents the atmospheric pressure, P_0, at the top surface. Then, from Eq. 13–6a, the pressure $P\ (= P_1)$ at a depth h in the fluid is

$$P = P_0 + \rho g h. \qquad [h \text{ is depth in liquid}] \quad \textbf{(13–6b)}$$

Note that Eq. 13–6b is simply the liquid pressure (Eq. 13–3) plus the pressure P_0 due to the atmosphere above.

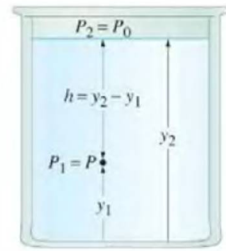


FIGURE 13–5 Pressure at a depth $h = (y_2 - y_1)$ in a liquid of density ρ is $P = P_0 + \rho g h$, where P_0 is the external pressure at the liquid's top surface.

EXAMPLE 13–3 Pressure at a faucet. The surface of the water in a storage tank is 30 m above a water faucet in the kitchen of a house, Fig. 13–6. Calculate the difference in water pressure between the faucet and the surface of the water in the tank.

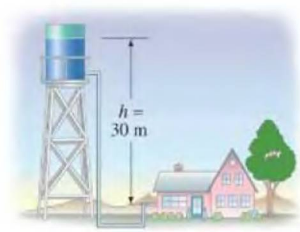
APPROACH Water is practically incompressible, so ρ is constant even for $h = 30$ m when used in Eq. 13–6b. Only h matters; we can ignore the "route" of the pipe and its bends.

SOLUTION We assume the atmospheric pressure at the surface of the water in the storage tank is the same as at the faucet. So, the water pressure difference between the faucet and the surface of the water in the tank is

$$\Delta P = \rho g h = (1.0 \times 10^3\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(30\,\text{m}) = 2.9 \times 10^5\,\text{N/m}^2.$$

NOTE The height h is sometimes called the **pressure head**. In this Example, the head of water is 30 m at the faucet. The very different diameters of the tank and faucet don't affect the result—only pressure does.

FIGURE 13–6 Example 13–3.



EXAMPLE 13–4 Force on aquarium window. Calculate the force due to water pressure exerted on a 1.0 m × 3.0 m aquarium viewing window whose top edge is 1.0 m below the water surface, Fig. 13–7.

APPROACH At a depth h, the pressure due to the water is given by Eq. 13–6b. Divide the window up into thin horizontal strips of length $\ell = 3.0$ m and thickness dy, as shown in Fig. 13–7. We choose a coordinate system with $y = 0$ at the surface of the water and y is positive downward. (With this choice, the minus sign in Eq. 13–6a becomes plus, or we use Eq. 13–6b with $y = h$.) The force due to water pressure on each strip is $dF = P\,dA = \rho g y \ell\, dy$.

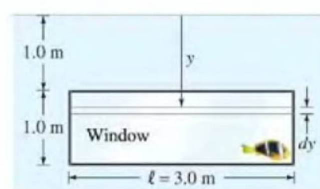
SOLUTION The total force on the window is given by the integral:

$$\int_{y_1 = 1.0\,\text{m}}^{y_2 = 2.0\,\text{m}} \rho g y \ell\, dy = \tfrac{1}{2}\rho g \ell (y_2^2 - y_1^2)$$

$$= \tfrac{1}{2}(1000\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(3.0\,\text{m})\left[(2.0\,\text{m})^2 - (1.0\,\text{m})^2\right] = 44{,}000\,\text{N}.$$

NOTE To check our answer, we can do an estimate: multiply the area of the window $(3.0\,\text{m}^2)$ times the pressure at the middle of the window $(h = 1.5\,\text{m})$ using Eq. 13–3, $P = \rho g h = (1000\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(1.5\,\text{m}) \approx 1.5 \times 10^4\,\text{N/m}^2$. So $F = PA \approx (1.5 \times 10^4\,\text{N/m}^2)(3.0\,\text{m})(1.0\,\text{m}) \approx 4.5 \times 10^4$ N. Good!

FIGURE 13–7 Example 13–4.



EXERCISE B A dam holds back a lake that is 85 m deep at the dam. If the lake is 20 km long, how much thicker should the dam be than if the lake were smaller, only 1.0 km long?

Now let us apply Eq. 13–4 or 13–5 to gases. The density of gases is normally quite small, so the difference in pressure at different heights can usually be ignored if $y_2 - y_1$ is not large (which is why, in Example 13–3, we could ignore the difference in air pressure between the faucet and the top of the storage tank). Indeed, for most ordinary containers of gas, we can assume that the pressure is the same throughout. However, if $y_2 - y_1$ is very large, we cannot make this assumption. An interesting example is the air of Earth's atmosphere, whose pressure at sea level is about $1.013 \times 10^5\,\mathrm{N/m^2}$ and decreases slowly with altitude.

EXAMPLE 13–5 **Elevation effect on atmospheric pressure.** (*a*) Determine the variation in pressure in the Earth's atmosphere as a function of height y above sea level, assuming g is constant and that the density of the air is proportional to the pressure. (This last assumption is not terribly accurate, in part because temperature and other weather effects are important.) (*b*) At what elevation is the air pressure equal to half the pressure at sea level?

APPROACH We start with Eq. 13–4 and integrate it from the surface of the Earth where $y = 0$ and $P = P_0$, up to height y at pressure P. In (*b*) we choose $P = \frac{1}{2}P_0$.

SOLUTION (*a*) We are assuming that ρ is proportional to P, so we can write

$$\frac{\rho}{\rho_0} = \frac{P}{P_0},$$

where $P_0 = 1.013 \times 10^5\,\mathrm{N/m^2}$ is atmospheric pressure at sea level and $\rho_0 = 1.29\,\mathrm{kg/m^3}$ is the density of air at sea level at 0°C (Table 13–1). From the differential change in pressure with height, Eq. 13–4, we have

$$\frac{dP}{dy} = -\rho g = -P\left(\frac{\rho_0}{P_0}\right)g,$$

so

$$\frac{dP}{P} = -\frac{\rho_0}{P_0}g\,dy.$$

We integrate this from $y = 0$ (Earth's surface) and $P = P_0$, to the height y where the pressure is P:

$$\int_{P_0}^{P}\frac{dP}{P} = -\frac{\rho_0}{P_0}g\int_0^y dy$$

$$\ln\frac{P}{P_0} = -\frac{\rho_0}{P_0}gy,$$

since $\ln P - \ln P_0 = \ln(P/P_0)$. Then

$$P = P_0 e^{-(\rho_0 g/P_0)y}.$$

So, based on our assumptions, we find that the air pressure in our atmosphere decreases approximately exponentially with height.

NOTE The atmosphere does not have a distinct top surface, so there is no natural point from which to measure depth in the atmosphere, as we can do for a liquid.

(*b*) The constant $(\rho_0 g/P_0)$ has the value

$$\frac{\rho_0 g}{P_0} = \frac{(1.29\,\mathrm{kg/m^3})(9.80\,\mathrm{m/s^2})}{(1.013 \times 10^5\,\mathrm{N/m^2})} = 1.25 \times 10^{-4}\,\mathrm{m^{-1}}.$$

Then, when we set $P = \frac{1}{2}P_0$ in our expression derived in (*a*), we obtain

$$\tfrac{1}{2} = e^{-(1.25\times10^{-4}\,\mathrm{m^{-1}})y}$$

or, taking natural logarithms of both sides,

$$\ln\tfrac{1}{2} = (-1.25 \times 10^{-4}\,\mathrm{m^{-1}})y$$

so (recall $\ln\frac{1}{2} = -\ln 2$, Appendix A–7, Eq. ii)

$$y = (\ln 2.00)/(1.25 \times 10^{-4}\,\mathrm{m^{-1}}) = 5550\,\mathrm{m}.$$

Thus, at an elevation of about 5500 m (about 18,000 ft), atmospheric pressure drops to half what it is at sea level. It is not surprising that mountain climbers often use oxygen tanks at very high altitudes.

13–4 Atmospheric Pressure and Gauge Pressure

Atmospheric Pressure

The pressure of the air at a given place on Earth varies slightly according to the weather. At sea level, the pressure of the atmosphere on average is $1.013 \times 10^5\,\mathrm{N/m^2}$ (or $14.7\,\mathrm{lb/in.^2}$). This value lets us define a commonly used unit of pressure, the **atmosphere** (abbreviated atm):

$$1\ \mathrm{atm} = 1.013 \times 10^5\,\mathrm{N/m^2} = 101.3\,\mathrm{kPa}.$$

Another unit of pressure sometimes used (in meteorology and on weather maps) is the **bar**, which is defined as

$$1\ \mathrm{bar} = 1.000 \times 10^5\,\mathrm{N/m^2}.$$

Thus standard atmospheric pressure is slightly more than 1 bar.

The pressure due to the weight of the atmosphere is exerted on all objects immersed in this great sea of air, including our bodies. How does a human body withstand the enormous pressure on its surface? The answer is that living cells maintain an internal pressure that closely equals the external pressure, just as the pressure inside a balloon closely matches the outside pressure of the atmosphere. An automobile tire, because of its rigidity, can maintain internal pressures much greater than the external pressure.

PHYSICS APPLIED
Pressure on living cells

CONCEPTUAL EXAMPLE 13–6 **Finger holds water in a straw.** You insert a straw of length ℓ into a tall glass of water. You place your finger over the top of the straw, capturing some air above the water but preventing any additional air from getting in or out, and then you lift the straw from the water. You find that the straw retains most of the water (see Fig. 13–8a). Does the air in the space between your finger and the top of the water have a pressure P that is greater than, equal to, or less than the atmospheric pressure P_0 outside the straw?

RESPONSE Consider the forces on the column of water (Fig. 13–8b). Atmospheric pressure outside the straw pushes upward on the water at the bottom of the straw, gravity pulls the water downward, and the air pressure inside the top of the straw pushes downward on the water. Since the water is in equilibrium, the upward force due to atmospheric pressure P_0 must balance the two downward forces. The only way this is possible is for the air pressure inside the straw to be *less than* the atmosphere pressure outside the straw. (When you initially remove the straw from the glass of water, a little water may leave the bottom of the straw, thus increasing the volume of trapped air and reducing its density and pressure.)

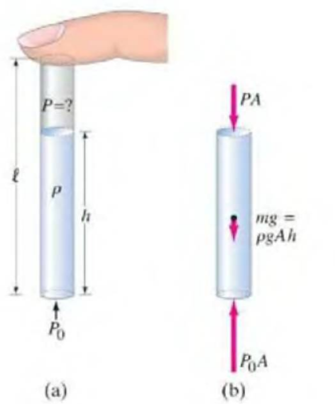


FIGURE 13–8 Example 13–6.

Gauge Pressure

It is important to note that tire gauges, and most other pressure gauges, register the pressure above and beyond atmospheric pressure. This is called **gauge pressure**. Thus, to get the **absolute pressure**, P, we must add the atmospheric pressure, P_0, to the gauge pressure, P_G:

$$P = P_0 + P_G.$$

If a tire gauge registers 220 kPa, the absolute pressure within the tire is 220 kPa + 101 kPa = 321 kPa, equivalent to about 3.2 atm (2.2 atm gauge pressure).

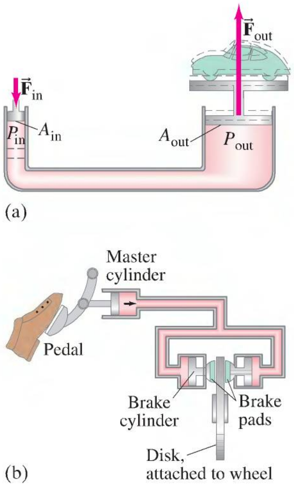


FIGURE 13–9 Applications of Pascal's principle: (a) hydraulic lift; (b) hydraulic brakes in a car.

13–5 Pascal's Principle

The Earth's atmosphere exerts a pressure on all objects with which it is in contact, including other fluids. External pressure acting on a fluid is transmitted throughout that fluid. For instance, according to Eq. 13–3, the pressure due to the water at a depth of 100 m below the surface of a lake is $P = \rho g h = (1000\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(100\,\text{m}) = 9.8 \times 10^5\,\text{N/m}^2$, or 9.7 atm. However, the total pressure at this point is due to the pressure of water plus the pressure of the air above it. Hence the total pressure (if the lake is near sea level) is $9.7\,\text{atm} + 1.0\,\text{atm} = 10.7\,\text{atm}$. This is just one example of a general principle attributed to the French philosopher and scientist Blaise Pascal (1623–1662). **Pascal's principle** states that *if an external pressure is applied to a confined fluid, the pressure at every point within the fluid increases by that amount.*

PHYSICS APPLIED
Hydraulic lift

A number of practical devices make use of Pascal's principle. One example is the hydraulic lift, illustrated in Fig. 13–9a, in which a small input force is used to exert a large output force by making the area of the output piston larger than the area of the input piston. To see how this works, we assume the input and output pistons are at the same height (at least approximately). Then the external input force F_{in}, by Pascal's principle, increases the pressure equally throughout. Therefore, at the same level (see Fig. 13–9a),

$$P_{\text{out}} = P_{\text{in}}$$

where the input quantities are represented by the subscript "in" and the output by "out." Since $P = F/A$, we write the above equality as

$$\frac{F_{\text{out}}}{A_{\text{out}}} = \frac{F_{\text{in}}}{A_{\text{in}}},$$

or

$$\frac{F_{\text{out}}}{F_{\text{in}}} = \frac{A_{\text{out}}}{A_{\text{in}}}.$$

The quantity $F_{\text{out}}/F_{\text{in}}$ is called the **mechanical advantage** of the hydraulic lift, and it is equal to the ratio of the areas. For example, if the area of the output piston is 20 times that of the input cylinder, the force is multiplied by a factor of 20. Thus a force of 200 lb could lift a 4000-lb car.

PHYSICS APPLIED
Hydraulic brakes

Figure 13–9b illustrates the brake system of a car. When the driver presses the brake pedal, the pressure in the master cylinder increases. This pressure increase occurs throughout the brake fluid, thus pushing the brake pads against the disk attached to the car's wheel.

13–6 Measurement of Pressure; Gauges and the Barometer

Many devices have been invented to measure pressure, some of which are shown in Fig. 13–10. The simplest is the open-tube *manometer* (Fig 13–10a) which is a U-shaped tube partially filled with a liquid, usually mercury or water. The pressure P being measured is related to the difference in height Δh of the two levels of the liquid by the relation

$$P = P_0 + \rho g\,\Delta h,$$

where P_0 is atmospheric pressure (acting on the top of the liquid in the left-hand tube), and ρ is the density of the liquid. Note that the quantity $\rho g\,\Delta h$ is the gauge pressure—the amount by which P exceeds atmospheric pressure P_0. If the liquid in the left-hand column were lower than that in the right-hand column, P would have to be less than atmospheric pressure (and Δh would be negative).

Instead of calculating the product $\rho g\,\Delta h$, sometimes only the change in height Δh is specified. In fact, pressures are sometimes specified as so many "millimeters of mercury" (mm-Hg) or "mm of water" (mm-H_2O). The unit mm-Hg is equivalent to a pressure of $133\,\text{N/m}^2$, since $\rho g\,\Delta h$ for $1\,\text{mm} = 1.0 \times 10^{-3}\,\text{m}$ of mercury gives

$$\rho g\,\Delta h = (13.6 \times 10^3\,\text{kg/m}^3)(9.80\,\text{m/s}^2)(1.00 \times 10^{-3}\,\text{m}) = 1.33 \times 10^2\,\text{N/m}^2.$$

The unit mm-Hg is also called the **torr** in honor of Evangelista Torricelli (1608–1647), a student of Galileo's who invented the barometer (see next page).

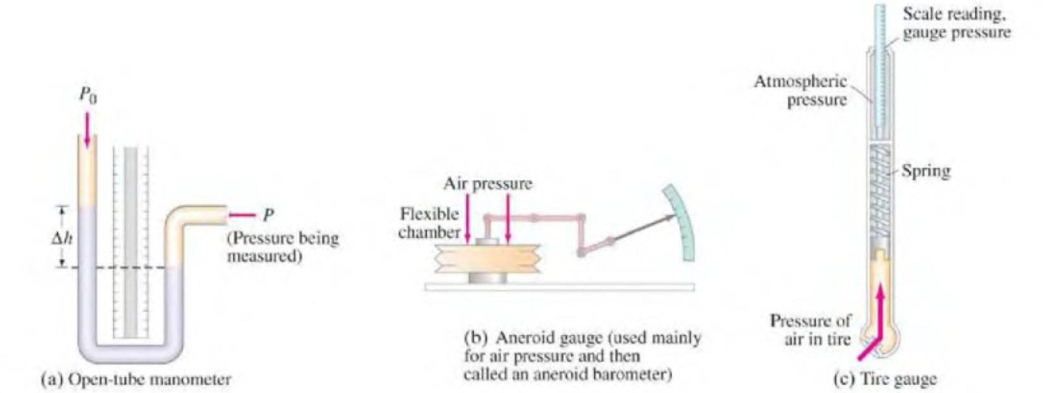


FIGURE 13–10 Pressure gauges: (a) open-tube manometer, (b) aneroid gauge, and (c) common tire-pressure gauge.

Conversion factors among the various units of pressure (an incredible nuisance!) are given in Table 13–2. It is important that only $N/m^2 = Pa$, the proper SI unit, be used in calculations involving other quantities specified in SI units.

PROBLEM SOLVING
In calculations, use SI units:
$1 \text{ Pa} = 1 \text{ N/m}^2$

TABLE 13–2 Conversion Factors Between Different Units of Pressure

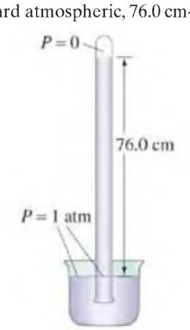
In Terms of $1 \text{ Pa} = 1 \text{ N/m}^2$	1 atm in Different Units
$1 \text{ atm} = 1.013 \times 10^5 \text{ N/m}^2$	$1 \text{ atm} = 1.013 \times 10^5 \text{ N/m}^2$
$= 1.013 \times 10^5 \text{ Pa} = 101.3 \text{ kPa}$	
$1 \text{ bar} = 1.000 \times 10^5 \text{ N/m}^2$	$1 \text{ atm} = 1.013 \text{ bar}$
$1 \text{ dyne/cm}^2 = 0.1 \text{ N/m}^2$	$1 \text{ atm} = 1.013 \times 10^6 \text{ dyne/cm}^2$
$1 \text{ lb/in.}^2 = 6.90 \times 10^3 \text{ N/m}^2$	$1 \text{ atm} = 14.7 \text{ lb/in.}^2$
$1 \text{ lb/ft}^2 = 47.9 \text{ N/m}^2$	$1 \text{ atm} = 2.12 \times 10^3 \text{ lb/ft}^2$
$1 \text{ cm-Hg} = 1.33 \times 10^3 \text{ N/m}^2$	$1 \text{ atm} = 76.0 \text{ cm-Hg}$
$1 \text{ mm-Hg} = 133 \text{ N/m}^2$	$1 \text{ atm} = 760 \text{ mm-Hg}$
$1 \text{ torr} = 133 \text{ N/m}^2$	$1 \text{ atm} = 760 \text{ torr}$
$1 \text{ mm-H}_2\text{O (4°C)} = 9.80 \text{ N/m}^2$	$1 \text{ atm} = 1.03 \times 10^4 \text{ mm-H}_2\text{O (4°C)}$

Another type of pressure gauge is the aneroid gauge (Fig. 13–10b) in which the pointer is linked to the flexible ends of an evacuated thin metal chamber. In an electronic gauge, the pressure may be applied to a thin metal diaphragm whose resulting distortion is translated into an electrical signal by a transducer. A common tire gauge is shown in Fig. 13–10c.

Atmospheric pressure can be measured by a modified kind of mercury manometer with one end closed, called a mercury **barometer** (Fig. 13–11). The glass tube is completely filled with mercury and then inverted into the bowl of mercury. If the tube is long enough, the level of the mercury will drop, leaving a vacuum at the top of the tube, since atmospheric pressure can support a column of mercury only about 76 cm high (exactly 76.0 cm at standard atmospheric pressure). That is, a column of mercury 76 cm high exerts the same pressure as the atmosphere†:

$$\begin{aligned} P &= \rho g \, \Delta h \\ &= (13.6 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(0.760 \text{ m}) = 1.013 \times 10^5 \text{ N/m}^2 = 1.00 \text{ atm}. \end{aligned}$$

FIGURE 13–11 A mercury barometer, invented by Torricelli, is shown here when the air pressure is standard atmospheric, 76.0 cm-Hg.



†This calculation confirms the entry in Table 13–2, $1 \text{ atm} = 76.0 \text{ cm-Hg}$.

A calculation similar to what we just did will show that atmospheric pressure can maintain a column of water 10.3 m high in a tube whose top is under vacuum (Fig. 13–12). No matter how good a vacuum pump is, water cannot be made to rise more than about 10 m using normal atmospheric pressure. To pump water out of deep mine shafts with a vacuum pump requires multiple stages for depths greater than 10 m. Galileo studied this problem, and his student Torricelli was the first to explain it. The point is that a pump does not really suck water up a tube—it merely reduces the pressure at the top of the tube. Atmospheric air pressure *pushes* the water up the tube if the top end is at low pressure (under a vacuum), just as it is air pressure that pushes (or maintains) the mercury 76 cm high in a barometer. [Force pumps (Section 13–14) that push up from the bottom can exert higher pressure to push water more than 10 m high.]

FIGURE 13–12 A water barometer: a full tube of water is inserted into a tub of water, keeping the tube's spigot at the top closed. When the bottom end of the tube is unplugged, some water flows out of the tube into the tub, leaving a vacuum between the water's upper surface and the spigot. Why? Because air pressure can not support a column of water more than 10 m high.

CONCEPTUAL EXAMPLE 13–7 **Suction.** A student suggests suction-cup shoes for Space Shuttle astronauts working on the exterior of a spacecraft. Having just studied this Chapter, you gently remind him of the fallacy of this plan. What is it?

RESPONSE Suction cups work by pushing out the air underneath the cup. What holds the suction cup in place is the air pressure outside it. (This can be a substantial force when on Earth. For example, a 10-cm-diameter suction cup has an area of $7.9 \times 10^{-3}\,\mathrm{m}^2$. The force of the atmosphere on it is $(7.9 \times 10^{-3}\,\mathrm{m}^2)(1.0 \times 10^5\,\mathrm{N/m}^2) \approx 800\,\mathrm{N}$, about 180 lbs!) But in outer space, there is no air pressure to push the suction cup onto the spacecraft.

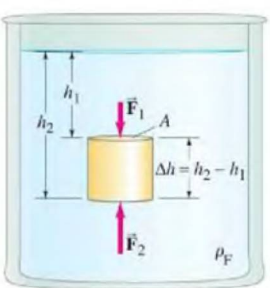
We sometimes mistakenly think of suction as something we actively do. For example, we intuitively think that we pull the soda up through a straw. Instead, what we do is lower the pressure at the top of the straw, and the atmosphere *pushes* the soda up the straw.

13–7 Buoyancy and Archimedes' Principle

Objects submerged in a fluid appear to weigh less than they do when outside the fluid. For example, a large rock that you would have difficulty lifting off the ground can often be easily lifted from the bottom of a stream. When the rock breaks through the surface of the water, it suddenly seems to be much heavier. Many objects, such as wood, float on the surface of water. These are two examples of *buoyancy*. In each example, the force of gravity is acting downward. But in addition, an upward *buoyant force* is exerted by the liquid. The buoyant force on fish and underwater divers (as in the Chapter-Opening photo) almost exactly balances the force of gravity downward, and allows them to "hover" in equilibrium.

The buoyant force occurs because the pressure in a fluid increases with depth. Thus the upward pressure on the bottom surface of a submerged object is greater than the downward pressure on its top surface. To see this effect, consider a cylinder of height Δh whose top and bottom ends have an area A and which is completely submerged in a fluid of density ρ_F, as shown in Fig. 13–13. The fluid exerts a pressure $P_1 = \rho_F g h_1$ at the top surface of the cylinder (Eq. 13–3). The force due to this pressure on top of the cylinder is $F_1 = P_1 A = \rho_F g h_1 A$, and it is directed downward. Similarly, the fluid exerts an upward force on the bottom of the cylinder equal to $F_2 = P_2 A = \rho_F g h_2 A$. The net force on the cylinder exerted by the fluid pressure, which is the **buoyant force**, $\vec{\mathbf{F}}_B$, acts upward and has the magnitude

FIGURE 13–13 Determination of the buoyant force.



$$\begin{aligned} F_B = F_2 - F_1 &= \rho_F g A(h_2 - h_1) \\ &= \rho_F g A\,\Delta h \\ &= \rho_F V g \\ &= m_F g, \end{aligned}$$

where $V = A\,\Delta h$ is the volume of the cylinder, the product $\rho_F V$ is the mass of the fluid displaced, and $\rho_F V g = m_F g$ is the weight of fluid which takes up a volume equal to the volume of the cylinder. Thus the buoyant force on the cylinder is equal to the weight of fluid displaced by the cylinder.

This result is valid no matter what the shape of the object. Its discovery is credited to Archimedes (287?–212 B.C.), and it is called **Archimedes' principle**: *the buoyant force on an object immersed in a fluid is equal to the weight of the fluid displaced by that object.*

By "fluid displaced," we mean a volume of fluid equal to the submerged volume of the object (or that part of the object that is submerged). If the object is placed in a glass or tub initially filled to the brim with water, the water that flows over the top represents the water displaced by the object.

We can derive Archimedes' principle in general by the following simple but elegant argument. The irregularly shaped object D shown in Fig. 13–14a is acted on by the force of gravity (its weight, $m\vec{\mathbf{g}}$, downward) and the buoyant force, $\vec{\mathbf{F}}_B$, upward. We wish to determine F_B. To do so, we next consider a body (D′ in Fig. 13–14b), this time made of the fluid itself, with the same shape and size as the original object, and located at the same depth. You might think of this body of fluid as being separated from the rest of the fluid by an imaginary membrane. The buoyant force F_B on this body of fluid will be exactly the same as that on the original object since the surrounding fluid, which exerts F_B, is in exactly the same configuration. This body of fluid D′ is in equilibrium (the fluid as a whole is at rest). Therefore, $F_B = m'g$, where $m'g$ is the weight of the body of fluid. Hence the buoyant force F_B is equal to the weight of the body of fluid whose volume equals the volume of the original submerged object, which is Archimedes' principle.

Archimedes' discovery was made by experiment. What we have done in the last two paragraphs is to show that Archimedes' principle can be derived from Newton's laws.

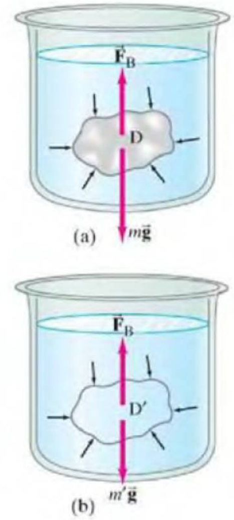


FIGURE 13–14 Archimedes' principle.

CONCEPTUAL EXAMPLE 13–8 **Two pails of water.** Consider two identical pails of water filled to the brim. One pail contains only water, the other has a piece of wood floating in it. Which pail has the greater weight?

RESPONSE Both pails weigh the same. Recall Archimedes' principle: the wood displaces a volume of water with weight equal to the weight of the wood. Some water will overflow the pail, but Archimedes' principle tells us the spilled water has weight equal to the weight of the wood object; so the pails have the same weight.

EXAMPLE 13–9 **Recovering a submerged statue.** A 70-kg ancient statue lies at the bottom of the sea. Its volume is $3.0 \times 10^4\,\text{cm}^3$. How much force is needed to lift it?

APPROACH The force F needed to lift the statue is equal to the statue's weight mg minus the buoyant force F_B. Figure 13–15 is the free-body diagram.

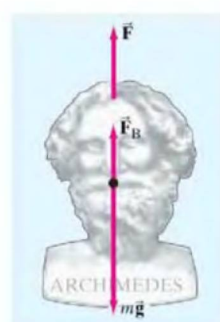
SOLUTION The buoyant force on the statue due to the water is equal to the weight of $3.0 \times 10^4\,\text{cm}^3 = 3.0 \times 10^{-2}\,\text{m}^3$ of water (for seawater, $\rho = 1.025 \times 10^3\,\text{kg/m}^3$):

$$\begin{aligned} F_B = m_{H_2O}g &= \rho_{H_2O}Vg \\ &= (1.025 \times 10^3\,\text{kg/m}^3)(3.0 \times 10^{-2}\,\text{m}^3)(9.8\,\text{m/s}^2) \\ &= 3.0 \times 10^2\,\text{N}. \end{aligned}$$

The weight of the statue is $mg = (70\,\text{kg})(9.8\,\text{m/s}^2) = 6.9 \times 10^2\,\text{N}$. Hence the force F needed to lift it is $690\,\text{N} - 300\,\text{N} = 390\,\text{N}$. It is as if the statue had a mass of only $(390\,\text{N})/(9.8\,\text{m/s}^2) = 40\,\text{kg}$.

NOTE Here $F = 390\,\text{N}$ is the force needed to lift the statue without acceleration when it is under water. As the statue comes *out* of the water, the force F increases, reaching 690 N when the statue is fully out of the water.

FIGURE 13–15 Example 13–9. The force needed to lift the statue is $\vec{\mathbf{F}}$.



Archimedes, is said to have discovered his principle in his bath while thinking how he might determine whether the king's new crown was pure gold or a fake. Gold has a specific gravity of 19.3, somewhat higher than that of most metals, but a determination of specific gravity or density is not readily done directly because, even if the mass is known, the volume of an irregularly shaped object is not easily calculated. However, if the object is weighed in air $(= w)$ and also "weighed" while it is under water $(= w')$, the density can be determined using Archimedes' principle, as the following Example shows. The quantity w' is called the *apparent weight* in water, and is what a scale reads when the object is submerged in water (see Fig. 13–16); w' equals the true weight $(w = mg)$ minus the buoyant force.

EXAMPLE 13–10 **Archimedes: Is the crown gold?** When a crown of mass 14.7 kg is submerged in water, an accurate scale reads only 13.4 kg. Is the crown made of gold?

APPROACH If the crown is gold, its density and specific gravity must be very high, $\text{SG} = 19.3$ (see Section 13–2 and Table 13–1). We determine the specific gravity using Archimedes' principle and the two free-body diagrams shown in Fig. 13–16.

SOLUTION The apparent weight of the submerged object (the crown) is w' (what the scale reads), and is the force pulling down on the scale hook. By Newton's third law, w' equals the force F'_{T} that the scale exerts on the crown in Fig. 13–16b. The sum of the forces on the crown is zero, so w' equals the actual weight $w\ (= mg)$ minus the buoyant force F_{B}:

$$w' = F'_{\text{T}} = w - F_{\text{B}}$$

so

$$w - w' = F_{\text{B}}.$$

Let V be the volume of the completely submerged object and ρ_{O} its density (so $\rho_{\text{O}} V$ is its mass), and let ρ_{F} be the density of the fluid (water). Then $(\rho_{\text{F}} V)g$ is the weight of fluid displaced $(= F_{\text{B}})$. Now we can write

$$w = mg = \rho_{\text{O}} V g$$

$$w - w' = F_{\text{B}} = \rho_{\text{F}} V g.$$

We divide these two equations and obtain

$$\frac{w}{w - w'} = \frac{\rho_{\text{O}} V g}{\rho_{\text{F}} V g} = \frac{\rho_{\text{O}}}{\rho_{\text{F}}}.$$

We see that $w/(w - w')$ is equal to the specific gravity of the object if the fluid in which it is submerged is water $(\rho_{\text{F}} = 1.00 \times 10^3\ \text{kg/m}^3)$. Thus

$$\frac{\rho_{\text{O}}}{\rho_{\text{H}_2\text{O}}} = \frac{w}{w - w'} = \frac{(14.7\ \text{kg})g}{(14.7\ \text{kg} - 13.4\ \text{kg})g} = \frac{14.7\ \text{kg}}{1.3\ \text{kg}} = 11.3.$$

This corresponds to a density of $11{,}300\ \text{kg/m}^3$. The crown is not gold, but seems to be made of lead (see Table 13–1).

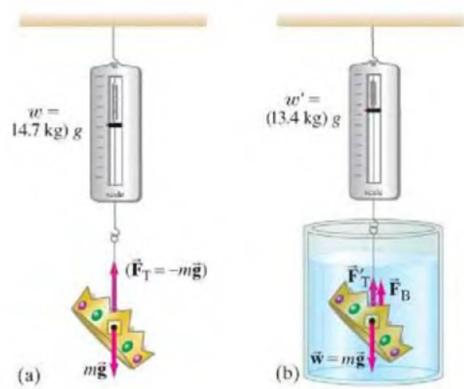


FIGURE 13–16 (a) A scale reads the mass of an object in air—in this case the crown of Example 13–10. All objects are at rest, so the tension F_{T} in the connecting cord equals the weight w of the object: $F_{\text{T}} = mg$. We show the free-body diagram of the crown, and F_{T} is what causes the scale reading (it is equal to the net downward force on the scale, by Newton's third law). (b) Submerged, the crown has an additional force on it, the buoyant force F_{B}. The net force is zero, so $F'_{\text{T}} + F_{\text{B}} = mg\ (= w)$. The scale now reads $m' = 13.4$ kg, where m' is related to the effective weight by $w' = m'g$. Thus $F'_{\text{T}} = w' = w - F_{\text{B}}$.

Archimedes' principle applies equally well to objects that float, such as wood. In general, *an object floats on a fluid if its density* (ρ_O) *is less than that of the fluid* (ρ_F). This is readily seen from Fig. 13–17a, where a submerged log will experience a net upward force and float to the surface if $F_B > mg$; that is, if $\rho_F Vg > \rho_O Vg$ or $\rho_F > \rho_O$. At equilibrium—that is, when floating—the buoyant force on an object has magnitude equal to the weight of the object. For example, a log whose specific gravity is 0.60 and whose volume is 2.0 m^3 has a mass $m = \rho_O V = (0.60 \times 10^3\,\text{kg/m}^3)(2.0\,\text{m}^3) = 1200\,\text{kg}$. If the log is fully submerged, it will displace a mass of water $m_F = \rho_F V = (1000\,\text{kg/m}^3)(2.0\,\text{m}^3) = 2000\,\text{kg}$. Hence the buoyant force on the log will be greater than its weight, and it will float upward to the surface (Fig. 13–17). The log will come to equilibrium when it displaces 1200 kg of water, which means that 1.2 m^3 of its volume will be submerged. This 1.2 m^3 corresponds to 60% of the volume of the log (1.2/2.0 = 0.60), so 60% of the log is submerged.

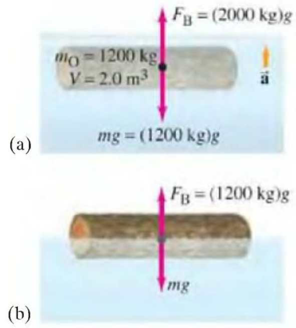


FIGURE 13–17 (a) The fully submerged log accelerates upward because $F_B > mg$. It comes to equilibrium (b) when $\Sigma F = 0$, so $F_B = mg = (1200\,\text{kg})g$. Thus 1200 kg, or 1.2 m^3, of water is displaced.

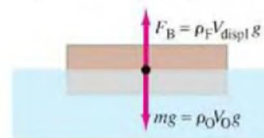
In general when an object floats, we have $F_B = mg$, which we can write as (see Fig. 13–18)

$$F_B = mg$$
$$\rho_F V_{\text{displ}}\, g = \rho_O V_O\, g,$$

where V_O is the full volume of the object and V_{displ} is the volume of fluid it displaces (= volume submerged). Thus

$$\frac{V_{\text{displ}}}{V_O} = \frac{\rho_O}{\rho_F}.$$

FIGURE 13–18 An object floating in equilibrium: $F_B = mg$.



That is, the fraction of the object submerged is given by the ratio of the object's density to that of the fluid. If the fluid is water, this fraction equals the specific gravity of the object.

EXAMPLE 13–11 **Hydrometer calibration.** A **hydrometer** is a simple instrument used to measure the specific gravity of a liquid by indicating how deeply the instrument sinks in the liquid. A particular hydrometer (Fig. 13–19) consists of a glass tube, weighted at the bottom, which is 25.0 cm long and 2.00 cm^2 in cross-sectional area, and has a mass of 45.0 g. How far from the end should the 1.000 mark be placed?

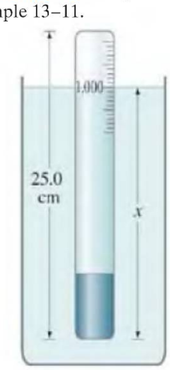
APPROACH The hydrometer will float in water if its density ρ is less than $\rho_w = 1.000\,\text{g/cm}^3$, the density of water. The fraction of the hydrometer submerged $(V_{\text{displaced}}/V_{\text{total}})$ is equal to the density ratio ρ/ρ_w.

SOLUTION The hydrometer has an overall density

$$\rho = \frac{m}{V} = \frac{45.0\,\text{g}}{(2.00\,\text{cm}^2)(25.0\,\text{cm})} = 0.900\,\text{g/cm}^3.$$

Thus, when placed in water, it will come to equilibrium when 0.900 of its volume is submerged. Since it is of uniform cross section, $(0.900)(25.0\,\text{cm}) = 22.5\,\text{cm}$ of its length will be submerged. The specific gravity of water is defined to be 1.000, so the mark should be placed 22.5 cm from the weighted end.

FIGURE 13–19 A hydrometer. Example 13–11.



EXERCISE C On the hydrometer of Example 13–11, will the marks above the 1.000 mark represent higher or lower values of density of the liquid in which it is submerged?

Archimedes' principle is also useful in geology. According to the theories of plate tectonics and continental drift, the continents float on a fluid "sea" of slightly deformable rock (mantle rock). Some interesting calculations can be done using very simple models, which we consider in the Problems at the end of the Chapter.

PHYSICS APPLIED
Continental drift—plate tectonics

Air is a fluid, and it too exerts a buoyant force. Ordinary objects weigh less in air than they do if weighed in a vacuum. Because the density of air is so small, the effect for ordinary solids is slight. There are objects, however, that *float* in air—helium-filled balloons, for example, because the density of helium is less than the density of air.

EXERCISE D Which of the following objects, submerged in water, experiences the largest magnitude of the buoyant force? (*a*) A 1-kg helium balloon; (*b*) 1 kg of wood; (*c*) 1 kg of ice; (*d*) 1 kg of iron; (*e*) all the same.

EXERCISE E Which of the following objects, submerged in water, experiences the largest magnitude of the buoyant force? (*a*) A 1-m^3 helium balloon; (*b*) 1 m^3 of wood; (*c*) 1 m^3 of ice; (*d*) 1 m^3 of iron; (*e*) all the same.

EXAMPLE 13–12 **Helium balloon.** What volume V of helium is needed if a balloon is to lift a load of 180 kg (including the weight of the empty balloon)?

APPROACH The buoyant force on the helium balloon, F_B, which is equal to the weight of displaced air, must be at least equal to the weight of the helium plus the weight of the balloon and load (Fig. 13–20). Table 13–1 gives the density of helium as $0.179\ \mathrm{kg/m^3}$.

SOLUTION The buoyant force must have a minimum value of

$$F_B = (m_{He} + 180\ \mathrm{kg})g.$$

This equation can be written in terms of density using Archimedes' principle:

$$\rho_{air} V g = (\rho_{He} V + 180\ \mathrm{kg})g.$$

Solving now for V, we find

$$V = \frac{180\ \mathrm{kg}}{\rho_{air} - \rho_{He}} = \frac{180\ \mathrm{kg}}{(1.29\ \mathrm{kg/m^3} - 0.179\ \mathrm{kg/m^3})} = 160\ \mathrm{m^3}.$$

NOTE This is the minimum volume needed near the Earth's surface, where $\rho_{air} = 1.29\ \mathrm{kg/m^3}$. To reach a high altitude, a greater volume would be needed since the density of air decreases with altitude.

FIGURE 13–20 Example 13–12.

13–8 Fluids in Motion; Flow Rate and the Equation of Continuity

We now turn to the subject of fluids in motion, which is called **fluid dynamics**, or (especially if the fluid is water) **hydrodynamics**.

We can distinguish two main types of fluid flow. If the flow is smooth, such that neighboring layers of the fluid slide by each other smoothly, the flow is said to be **streamline** or **laminar flow**.† In streamline flow, each particle of the fluid follows a smooth path, called a **streamline**, and these paths do not cross one another (Fig. 13–21a). Above a certain speed, the flow becomes turbulent. **Turbulent flow** is characterized by erratic, small, whirlpool-like circles called *eddy currents* or *eddies* (Fig. 13–21b). Eddies absorb a great deal of energy, and although a certain amount of internal friction called **viscosity** is present even during streamline flow, it is much greater when the flow is turbulent. A few tiny drops of ink or food coloring dropped into a moving liquid can quickly reveal whether the flow is streamline or turbulent.

†The word *laminar* means "in layers."

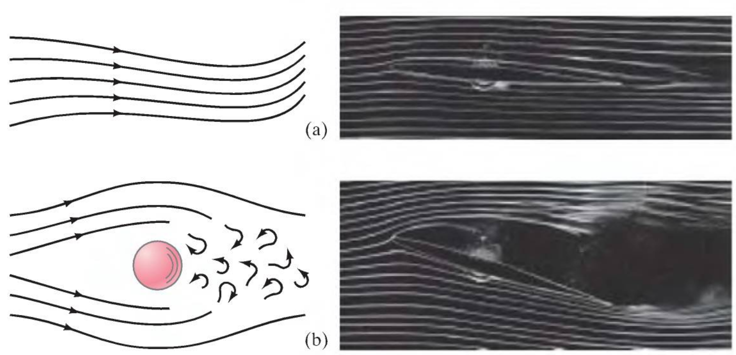


FIGURE 13–21 (a) Streamline, or laminar, flow; (b) turbulent flow. The photos show airflow around an airfoil or airplane wing (more in Section 13–10).

Let us consider the steady laminar flow of a fluid through an enclosed tube or pipe as shown in Fig. 13–22. First we determine how the speed of the fluid changes when the size of the tube changes. The mass **flow rate** is defined as the mass Δm of fluid that passes a given point per unit time Δt:

$$\text{mass flow rate} = \frac{\Delta m}{\Delta t}.$$

In Fig. 13–22, the volume of fluid passing point 1 (that is, through area A_1) in a time Δt is $A_1\,\Delta\ell_1$, where $\Delta\ell_1$ is the distance the fluid moves in time Δt. Since the velocity† of fluid passing point 1 is $v_1 = \Delta\ell_1/\Delta t$, the mass flow rate through area A_1 is

$$\frac{\Delta m_1}{\Delta t} = \frac{\rho_1\,\Delta V_1}{\Delta t} = \frac{\rho_1 A_1\,\Delta\ell_1}{\Delta t} = \rho_1 A_1 v_1,$$

where $\Delta V_1 = A_1\,\Delta\ell_1$ is the volume of mass Δm_1, and ρ_1 is the fluid density. Similarly, at point 2 (through area A_2), the flow rate is $\rho_2 A_2 v_2$. Since no fluid flows in or out the sides, the flow rates through A_1 and A_2 must be equal. Thus, since

$$\frac{\Delta m_1}{\Delta t} = \frac{\Delta m_2}{\Delta t},$$

then

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2. \qquad \textbf{(13–7a)}$$

This is called the **equation of continuity**.

If the fluid is incompressible (ρ doesn't change with pressure), which is an excellent approximation for liquids under most circumstances (and sometimes for gases as well), then $\rho_1 = \rho_2$, and the equation of continuity becomes

$$A_1 v_1 = A_2 v_2. \qquad [\rho = \text{constant}] \quad \textbf{(13–7b)}$$

The product Av represents the *volume rate of flow* (volume of fluid passing a given point per second), since $\Delta V/\Delta t = A\,\Delta\ell/\Delta t = Av$, which in SI units is m^3/s. Equation 13–7b tells us that where the cross-sectional area is large, the velocity is small, and where the area is small, the velocity is large. That this is reasonable can be seen by looking at a river. A river flows slowly through a meadow where it is broad, but speeds up to torrential speed when passing through a narrow gorge.

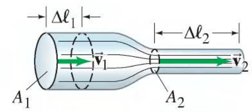


FIGURE 13–22 Fluid flow through a pipe of varying diameter.

EXAMPLE 13–13 ESTIMATE Blood flow. In humans, blood flows from the heart into the aorta, from which it passes into the major arteries. These branch into the small arteries (arterioles), which in turn branch into myriads of tiny capillaries, Fig. 13–23. The blood returns to the heart via the veins. The radius of the aorta is about 1.2 cm, and the blood passing through it has a speed of about 40 cm/s. A typical capillary has a radius of about 4×10^{-4} cm, and blood flows through it at a speed of about 5×10^{-4} m/s. Estimate the number of capillaries that are in the body.

APPROACH We assume the density of blood doesn't vary significantly from the aorta to the capillaries. By the equation of continuity, the volume flow rate in the aorta must equal the volume flow rate through *all* the capillaries. The total area of all the capillaries is given by the area of one capillary multiplied by the total number N of capillaries.

SOLUTION Let A_1 be the area of the aorta and A_2 be the area of *all* the capillaries through which blood flows. Then $A_2 = N\pi r_{\text{cap}}^2$, where $r_{\text{cap}} \approx 4 \times 10^{-4}$ cm is the estimated average radius of one capillary. From the equation of continuity (Eq. 13–7b), we have

$$v_2 A_2 = v_1 A_1$$
$$v_2 N\pi r_{\text{cap}}^2 = v_1 \pi r_{\text{aorta}}^2$$

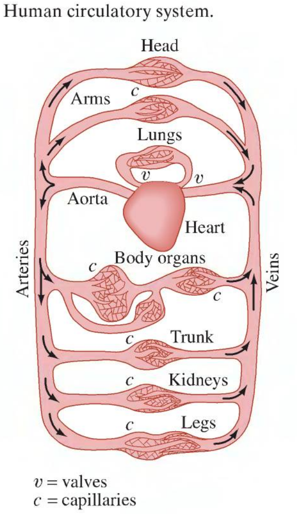
so

$$N = \frac{v_1}{v_2}\frac{r_{\text{aorta}}^2}{r_{\text{cap}}^2} = \left(\frac{0.40\ \text{m/s}}{5 \times 10^{-4}\ \text{m/s}}\right)\left(\frac{1.2 \times 10^{-2}\ \text{m}}{4 \times 10^{-6}\ \text{m}}\right)^2 \approx 7 \times 10^9,$$

or on the order of 10 billion capillaries.

PHYSICS APPLIED
Blood flow

FIGURE 13–23 Human circulatory system.



† If there were no viscosity, the velocity would be the same across a cross section of the tube. Real fluids have viscosity, and this internal friction causes different layers of the fluid to flow at different speeds. In this case v_1 and v_2 represent the average speeds at each cross section.

PHYSICS APPLIED
Heating duct

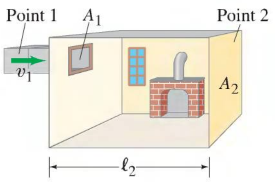


FIGURE 13–24 Example 13–14.

EXAMPLE 13–14 **Heating duct to a room.** What area must a heating duct have if air moving 3.0 m/s along it can replenish the air every 15 minutes in a room of volume 300 m^3? Assume the air's density remains constant.

APPROACH We apply the equation of continuity at constant density, Eq. 13–7b, to the air that flows through the duct (point 1 in Fig. 13–24) and then into the room (point 2). The volume flow rate in the room equals the volume of the room divided by the 15-min replenishing time.

SOLUTION Consider the room as a large section of the duct, Fig. 13–24, and think of air equal to the volume of the room as passing by point 2 in $t = 15\text{ min} = 900\text{ s}$. Reasoning in the same way we did to obtain Eq. 13–7a (changing Δt to t), we write $v_2 = \ell_2/t$ so $A_2 v_2 = A_2 \ell_2/t = V_2/t$, where V_2 is the volume of the room. Then the equation of continuity becomes $A_1 v_1 = A_2 v_2 = V_2/t$ and

$$A_1 = \frac{V_2}{v_1 t} = \frac{300\text{ m}^3}{(3.0\text{ m/s})(900\text{ s})} = 0.11\text{ m}^2.$$

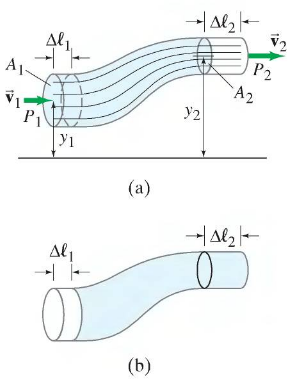
If the duct is square, then each side has length $\ell = \sqrt{A} = 0.33\text{ m}$, or 33 cm. A rectangular duct 20 cm × 55 cm will also do.

13–9 Bernoulli's Equation

Have you ever wondered why an airplane can fly, or how a sailboat can move against the wind? These are examples of a principle worked out by Daniel Bernoulli (1700–1782) concerning fluids in motion. In essence, **Bernoulli's principle** states that *where the velocity of a fluid is high, the pressure is low, and where the velocity is low, the pressure is high*. For example, if the pressures in the fluid at points 1 and 2 of Fig. 13–22 are measured, it will be found that the pressure is lower at point 2, where the velocity is greater, than it is at point 1, where the velocity is smaller. At first glance, this might seem strange; you might expect that the greater speed at point 2 would imply a higher pressure. But this cannot be the case. For if the pressure in the fluid at point 2 were higher than at point 1, this higher pressure would slow the fluid down, whereas in fact it has sped up in going from point 1 to point 2. Thus the pressure at point 2 must be less than at point 1, to be consistent with the fact that the fluid accelerates.

To help clarify any misconceptions, a faster fluid *would* exert a greater force on an obstacle placed in its path. But that is not what we mean by the pressure in a fluid, and besides we are not considering obstacles that interrupt the flow. We are examining smooth streamline flow. The fluid pressure is exerted on the walls of a tube or pipe, or on the surface of any material the fluid passes over.

FIGURE 13–25 Fluid flow: for derivation of Bernoulli's equation.



Bernoulli developed an equation that expresses this principle quantitatively. To derive Bernoulli's equation, we assume the flow is steady and laminar, the fluid is incompressible, and the viscosity is small enough to be ignored. To be general, we assume the fluid is flowing in a tube of nonuniform cross section that varies in height above some reference level, Fig. 13–25. We will consider the volume of fluid shown in color and calculate the work done to move it from the position shown in Fig. 13–25a to that shown in Fig. 13–25b. In this process, fluid entering area A_1 flows a distance $\Delta\ell_1$ and forces the fluid at area A_2 to move a distance $\Delta\ell_2$. The fluid to the left of area A_1 exerts a pressure P_1 on our section of fluid and does an amount of work

$$W_1 = F_1 \Delta\ell_1 = P_1 A_1 \Delta\ell_1.$$

At area A_2, the work done on our cross section of fluid is

$$W_2 = -P_2 A_2 \Delta\ell_2.$$

The negative sign is present because the force exerted on the fluid is opposite to the motion (thus the fluid shown in color does work on the fluid to the right of point 2). Work is also done on the fluid by the force of gravity. The net effect of the process shown in Fig. 13–25 is to move a mass m of volume $A_1 \Delta\ell_1$ (= $A_2 \Delta\ell_2$, since the fluid is incompressible) from point 1 to point 2, so the work done by gravity is

$$W_3 = -mg(y_2 - y_1),$$

where y_1 and y_2 are heights of the center of the tube above some (arbitrary) reference level. In the case shown in Fig. 13–25, this term is negative since the motion is uphill against the force of gravity. The net work W done on the fluid is thus

$$W = W_1 + W_2 + W_3$$
$$W = P_1 A_1 \Delta\ell_1 - P_2 A_2 \Delta\ell_2 - mgy_2 + mgy_1.$$

According to the work-energy principle (Section 7–4), the net work done on a system is equal to its change in kinetic energy. Hence

$$\tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 = P_1 A_1 \Delta\ell_1 - P_2 A_2 \Delta\ell_2 - mgy_2 + mgy_1.$$

The mass m has volume $A_1 \Delta\ell_1 = A_2 \Delta\ell_2$ for an incompressible fluid. Thus we can substitute $m = \rho A_1 \Delta\ell_1 = \rho A_2 \Delta\ell_2$, and then divide through by $A_1 \Delta\ell_1 = A_2 \Delta\ell_2$, to obtain

$$\tfrac{1}{2}\rho v_2^2 - \tfrac{1}{2}\rho v_1^2 = P_1 - P_2 - \rho g y_2 + \rho g y_1,$$

which we rearrange to get

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2. \qquad \textbf{(13–8)}$$

Bernoulli's equation

This is **Bernoulli's equation**. Since points 1 and 2 can be any two points along a tube of flow, Bernoulli's equation can be written as

$$P + \tfrac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

at every point in the fluid, where y is the height of the center of the tube above a fixed reference level. [Note that if there is no flow $(v_1 = v_2 = 0)$, then Eq. 13–8 reduces to the hydrostatic equation, Eq. 13–6a: $P_2 - P_1 = -\rho g(y_2 - y_1)$.]

Bernoulli's equation is an expression of the law of energy conservation, since we derived it from the work-energy principle.

EXERCISE F As water in a level pipe passes from a narrow cross section of pipe to a wider cross section, how does the pressure against the walls change?

EXAMPLE 13–15 **Flow and pressure in a hot-water heating system.** Water circulates throughout a house in a hot-water heating system. If the water is pumped at a speed of 0.50 m/s through a 4.0-cm-diameter pipe in the basement under a pressure of 3.0 atm, what will be the flow speed and pressure in a 2.6-cm-diameter pipe on the second floor 5.0 m above? Assume the pipes do not divide into branches.

APPROACH We use the equation of continuity at constant density to determine the flow speed on the second floor, and then Bernoulli's equation to find the pressure.

SOLUTION We take v_2 in the equation of continuity, Eq. 13–7, as the flow speed on the second floor, and v_1 as the flow speed in the basement. Noting that the areas are proportional to the radii squared $(A = \pi r^2)$, we obtain

$$v_2 = \frac{v_1 A_1}{A_2} = \frac{v_1 \pi r_1^2}{\pi r_2^2} = (0.50\,\text{m/s})\frac{(0.020\,\text{m})^2}{(0.013\,\text{m})^2} = 1.2\,\text{m/s}.$$

To find the pressure on the second floor, we use Bernoulli's equation (Eq. 13–8):

$$\begin{aligned} P_2 &= P_1 + \rho g(y_1 - y_2) + \tfrac{1}{2}\rho(v_1^2 - v_2^2) \\ &= (3.0 \times 10^5\,\text{N/m}^2) + (1.0 \times 10^3\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(-5.0\,\text{m}) \\ &\quad + \tfrac{1}{2}(1.0 \times 10^3\,\text{kg/m}^3)\big[(0.50\,\text{m/s})^2 - (1.2\,\text{m/s})^2\big] \\ &= (3.0 \times 10^5\,\text{N/m}^2) - (4.9 \times 10^4\,\text{N/m}^2) - (6.0 \times 10^2\,\text{N/m}^2) \\ &= 2.5 \times 10^5\,\text{N/m}^2 = 2.5\,\text{atm}. \end{aligned}$$

NOTE The velocity term contributes very little in this case.

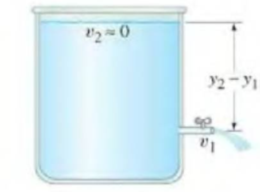


FIGURE 13–26 Torricelli's theorem: $v_1 = \sqrt{2g(y_2 - y_1)}$.

13–10 Applications of Bernoulli's Principle: Torricelli, Airplanes, Baseballs, TIA

Bernoulli's equation can be applied to many situations. One example is to calculate the velocity, v_1, of a liquid flowing out of a spigot at the bottom of a reservoir, Fig. 13–26. We choose point 2 in Eq. 13–8 to be the top surface of the liquid. Assuming the diameter of the reservoir is large compared to that of the spigot, v_2 will be almost zero. Points 1 (the spigot) and 2 (top surface) are open to the atmosphere, so the pressure at both points is equal to atmospheric pressure: $P_1 = P_2$. Then Bernoulli's equation becomes

$$\tfrac{1}{2}\rho v_1^2 + \rho g y_1 = \rho g y_2$$

or

$$v_1 = \sqrt{2g(y_2 - y_1)}. \qquad \textbf{(13–9)}$$

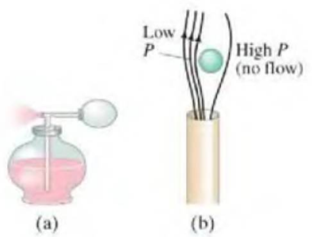
This result is called **Torricelli's theorem**. Although it is seen to be a special case of Bernoulli's equation, it was discovered a century earlier by Evangelista Torricelli. Equation 13–9 tells us that the liquid leaves the spigot with the same speed that a freely falling object would attain if falling from the same height. This should not be too surprising since Bernoulli's equation relies on the conservation of energy.

Another special case of Bernoulli's equation arises when a fluid is flowing horizontally with no significant change in height; that is, $y_1 = y_2$. Then Eq. 13–8 becomes

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2, \qquad \textbf{(13–10)}$$

which tells us quantitatively that the speed is high where the pressure is low, and vice versa. It explains many common phenomena, some of which are illustrated in Figs. 13–27 to 13–32. The pressure in the air blown at high speed across the top of the vertical tube of a perfume atomizer (Fig. 13–27a) is less than the normal air pressure acting on the surface of the liquid in the bowl. Thus atmospheric pressure in the bowl pushes the perfume up the tube because of the lower pressure at the top. A Ping-Pong ball can be made to float above a blowing jet of air (some vacuum cleaners can blow air), Fig. 13–27b; if the ball begins to leave the jet of air, the higher pressure in the still air outside the jet pushes the ball back in.

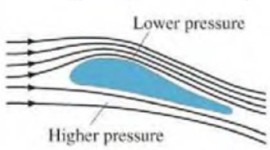
FIGURE 13–27 Examples of Bernoulli's principle: (a) atomizer, (b) Ping-Pong ball in jet of air.



EXERCISE G Return to Chapter-Opening Question 2, page 339, and answer it again now. Try to explain why you may have answered differently the first time. Try it and see.

Airplane Wings and Dynamic Lift

FIGURE 13–28 Lift on an airplane wing. We are in the reference frame of the wing, seeing the air flow by.



PHYSICS APPLIED
Airplanes and dynamic lift

Airplanes experience a "lift" force on their wings, keeping them up in the air, if they are moving at a sufficiently high speed relative to the air and the wing is tilted upward at a small angle (the "attack angle"), as in Fig. 13–28, where streamlines of air are shown rushing by the wing. (We are in the reference frame of the wing, as if sitting on the wing.) The upward tilt, as well as the rounded upper surface of the wing, causes the streamlines to be forced upward and to be crowded together above the wing. The area for air flow between any two streamlines is reduced as the streamlines get closer together, so from the equation of continuity $(A_1 v_1 = A_2 v_2)$, the air speed increases above the wing where the streamlines are squished together. (Recall also how the crowded streamlines in a pipe constriction, Fig. 13–22, indicate the velocity is higher in the constriction.) Because the air speed is greater above the wing than below it, the pressure above the wing is less than the pressure below the wing (Bernoulli's principle). Hence there is a net upward force on the wing called **dynamic lift**. Experiments show that the speed of air above the wing can even be double the speed of the air below it. (Friction between the air and wing exerts a *drag force*, toward the rear, which must be overcome by the plane's engines.)

A flat wing, or one with symmetric cross section, will experience lift as long as the front of the wing is tilted upward (attack angle). The wing shown in Fig. 13–28 can experience lift even if the attack angle is zero, because the rounded upper surface deflects air up, squeezing the streamlines together. Airplanes can fly upside down, experiencing lift, if the attack angle is sufficient to deflect streamlines up and closer together.

Our picture considers streamlines; but if the attack angle is larger than about 15°, turbulence sets in (Fig. 13–21b) leading to greater drag and less lift, causing the wing to "stall" and the plane to drop.

From another point of view, the upward tilt of a wing means the air moving horizontally in front of the wing is deflected downward; the change in momentum of the rebounding air molecules results in an upward force on the wing (Newton's third law).

Sailboats

A sailboat can move *against* the wind, with the aid of the Bernoulli effect, by setting the sails at an angle, as shown in Fig. 13–29. The air travels rapidly over the bulging front surface of the sail, and the relatively still air filling the sail exerts a greater pressure behind the sail, resulting in a net force on the sail, $\vec{\mathbf{F}}_{\text{wind}}$. This force would tend to make the boat move sideways if it weren't for the keel that extends vertically downward beneath the water: the water exerts a force $(\vec{\mathbf{F}}_{\text{water}})$ on the keel nearly perpendicular to the keel. The resultant of these two forces $(\vec{\mathbf{F}}_{\text{R}})$ is almost directly forward as shown.

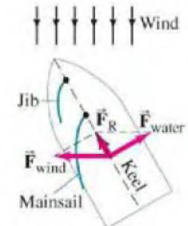


FIGURE 13–29 Sailboat sailing against the wind.

Baseball Curve

Why a spinning pitched baseball (or tennis ball) curves can also be explained using Bernoulli's principle. It is simplest if we put ourselves in the reference frame of the ball, with the air rushing by, just as we did for the airplane wing. Suppose the ball is rotating counterclockwise as seen from above, Fig. 13–30. A thin layer of air ("boundary layer") is being dragged around by the ball. We are looking down on the ball, and at point A in Fig. 13–30, this boundary layer tends to slow down the oncoming air. At point B, the air rotating with the ball adds its speed to that of the oncoming air, so the air speed is higher at B than at A. The higher speed at B means the pressure is lower at B than at A, resulting in a net force toward B. The ball's path curves toward the left (as seen by the pitcher).

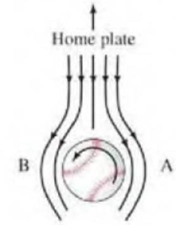


FIGURE 13–30 Looking down on a pitched baseball heading toward home plate. We are in the reference frame of the baseball, with the air flowing by.

Lack of Blood to the Brain—TIA

In medicine, one of many applications of Bernoulli's principle is to explain a TIA, a transient ischemic attack (meaning a temporary lack of blood supply to the brain). A person suffering a TIA may experience symptoms such as dizziness, double vision, headache, and weakness of the limbs. A TIA can occur as follows. Blood normally flows up to the brain at the back of the head via the two vertebral arteries—one going up each side of the neck—which meet to form the basilar artery just below the brain, as shown in Fig. 13–31. The vertebral arteries issue from the subclavian arteries, as shown, before the latter pass to the arms. When an arm is exercised vigorously, blood flow increases to meet the needs of the arm's muscles. If the subclavian artery on one side of the body is partially blocked, however, as in arteriosclerosis (hardening of the arteries), the blood velocity will have to be higher on that side to supply the needed blood. (Recall the equation of continuity: smaller area means larger velocity for the same flow rate, Eqs. 13–7.) The increased blood velocity past the opening to the vertebral artery results in lower pressure (Bernoulli's principle). Thus blood rising in the vertebral artery on the "good" side at normal pressure can be *diverted down* into the other vertebral artery because of the low pressure on that side, instead of passing upward to the brain. Hence the blood supply to the brain is reduced.

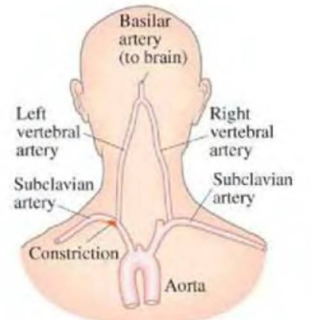


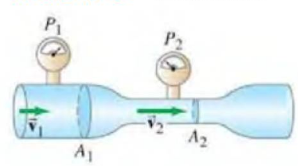
FIGURE 13–31 Rear of the head and shoulders showing arteries leading to the brain and to the arms. High blood velocity past the constriction in the left subclavian artery causes low pressure in the left vertebral artery, in which a reverse (downward) blood flow can then occur, resulting in a TIA, a loss of blood to the brain.

Other Applications

A **venturi tube** is essentially a pipe with a narrow constriction (the throat). The flowing fluid speeds up as it passes through this constriction, so the pressure is lower in the throat. A *venturi meter*, Fig. 13–32, is used to measure the flow speed of gases and liquids, including blood velocity in arteries.

Why does smoke go up a chimney? It's partly because hot air rises (it's less dense and therefore buoyant). But Bernoulli's principle also plays a role. When wind blows across the top of a chimney, the pressure is less there than inside the house. Hence, air and smoke are pushed up the chimney by the higher indoor pressure. Even on an apparently still night there is usually enough ambient air flow at the top of a chimney to assist upward flow of smoke.

FIGURE 13–32 Venturi meter.



Bernoulli's equation ignores the effects of friction (viscosity) and the compressibility of the fluid. The energy that is transformed to internal (or potential) energy due to compression and to thermal energy by friction can be taken into account by adding terms to Eq. 13–8. These terms are difficult to calculate theoretically and are normally determined empirically. They do not significantly alter the explanations for the phenomena described above.

* 13–11 Viscosity

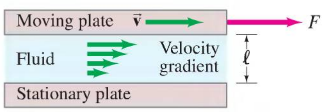


FIGURE 13–33
Determination of viscosity.

Real fluids have a certain amount of internal friction called **viscosity**, as mentioned in Section 13–8. Viscosity exists in both liquids and gases, and is essentially a frictional force between adjacent layers of fluid as the layers move past one another. In liquids, viscosity is due to the electrical cohesive forces between the molecules. In gases, it arises from collisions between the molecules.

The viscosity of different fluids can be expressed quantitatively by a *coefficient of viscosity*, η (the Greek lowercase letter eta), which is defined in the following way. A thin layer of fluid is placed between two flat plates. One plate is stationary and the other is made to move, Fig. 13–33. The fluid directly in contact with each plate is held to the surface by the adhesive force between the molecules of the liquid and those of the plate. Thus the upper surface of the fluid moves with the same speed v as the upper plate, whereas the fluid in contact with the stationary plate remains stationary. The stationary layer of fluid retards the flow of the layer just above it, which in turn retards the flow of the next layer, and so on. Thus the velocity varies continuously from 0 to v, as shown. The increase in velocity divided by the distance over which this change is made—equal to v/ℓ—is called the *velocity gradient*. To move the upper plate requires a force, which you can verify by moving a flat plate across a puddle of syrup on a table. For a given fluid, it is found that the force required, F, is proportional to the area of fluid in contact with each plate, A, and to the speed, v, and is inversely proportional to the separation, ℓ, of the plates: $F \propto vA/\ell$. For different fluids, the more viscous the fluid, the greater is the required force. Hence the proportionality constant for this equation is defined as the coefficient of viscosity, η:

$$F = \eta A \frac{v}{\ell}. \qquad \textbf{(13–11)}$$

Solving for η, we find $\eta = F\ell/vA$. The SI unit for η is $\mathrm{N \cdot s/m^2} = \mathrm{Pa \cdot s}$ (pascal·second). In the cgs system, the unit is $\mathrm{dyne \cdot s/cm^2}$, which is called a *poise* (P). Viscosities are often given in centipoise $(1\,\mathrm{cP} = 10^{-2}\,\mathrm{P})$. Table 13–3 lists the coefficient of viscosity for various fluids. The temperature is also specified, since it has a strong effect; the viscosity of liquids such as motor oil, for example, decreases rapidly as temperature increases.†

TABLE 13–3
Coefficients of Viscosity

Fluid (temperature in °C)	Coefficient of Viscosity, η (Pa·s)†
Water (0°)	1.8×10^{-3}
(20°)	1.0×10^{-3}
(100°)	0.3×10^{-3}
Whole blood (37°)	$\approx 4 \times 10^{-3}$
Blood plasma (37°)	$\approx 1.5 \times 10^{-3}$
Ethyl alcohol (20°)	1.2×10^{-3}
Engine oil (30°) (SAE 10)	200×10^{-3}
Glycerine (20°)	1500×10^{-3}
Air (20°)	0.018×10^{-3}
Hydrogen (0°)	0.009×10^{-3}
Water vapor (100°)	0.013×10^{-3}

† 1 Pa·s = 10 P = 1000 cP.

* 13–12 Flow in Tubes: Poiseuille's Equation, Blood Flow

If a fluid had no viscosity, it could flow through a level tube or pipe without a force being applied. Viscosity acts like a sort of friction (between fluid layers moving at slightly different speeds), so a pressure difference between the ends of a level tube is necessary for the steady flow of any real fluid, be it water or oil in a pipe, or blood in the circulatory system of a human.

The French scientist J. L. Poiseuille (1799–1869), who was interested in the physics of blood circulation (and after whom the poise is named), determined how the variables affect the flow rate of an incompressible fluid undergoing laminar flow in a cylindrical tube. His result, known as *Poiseuille's equation*, is:

$$Q = \frac{\pi R^4 (P_1 - P_2)}{8 \eta \ell}, \qquad \textbf{(13–12)}$$

where R is the inside radius of the tube, ℓ is the tube length, $P_1 - P_2$ is the pressure

†The Society of Automotive Engineers assigns numbers to represent the viscosity of oils: 30 weight (SAE 30) is more viscous than 10 weight. Multigrade oils, such as 20–50, are designed to maintain viscosity as temperature increases; 20–50 means the oil is 20 wt when cool but is like a 50-wt pure oil when it is hot (engine running temperature).

difference between the ends, η is the coefficient of viscosity, and Q is the volume rate of flow (volume of fluid flowing past a given point per unit time which in SI has units of m^3/s). Equation 13–12 applies only to laminar flow.

Poiseuille's equation tells us that the flow rate Q is directly proportional to the "pressure gradient," $(P_1 - P_2)/\ell$, and it is inversely proportional to the viscosity of the fluid. This is just what we might expect. It may be surprising, however, that Q also depends on the *fourth* power of the tube's radius. This means that for the same pressure gradient, if the tube radius is halved, the flow rate is decreased by a factor of 16! Thus the rate of flow, or alternately the pressure required to maintain a given flow rate, is greatly affected by only a small change in tube radius.

PHYSICS APPLIED
Blood flow

An interesting example of this R^4 dependence is *blood flow* in the human body. Poiseuille's equation is valid only for the streamline flow of an incompressible fluid. So it cannot be precisely accurate for blood whose flow is not without turbulence and that contains blood cells (whose diameter is almost equal to that of a capillary). Nonetheless, Poiseuille's equation does give a reasonable first approximation. Because the radius of arteries is reduced as a result of arteriosclerosis (thickening and hardening of artery walls) and by cholesterol buildup, the pressure gradient must be increased to maintain the same flow rate. If the radius is reduced by half, the heart would have to increase the pressure by a factor of about $2^4 = 16$ in order to maintain the same blood-flow rate. The heart must work much harder under these conditions, but usually cannot maintain the original flow rate. Thus, high blood pressure is an indication both that the heart is working harder and that the blood-flow rate is reduced.

*13–13 Surface Tension and Capillarity

The *surface* of a liquid at rest behaves in an interesting way, almost as if it were a stretched membrane under tension. For example, a drop of water on the end of a dripping faucet, or hanging from a thin branch in the early morning dew (Fig. 13–34), forms into a nearly spherical shape as if it were a tiny balloon filled with water. A steel needle can be made to float on the surface of water even though it is denser than the water. The surface of a liquid acts like it is under tension, and this tension, acting along the surface, arises from the attractive forces between the molecules. This effect is called **surface tension**. More specifically, a quantity called the *surface tension*, γ (the Greek letter gamma), is defined as the force F per unit length ℓ that acts perpendicular to any line or cut in a liquid surface, tending to pull the surface closed:

$$\gamma = \frac{F}{\ell}. \qquad \textbf{(13–13)}$$

FIGURE 13–34 Spherical water droplets, dew on a blade of grass.

To understand this, consider the U-shaped apparatus shown in Fig. 13–35 which encloses a thin film of liquid. Because of surface tension, a force F is required to pull the movable wire and thus increase the surface area of the liquid. The liquid contained by the wire apparatus is a thin film having both a top and a bottom surface. Hence the total length of the surface being increased is 2ℓ, and the surface tension is $\gamma = F/2\ell$. A delicate apparatus of this type can be used to measure the surface tension of various liquids. The surface tension of water is 0.072 N/m at 20°C. Table 13–4 gives the values for several substances. Note that temperature has a considerable effect on the surface tension.

TABLE 13–4 Surface Tension of Some Substances

Substance (temperature in °C)	Surface Tension (N/m)
Mercury (20°)	0.44
Blood, whole (37°)	0.058
Blood, plasma (37°)	0.073
Alcohol, ethyl (20°)	0.023
Water (0°)	0.076
(20°)	0.072
(100°)	0.059
Benzene (20°)	0.029
Soap solution (20°)	≈0.025
Oxygen (−193°)	0.016

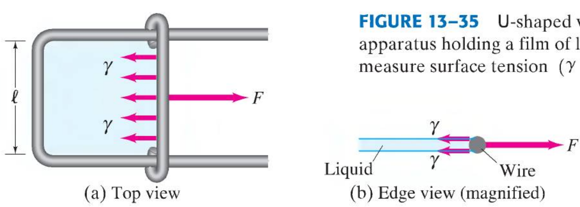


FIGURE 13–35 U-shaped wire apparatus holding a film of liquid to measure surface tension ($\gamma = F/2\ell$).

FIGURE 13–36 A water strider.

Because of surface tension, some insects (Fig. 13–36) can walk on water, and objects more dense than water, such as a steel needle, can float on the surface. Figure 13–37a shows how the surface tension can support the weight w of an object. Actually, the object sinks slightly into the fluid, so w is the "effective weight" of that object—its true weight less the buoyant force.

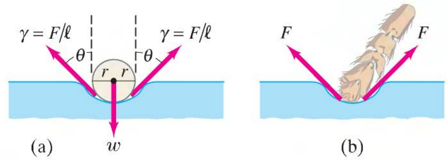


FIGURE 13–37 Surface tension acting on (a) a sphere, and (b) an insect leg. Example 13–16.

EXAMPLE 13–16 ESTIMATE **Insect walks on water.** The base of an insect's leg is approximately spherical in shape, with a radius of about 2.0×10^{-5} m. The 0.0030-g mass of the insect is supported equally by its six legs. Estimate the angle θ (see Fig. 13–37) for an insect on the surface of water. Assume the water temperature is 20°C.

APPROACH Since the insect is in equilibrium, the upward surface tension force is equal to the pull of gravity downward on each leg. We ignore the buoyant force for this estimate.

SOLUTION For each leg, we assume the surface tension force acts all around a circle of radius r, at an angle θ, as shown in Fig. 13–37a. Only the vertical component, $\gamma\cos\theta$, acts to balance the weight mg. So we set the length ℓ in Eq. 13–13 equal to the circumference of the circle, $\ell \approx 2\pi r$. Then the net upward force due to surface tension is $F_y \approx (\gamma\cos\theta)\ell \approx 2\pi r\gamma\cos\theta$. We set this surface tension force equal to one-sixth the weight of the insect since it has six legs:

$$2\pi r\gamma\cos\theta \approx \tfrac{1}{6}mg$$

$$(6.28)(2.0 \times 10^{-5}\,\text{m})(0.072\,\text{N/m})\cos\theta \approx \tfrac{1}{6}(3.0 \times 10^{-6}\,\text{kg})(9.8\,\text{m/s}^2)$$

$$\cos\theta \approx \frac{0.49}{0.90} = 0.54.$$

So $\theta \approx 57°$. If $\cos\theta$ had come out greater than 1, the surface tension would not be great enough to support the insect's weight.

NOTE Our estimate ignored the buoyant force and ignored any difference between the radius of the insect's "foot" and the radius of the surface depression.

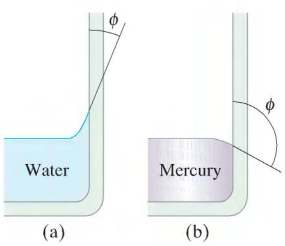
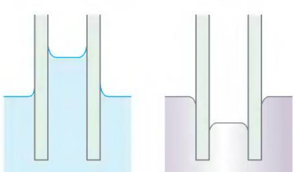


FIGURE 13–38 (a) Water "wets" the surface of glass, whereas (b) mercury does not "wet" the glass.

Soaps and detergents lower the surface tension of water. This is desirable for washing and cleaning since the high surface tension of pure water prevents it from penetrating easily between the fibers of material and into tiny crevices. Substances that reduce the surface tension of a liquid are called *surfactants*.

Surface tension plays a role in another interesting phenomenon, capillarity. It is a common observation that water in a glass container rises up slightly where it touches the glass, Fig. 13–38a. The water is said to "wet" the glass. Mercury, on the other hand, is depressed when it touches the glass, Fig. 13–38b; the mercury does not wet the glass. Whether a liquid wets a solid surface is determined by the relative strength of the cohesive forces between the molecules of the liquid compared to the adhesive forces between the molecules of the liquid and those of the container. *Cohesion* refers to the force between molecules of the same type, whereas *adhesion* refers to the force between molecules of different types. Water wets glass because the water molecules are more strongly attracted to the glass molecules than they are to other water molecules. The opposite is true for mercury: the cohesive forces are stronger than the adhesive forces.

FIGURE 13–39 Capillarity.



(a) Glass tube in water

(b) Glass tube in mercury

In tubes having very small diameters, liquids are observed to rise or fall relative to the level of the surrounding liquid. This phenomenon is called **capillarity**, and such thin tubes are called **capillaries**. Whether the liquid rises or falls (Fig. 13–39) depends on the relative strengths of the adhesive and cohesive forces. Thus water rises in a glass tube, whereas mercury falls. The actual amount of rise (or fall) depends on the surface tension—which is what keeps the liquid surface from breaking apart.

*13–14 Pumps, and the Heart

We conclude this Chapter with a brief discussion of pumps, including the heart. Pumps can be classified into categories according to their function. A *vacuum pump* is designed to reduce the pressure (usually of air) in a given vessel. A *force pump*, on the other hand, is a pump that is intended to increase the pressure—for example, to lift a liquid (such as water from a well) or to push a fluid through a pipe. Figure 13–40 illustrates the principle behind a simple reciprocating pump. It could be a vacuum pump, in which case the intake is connected to the vessel to be evacuated. A similar mechanism is used in some force pumps, and in this case the fluid is forced under increased pressure through the outlet.

A centrifugal pump (Fig. 13–41), or any force pump, can be used as a *circulating pump*—that is, to circulate a fluid around a closed path, such as the cooling water or lubricating oil in an automobile.

The heart of a human (and of other animals as well) is essentially a circulating pump. The action of a human heart is shown in Fig. 13–42. There are actually two separate paths for blood flow. The longer path takes blood to the parts of the body, via the arteries, bringing oxygen to body tissues and picking up carbon dioxide, which it carries back to the heart via veins. This blood is then pumped to the lungs (the second path), where the carbon dioxide is released and oxygen is taken up. The oxygen-laden blood is returned to the heart, where it is again pumped to the tissues of the body.

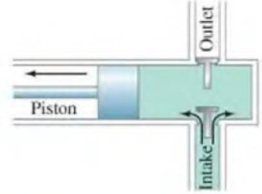


FIGURE 13–40 One kind of pump: the intake valve opens and air (or fluid that is being pumped) fills the empty space when the piston moves to the left. When the piston moves to the right (not shown), the outlet valve opens and fluid is forced out.

FIGURE 13–41 Centrifugal pump: the rotating blades force fluid through the outlet pipe; this kind of pump is used in vacuum cleaners and as a water pump in automobiles.

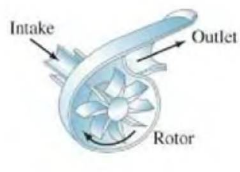
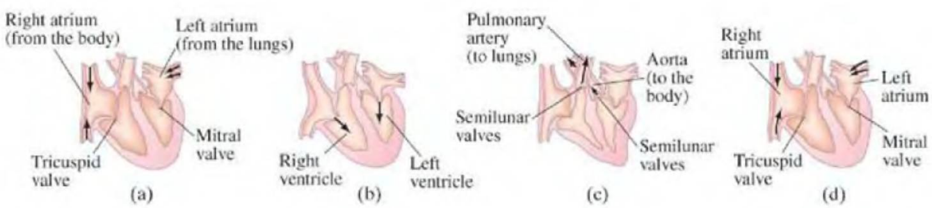


FIGURE 13–42 (a) In the diastole phase, the heart relaxes between beats. Blood moves into the heart; both atria fill rapidly. (b) When the atria contract, the systole or pumping phase begins. The contraction pushes the blood through the mitral and tricuspid valves into the ventricles. (c) The contraction of the ventricles forces the blood through the semilunar valves into the pulmonary artery, which leads to the lungs, and to the aorta (the body's largest artery), which leads to the arteries serving all the body. (d) When the heart relaxes, the semilunar valves close; blood fills the atria, beginning the cycle again.



Summary

The three common phases of matter are **solid**, **liquid**, and **gas**. Liquids and gases are collectively called **fluids**, meaning they have the ability to flow. The **density** of a material is defined as its mass per unit volume:

$$\rho = \frac{m}{V}. \tag{13–1}$$

Specific gravity is the ratio of the density of the material to the density of water (at 4°C).

Pressure is defined as force per unit area:

$$P = \frac{F}{A}. \tag{13–2}$$

The pressure P at a depth h in a liquid is given by

$$P = \rho g h, \tag{13–3}$$

where ρ is the density of the liquid and g is the acceleration due to gravity. If the density of a fluid is not uniform, the pressure P varies with height y as

$$\frac{dP}{dy} = -\rho g. \tag{13–4}$$

Pascal's principle says that an external pressure applied to a confined fluid is transmitted throughout the fluid.

Pressure is measured using a manometer or other type of gauge. A **barometer** is used to measure atmospheric pressure. Standard **atmospheric pressure** (average at sea level) is $1.013 \times 10^5\,\mathrm{N/m^2}$. **Gauge pressure** is the total (absolute) pressure less atmospheric pressure.

Archimedes' principle states that an object submerged wholly or partially in a fluid is buoyed up by a force equal to the weight of fluid it displaces $(F_B = m_F g = \rho_F V_{\mathrm{displ}}\, g)$.

Fluid flow can be characterized either as **streamline** (sometimes called **laminar**), in which the layers of fluid move smoothly and regularly along paths called **streamlines**, or as **turbulent**, in which case the flow is not smooth and regular but is characterized by irregularly shaped whirlpools.

Fluid flow rate is the mass or volume of fluid that passes a given point per unit time. The **equation of continuity** states that for an incompressible fluid flowing in an enclosed tube, the product of the velocity of flow and the cross-sectional area of the tube remains constant:

$$Av = \text{constant}. \qquad \textbf{(13–7b)}$$

Bernoulli's principle tells us that where the velocity of a fluid is high, the pressure in it is low, and where the velocity is low, the pressure is high. For steady laminar flow of an incompressible and nonviscous fluid, **Bernoulli's equation**, which is based on the law of conservation of energy, is

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2, \qquad \textbf{(13–8)}$$

for two points along the flow.

[***Viscosity** refers to friction within a fluid and is essentially a frictional force between adjacent layers of fluid as they move past one another.]

[*Liquid surfaces hold together as if under tension (**surface tension**), allowing drops to form and objects like needles and insects to stay on the surface.]

Questions

1. If one material has a higher density than another, must the molecules of the first be heavier than those of the second? Explain.
2. Airplane travelers sometimes note that their cosmetics bottles and other containers have leaked during a flight. What might cause this?
3. The three containers in Fig. 13–43 are filled with water to the same height and have the same surface area at the base; hence the water pressure, and the total force on the base of each, is the same. Yet the total weight of water is different for each. Explain this "hydrostatic paradox."

FIGURE 13–43 Question 3.

4. Consider what happens when you push both a pin and the blunt end of a pen against your skin with the same force. Decide what determines whether your skin is cut—the net force applied to it or the pressure.
5. A small amount of water is boiled in a 1-gallon metal can. The can is removed from the heat and the lid put on. As the can cools, it collapses. Explain.
6. When blood pressure is measured, why must the cuff be held at the level of the heart?
7. An ice cube floats in a glass of water filled to the brim. What can you say about the density of ice? As the ice melts, will the water overflow? Explain.
8. Will an ice cube float in a glass of alcohol? Why or why not?
9. A submerged can of Coke® will sink, but a can of Diet Coke® will float. (Try it!) Explain.
10. Why don't ships made of iron sink?
11. Explain how the tube in Fig. 13–44, known as a **siphon**, can transfer liquid from one container to a lower one even though the liquid must flow uphill for part of its journey. (Note that the tube must be filled with liquid to start with.)

FIGURE 13–44 Question 11. A siphon.

12. A barge filled high with sand approaches a low bridge over the river and cannot quite pass under it. Should sand be added to, or removed from, the barge? [*Hint*: Consider Archimedes' principle.]
13. Explain why helium weather balloons, which are used to measure atmospheric conditions at high altitudes, are normally released while filled to only 10–20% of their maximum volume.
14. A row boat floats in a swimming pool, and the level of the water at the edge of the pool is marked. Consider the following situations and explain whether the level of the water will rise, fall, or stay the same. (*a*) The boat is removed from the water. (*b*) The boat in the water holds an iron anchor which is removed from the boat and placed on the shore. (*c*) The iron anchor is removed from the boat and dropped in the pool.
15. Will an empty balloon have precisely the same apparent weight on a scale as a balloon filled with air? Explain.
16. Why do you float higher in salt water than in fresh water?
17. If you dangle two pieces of paper vertically, a few inches apart (Fig. 13–45), and blow between them, how do you think the papers will move? Try it and see. Explain.

FIGURE 13–45 Question 17.

FIGURE 13–46 Question 18. Water coming from a faucet.

18. Why does the stream of water from a faucet become narrower as it falls (Fig. 13–46)?
19. Children are told to avoid standing too close to a rapidly moving train because they might get sucked under it. Is this possible? Explain.
20. A tall Styrofoam cup is filled with water. Two holes are punched in the cup near the bottom, and water begins rushing out. If the cup is dropped so it falls freely, will the water continue to flow from the holes? Explain.
21. Why do airplanes normally take off into the wind?
22. Two ships moving in parallel paths close to one another risk colliding. Why?
23. Why does the canvas top of a convertible bulge out when the car is traveling at high speed? [*Hint*: The windshield deflects air upward, pushing streamlines closer together.]
24. Roofs of houses are sometimes "blown" off (or are they pushed off?) during a tornado or hurricane. Explain using Bernoulli's principle.

Problems

13–2 Density and Specific Gravity

1. (I) The approximate volume of the granite monolith known as El Capitan in Yosemite National Park (Fig. 13–47) is about $10^8\,m^3$. What is its approximate mass?

FIGURE 13–47 Problem 1.

2. (I) What is the approximate mass of air in a living room $5.6\,m \times 3.8\,m \times 2.8\,m$?
3. (I) If you tried to smuggle gold bricks by filling your backpack, whose dimensions are $56\,cm \times 28\,cm \times 22\,cm$, what would its mass be?
4. (I) State your mass and then estimate your volume. [*Hint*: Because you can swim on or just under the surface of the water in a swimming pool, you have a pretty good idea of your density.]
5. (II) A bottle has a mass of 35.00 g when empty and 98.44 g when filled with water. When filled with another fluid, the mass is 89.22 g. What is the specific gravity of this other fluid?
6. (II) If 5.0 L of antifreeze solution (specific gravity = 0.80) is added to 4.0 L of water to make a 9.0-L mixture, what is the specific gravity of the mixture?
7. (III) The Earth is not a uniform sphere, but has regions of varying density. Consider a simple model of the Earth divided into three regions—inner core, outer core, and mantle. Each region is taken to have a unique constant density (the average density of that region in the real Earth):

Region	Radius (km)	Density (kg/m^3)
Inner Core	0–1220	13,000
Outer Core	1220–3480	11,100
Mantle	3480–6371	4,400

(*a*) Use this model to predict the average density of the entire Earth. (*b*) The measured radius of the Earth is 6371 km and its mass is 5.98×10^{24} kg. Use these data to determine the actual average density of the Earth and compare it (as a percent difference) with the one you determined in (*a*).

13–3 to 13–6 Pressure; Pascal's Principle

8. (I) Estimate the pressure needed to raise a column of water to the same height as a 35-m-tall oak tree.
9. (I) Estimate the pressure exerted on a floor by (*a*) one pointed chair leg (66 kg on all four legs) of area = $0.020\,cm^2$, and (*b*) a 1300-kg elephant standing on one foot (area = $800\,cm^2$).
10. (I) What is the difference in blood pressure (mm-Hg) between the top of the head and bottom of the feet of a 1.70-m-tall person standing vertically?
11. (II) How high would the level be in an alcohol barometer at normal atmospheric pressure?
12. (II) In a movie, Tarzan evades his captors by hiding underwater for many minutes while breathing through a long, thin reed. Assuming the maximum pressure difference his lungs can manage and still breathe is −85 mm-Hg, calculate the deepest he could have been.
13. (II) The maximum gauge pressure in a hydraulic lift is 17.0 atm. What is the largest-size vehicle (kg) it can lift if the diameter of the output line is 22.5 cm?
14. (II) The gauge pressure in each of the four tires of an automobile is 240 kPa. If each tire has a "footprint" of $220\,cm^2$, estimate the mass of the car.
15. (II) (*a*) Determine the total force and the absolute pressure on the bottom of a swimming pool 28.0 m by 8.5 m whose uniform depth is 1.8 m. (*b*) What will be the pressure against the *side* of the pool near the bottom?
16. (II) A house at the bottom of a hill is fed by a full tank of water 5.0 m deep and connected to the house by a pipe that is 110 m long at an angle of 58° from the horizontal (Fig. 13–48). (*a*) Determine the water gauge pressure at the house. (*b*) How high could the water shoot if it came vertically out of a broken pipe in front of the house?

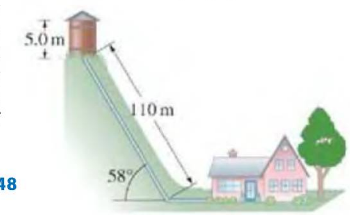


FIGURE 13–48 Problem 16.

17. (II) Water and then oil (which don't mix) are poured into a U-shaped tube, open at both ends. They come to equilibrium as shown in Fig. 13–49. What is the density of the oil? [*Hint*: Pressures at points a and b are equal. Why?]

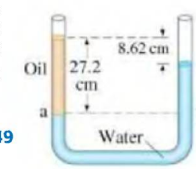


FIGURE 13–49 Problem 17.

18. (II) In working out his principle, Pascal showed dramatically how force can be multiplied with fluid pressure. He placed a long, thin tube of radius $r = 0.30$ cm vertically into a wine barrel of radius $R = 21$ cm, Fig. 13–50. He found that when the barrel was filled with water and the tube filled to a height of 12 m, the barrel burst. Calculate (*a*) the mass of water in the tube, and (*b*) the net force exerted by the water in the barrel on the lid just before rupture.

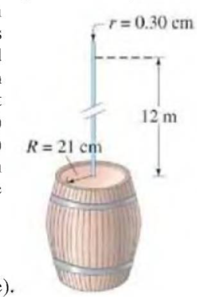


FIGURE 13–50 Problem 18 (not to scale).

19. (II) What is the normal pressure of the atmosphere at the summit of Mt. Everest, 8850 m above sea level?

20. (II) A hydraulic press for compacting powdered samples has a large cylinder which is 10.0 cm in diameter, and a small cylinder with a diameter of 2.0 cm (Fig. 13–51). A lever is attached to the small cylinder as shown. The sample, which is placed on the large cylinder, has an area of $4.0\,\text{cm}^2$. What is the pressure on the sample if 350 N is applied to the lever?

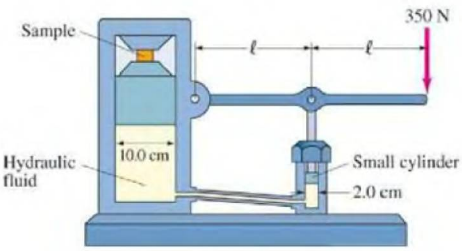


FIGURE 13–51 Problem 20.

21. (II) An open-tube mercury manometer is used to measure the pressure in an oxygen tank. When the atmospheric pressure is 1040 mbar, what is the absolute pressure (in Pa) in the tank if the height of the mercury in the open tube is (*a*) 21.0 cm higher, (*b*) 5.2 cm lower, than the mercury in the tube connected to the tank?

22. (III) A beaker of liquid accelerates from rest, on a horizontal surface, with acceleration a to the right. (*a*) Show that the surface of the liquid makes an angle $\theta = \tan^{-1}(a/g)$ with the horizontal. (*b*) Which edge of the water surface is higher? (*c*) How does the pressure vary with depth below the surface?

23. (III) Water stands at a height h behind a vertical dam of uniform width b. (*a*) Use integration to show that the total force of the water on the dam is $F = \frac{1}{2}\rho g h^2 b$. (*b*) Show that the torque about the base of the dam due to this force can be considered to act with a lever arm equal to $h/3$. (*c*) For a freestanding concrete dam of uniform thickness t and height h, what minimum thickness is needed to prevent overturning? Do you need to add in atmospheric pressure for this last part? Explain.

24. (III) Estimate the density of the water 5.4 km deep in the sea. (See Table 12–1 and Section 12–4 regarding bulk modulus.) By what fraction does it differ from the density at the surface?

25. (III) A cylindrical bucket of liquid (density ρ) is rotated about its symmetry axis, which is vertical. If the angular velocity is ω, show that the pressure at a distance r from the rotation axis is

$$P = P_0 + \tfrac{1}{2}\rho\omega^2 r^2,$$

where P_0 is the pressure at $r = 0$.

13–7 Buoyancy and Archimedes' Principle

26. (I) What fraction of a piece of iron will be submerged when it floats in mercury?

27. (I) A geologist finds that a Moon rock whose mass is 9.28 kg has an apparent mass of 6.18 kg when submerged in water. What is the density of the rock?

28. (II) A crane lifts the 16,000-kg steel hull of a sunken ship out of the water. Determine (*a*) the tension in the crane's cable when the hull is fully submerged in the water, and (*b*) the tension when the hull is completely out of the water.

29. (II) A spherical balloon has a radius of 7.35 m and is filled with helium. How large a cargo can it lift, assuming that the skin and structure of the balloon have a mass of 930 kg? Neglect the buoyant force on the cargo volume itself.

30. (II) A 74-kg person has an apparent mass of 54 kg (because of buoyancy) when standing in water that comes up to the hips. Estimate the mass of each leg. Assume the body has SG = 1.00.

31. (II) What is the likely identity of a metal (see Table 13–1) if a sample has a mass of 63.5 g when measured in air and an apparent mass of 55.4 g when submerged in water?

32. (II) Calculate the true mass (in vacuum) of a piece of aluminum whose apparent mass is 3.0000 kg when weighed in air.

33. (II) Because gasoline is less dense than water, drums containing gasoline will float in water. Suppose a 230-L steel drum is completely full of gasoline. What total volume of steel can be used in making the drum if the gasoline-filled drum is to float in fresh water?

34. (II) A scuba diver and her gear displace a volume of 65.0 L and have a total mass of 68.0 kg. (*a*) What is the buoyant force on the diver in seawater? (*b*) Will the diver sink or float?

35. (II) The specific gravity of ice is 0.917, whereas that of seawater is 1.025. What percent of an iceberg is above the surface of the water?

36. (II) Archimedes' principle can be used not only to determine the specific gravity of a solid using a known liquid (Example 13–10); the reverse can be done as well. (*a*) As an example, a 3.80-kg aluminum ball has an apparent mass of 2.10 kg when submerged in a particular liquid: calculate the density of the liquid. (*b*) Derive a formula for determining the density of a liquid using this procedure.

37. (II) (*a*) Show that the buoyant force F_B on a partially submerged object such as a ship acts at the center of gravity of the fluid before it is displaced. This point is called the **center of buoyancy**. (*b*) To ensure that a ship is in stable equilibrium, would it be better if its center of buoyancy was above, below, or at the same point as, its center of gravity? Explain. (See Fig. 13–52.)

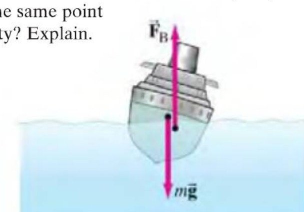


FIGURE 13–52 Problem 37.

38. (II) A cube of side length 10.0 cm and made of unknown material floats at the surface between water and oil. The oil has a density of $810\,\text{kg/m}^3$. If the cube floats so that it is 72% in the water and 28% in the oil, what is the mass of the cube and what is the buoyant force on the cube?

39. (II) How many helium-filled balloons would it take to lift a person? Assume the person has a mass of 75 kg and that each helium-filled balloon is spherical with a diameter of 33 cm.

40. (II) A scuba tank, when fully submerged, displaces 15.7 L of seawater. The tank itself has a mass of 14.0 kg and, when "full," contains 3.00 kg of air. Assuming only a weight and buoyant force act, determine the net force (magnitude and direction) on the fully submerged tank at the beginning of a dive (when it is full of air) and at the end of a dive (when it no longer contains any air).

41. (III) If an object floats in water, its density can be determined by tying a sinker to it so that both the object and the sinker are submerged. Show that the specific gravity is given by $w/(w_1 - w_2)$, where w is the weight of the object alone in air, w_1 is the apparent weight when a sinker is tied to it and the sinker only is submerged, and w_2 is the apparent weight when both the object and the sinker are submerged.

42. (III) A 3.25-kg piece of wood (SG = 0.50) floats on water. What minimum mass of lead, hung from the wood by a string, will cause it to sink?

13–8 to 13–10 Fluid Flow, Bernoulli's Equation

43. (I) A 15-cm-radius air duct is used to replenish the air of a room 8.2 m × 5.0 m × 3.5 m every 12 min. How fast does the air flow in the duct?

44. (I) Using the data of Example 13–13, calculate the average speed of blood flow in the major arteries of the body which have a total cross-sectional area of about 2.0 cm^2.

45. (I) How fast does water flow from a hole at the bottom of a very wide, 5.3-m-deep storage tank filled with water? Ignore viscosity.

46. (II) A fish tank has dimensions 36 cm wide by 1.0 m long by 0.60 m high. If the filter should process all the water in the tank once every 4.0 h, what should the flow speed be in the 3.0-cm-diameter input tube for the filter?

47. (II) What gauge pressure in the water mains is necessary if a firehose is to spray water to a height of 18 m?

48. (II) A $\frac{5}{8}$-in. (inside) diameter garden hose is used to fill a round swimming pool 6.1 m in diameter. How long will it take to fill the pool to a depth of 1.2 m if water flows from the hose at a speed of 0.40 m/s?

49. (II) A 180-km/h wind blowing over the flat roof of a house causes the roof to lift off the house. If the house is 6.2 m × 12.4 m in size, estimate the weight of the roof. Assume the roof is not nailed down.

50. (II) A 6.0-cm-diameter horizontal pipe gradually narrows to 4.5 cm. When water flows through this pipe at a certain rate, the gauge pressure in these two sections is 32.0 kPa and 24.0 kPa, respectively. What is the volume rate of flow?

51. (II) Estimate the air pressure inside a category 5 hurricane, where the wind speed is 300 km/h (Fig. 13–53).

FIGURE 13–53 Problem 51.

52. (II) What is the lift (in newtons) due to Bernoulli's principle on a wing of area 88 m^2 if the air passes over the top and bottom surfaces at speeds of 280 m/s and 150 m/s, respectively?

53. (II) Show that the power needed to drive a fluid through a pipe with uniform cross-section is equal to the volume rate of flow, Q, times the pressure difference, $P_1 - P_2$.

54. (II) Water at a gauge pressure of 3.8 atm at street level flows into an office building at a speed of 0.68 m/s through a pipe 5.0 cm in diameter. The pipe tapers down to 2.8 cm in diameter by the top floor, 18 m above (Fig. 13–54), where the faucet has been left open. Calculate the flow velocity and the gauge pressure in the pipe on the top floor. Assume no branch pipes and ignore viscosity.

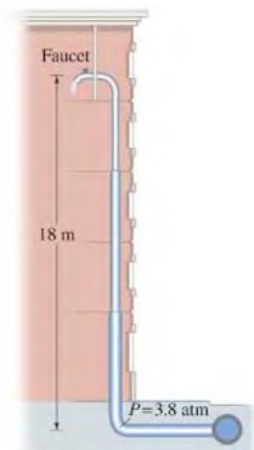


FIGURE 13–54 Problem 54.

55. (II) In Fig. 13–55, take into account the speed of the top surface of the tank and show that the speed of fluid leaving the opening at the bottom is

$$v_1 = \sqrt{\frac{2gh}{(1 - A_1^2/A_2^2)}},$$

where $h = y_2 - y_1$, and A_1 and A_2 are the areas of the opening and of the top surface, respectively. Assume $A_1 \ll A_2$ so that the flow remains nearly steady and laminar.

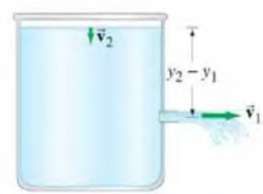


FIGURE 13–55 Problems 55, 56, 58, and 59.

56. (II) Suppose the top surface of the vessel in Fig. 13–55 is subjected to an external gauge pressure P_2. (*a*) Derive a formula for the speed, v_1, at which the liquid flows from the opening at the bottom into atmospheric pressure, P_0. Assume the velocity of the liquid surface, v_2, is approximately zero. (*b*) If $P_2 = 0.85$ atm and $y_2 - y_1 = 2.4$ m, determine v_1 for water.

57. (II) You are watering your lawn with a hose when you put your finger over the hose opening to increase the distance the water reaches. If you are pointing the hose at the same angle, and the distance the water reaches increases by a factor of 4, what fraction of the hose opening did you block?

58. (III) Suppose the opening in the tank of Fig. 13–55 is a height h_1 above the base and the liquid surface is a height h_2 above the base. The tank rests on level ground. (*a*) At what horizontal distance from the base of the tank will the fluid strike the ground? (*b*) At what other height, h_1', can a hole be placed so that the emerging liquid will have the same "range"? Assume $v_2 \approx 0$.

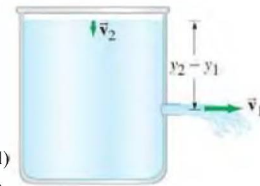


FIGURE 13–55 (repeated) Problems 55, 56, 58, and 59.

59. (III) (*a*) In Fig. 13–55, show that Bernoulli's principle predicts that the level of the liquid, $h = y_2 - y_1$, drops at a rate

$$\frac{dh}{dt} = -\sqrt{\frac{2ghA_1^2}{A_2^2 - A_1^2}},$$

where A_1 and A_2 are the areas of the opening and the top surface, respectively, assuming $A_1 \ll A_2$, and viscosity is ignored. (*b*) Determine h as a function of time by integrating. Let $h = h_0$ at $t = 0$. (*c*) How long would it take to empty a 10.6-cm-tall cylinder filled with 1.3 L of water if the opening is at the bottom and has a 0.50-cm diameter?

60. (III) (*a*) Show that the flow speed measured by a venturi meter (see Fig. 13–32) is given by the relation

$$v_1 = A_2\sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}.$$

(*b*) A venturi meter is measuring the flow of water; it has a main diameter of 3.0 cm tapering down to a throat diameter of 1.0 cm. If the pressure difference is measured to be 18 mm-Hg, what is the speed of the water entering the venturi throat?

61. (III) *Thrust of a rocket.* (*a*) Use Bernoulli's equation and the equation of continuity to show that the emission speed of the propelling gases of a rocket is

$$v = \sqrt{2(P - P_0)/\rho},$$

where ρ is the density of the gas, P is the pressure of the gas inside the rocket, and P_0 is atmospheric pressure just outside the exit orifice. Assume that the gas density stays approximately constant, and that the area of the exit orifice, A_0, is much smaller than the cross-sectional area, A, of the inside of the rocket (take it to be a large cylinder). Assume also that the gas speed is not so high that significant turbulence or nonsteady flow sets in. (*b*) Show that the thrust force on the rocket due to the emitted gases is

$$F = 2A_0(P - P_0).$$

62. (III) A fire hose exerts a force on the person holding it. This is because the water accelerates as it goes from the hose through the nozzle. How much force is required to hold a 7.0-cm-diameter hose delivering 450 L/min through a 0.75-cm-diameter nozzle?

*13–11 Viscosity

*63. (II) A viscometer consists of two concentric cylinders, 10.20 cm and 10.60 cm in diameter. A liquid fills the space between them to a depth of 12.0 cm. The outer cylinder is fixed, and a torque of 0.024 m·N keeps the inner cylinder turning at a steady rotational speed of 57 rev/min. What is the viscosity of the liquid?

*64. (III) A long vertical hollow tube with an inner diameter of 1.00 cm is filled with SAE 10 motor oil. A 0.900-cm-diameter, 30.0-cm-long 150-g rod is dropped vertically through the oil in the tube. What is the maximum speed attained by the rod as it falls?

*13–12 Flow in Tubes; Poiseuille's Equation

*65. (I) Engine oil (assume SAE 10, Table 13–3) passes through a fine 1.80-mm-diameter tube that is 8.6 cm long. What pressure difference is needed to maintain a flow rate of 6.2 mL/min?

*66. (I) A gardener feels it is taking too long to water a garden with a $\frac{3}{8}$-in.-diameter hose. By what factor will the time be cut using a $\frac{5}{8}$-in.-diameter hose instead? Assume nothing else is changed.

*67. (II) What diameter must a 15.5-m-long air duct have if the ventilation and heating system is to replenish the air in a room 8.0 m × 14.0 m × 4.0 m every 12.0 min? Assume the pump can exert a gauge pressure of 0.710×10^{-3} atm.

*68. (II) What must be the pressure difference between the two ends of a 1.9-km section of pipe, 29 cm in diameter, if it is to transport oil $(\rho = 950\ \mathrm{kg/m^3},\ \eta = 0.20\ \mathrm{Pa \cdot s})$ at a rate of 650 $\mathrm{cm^3/s}$?

*69. (II) Poiseuille's equation does not hold if the flow velocity is high enough that turbulence sets in. The onset of turbulence occurs when the **Reynolds number**, *Re*, exceeds approximately 2000. *Re* is defined as

$$Re = \frac{2\bar{v}r\rho}{\eta},$$

where $\bar{v}$ is the average speed of the fluid, ρ is its density, η is its viscosity, and r is the radius of the tube in which the fluid is flowing. (*a*) Determine if blood flow through the aorta is laminar or turbulent when the average speed of blood in the aorta $(r = 0.80\ \mathrm{cm})$ during the resting part of the heart's cycle is about 35 cm/s. (*b*) During exercise, the blood-flow speed approximately doubles. Calculate the Reynolds number in this case, and determine if the flow is laminar or turbulent.

*70. (II) Assuming a constant pressure gradient, if blood flow is reduced by 85%, by what factor is the radius of a blood vessel decreased?

*71. (III) A patient is to be given a blood transfusion. The blood is to flow through a tube from a raised bottle to a needle inserted in the vein (Fig. 13–56). The inside diameter of the 25-mm-long needle is 0.80 mm, and the required flow rate is 2.0 $\mathrm{cm^3}$ of blood per minute. How high h should the bottle be placed above the needle? Obtain ρ and η from the Tables. Assume the blood pressure is 78 torr above atmospheric pressure.

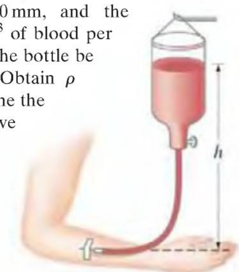


FIGURE 13–56 Problems 71 and 79.

*13–13 Surface Tension and Capillarity

*72. (I) If the force F needed to move the wire in Fig. 13–35 is 3.4×10^{-3} N, calculate the surface tension γ of the enclosed fluid. Assume $\ell = 0.070$ m.

*73. (I) Calculate the force needed to move the wire in Fig. 13–35 if it is immersed in a soapy solution and the wire is 24.5 cm long.

*74. (II) The surface tension of a liquid can be determined by measuring the force F needed to just lift a circular platinum ring of radius r from the surface of the liquid. (*a*) Find a formula for γ in terms of F and r. (*b*) At 30°C, if $F = 5.80 \times 10^{-3}$ N and $r = 2.8$ cm, calculate γ for the tested liquid.

*75. (III) Estimate the diameter of a steel needle that can just "float" on water due to surface tension.

*76. (III) Show that inside a soap bubble, there must be a pressure ΔP in excess of that outside equal to $\Delta P = 4\gamma/r$, where r is the radius of the bubble and γ is the surface tension. [*Hint*: Think of the bubble as two hemispheres in contact with each other; and remember that there are two surfaces to the bubble. Note that this result applies to any kind of membrane, where 2γ is the tension per unit length in that membrane.]

*77. (III) A common effect of surface tension is the ability of a liquid to rise up a narrow tube due to what is called capillary action. Show that for a narrow tube of radius r placed in a liquid of density ρ and surface tension γ, the liquid in the tube will reach a height $h = 2\gamma/\rho g r$ above the level of the liquid outside the tube, where g is the gravitational acceleration. Assume that the liquid "wets" the capillary (the liquid surface is vertical at the contact with the inside of the tube).

General Problems

78. A 2.8-N force is applied to the plunger of a hypodermic needle. If the diameter of the plunger is 1.3 cm and that of the needle 0.20 mm, (*a*) with what force does the fluid leave the needle? (*b*) What force on the plunger would be needed to push fluid into a vein where the gauge pressure is 75 mm-Hg? Answer for the instant just before the fluid starts to move.

79. Intravenous infusions are often made under gravity, as shown in Fig. 13–56. Assuming the fluid has a density of $1.00\ \mathrm{g/cm^3}$, at what height h should the bottle be placed so the liquid pressure is (*a*) 55 mm-Hg, and (*b*) 650 mm-H_2O? (*c*) If the blood pressure is 78 mm-Hg above atmospheric pressure, how high should the bottle be placed so that the fluid just barely enters the vein?

80. A beaker of water rests on an electronic balance that reads 998.0 g. A 2.6-cm-diameter solid copper ball attached to a string is submerged in the water, but does not touch the bottom. What are the tension in the string and the new balance reading?

81. Estimate the difference in air pressure between the top and the bottom of the Empire State building in New York City? It is 380 m tall and is located at sea level. Express as a fraction of atmospheric pressure at sea level.

82. A hydraulic lift is used to jack a 920-kg car 42 cm off the floor. The diameter of the output piston is 18 cm, and the input force is 350 N. (*a*) What is the area of the input piston? (*b*) What is the work done in lifting the car 42 cm? (*c*) If the input piston moves 13 cm in each stroke, how high does the car move up for each stroke? (*d*) How many strokes are required to jack the car up 42 cm? (*e*) Show that energy is conserved.

83. When you ascend or descend a great deal when driving in a car, your ears "pop," which means that the pressure behind the eardrum is being equalized to that outside. If this did not happen, what would be the approximate force on an eardrum of area $0.20\ \mathrm{cm^2}$ if a change in altitude of 950 m takes place?

84. Giraffes are a wonder of cardiovascular engineering. Calculate the difference in pressure (in atmospheres) that the blood vessels in a giraffe's head must accommodate as the head is lowered from a full upright position to ground level for a drink. The height of an average giraffe is about 6 m.

85. Suppose a person can reduce the pressure in his lungs to −75 mm-Hg gauge pressure. How high can water then be "sucked" up a straw?

86. Airlines are allowed to maintain a minimum air pressure within the passenger cabin equivalent to that at an altitude of 8000 ft (2400 m) to avoid adverse health effects among passengers due to oxygen deprivation. Estimate this minimum pressure (in atm).

87. A simple model (Fig. 13–57) considers a continent as a block (density $\approx 2800\ \mathrm{kg/m^3}$) floating in the mantle rock around it (density $\approx 3300\ \mathrm{kg/m^3}$). Assuming the continent is 35 km thick (the average thickness of the Earth's continental crust), estimate the height of the continent above the surrounding rock.

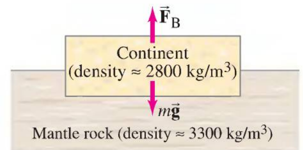


FIGURE 13–57 Problem 87.

88. A ship, carrying fresh water to a desert island in the Caribbean, has a horizontal cross-sectional area of $2240\ \mathrm{m^2}$ at the waterline. When unloaded, the ship rises 8.50 m higher in the sea. How many cubic meters of water was delivered?

89. During ascent, and especially during descent, volume changes of trapped air in the middle ear can cause ear discomfort until the middle-ear pressure and exterior pressure are equalized. (*a*) If a rapid descent at a rate of 7.0 m/s or faster commonly causes ear discomfort, what is the maximum rate of increase in atmospheric pressure (that is, dP/dt) tolerable to most people? (*b*) In a 350-m-tall building, what will be the fastest possible descent time for an elevator traveling from the top to ground floor, assuming the elevator is properly designed to account for human physiology?

90. A raft is made of 12 logs lashed together. Each is 45 cm in diameter and has a length of 6.1 m. How many people can the raft hold before they start getting their feet wet, assuming the average person has a mass of 68 kg? Do *not* neglect the weight of the logs. Assume the specific gravity of wood is 0.60.

91. Estimate the total mass of the Earth's atmosphere, using the known value of atmospheric pressure at sea level.

92. During each heartbeat, approximately $70\ \mathrm{cm^3}$ of blood is pushed from the heart at an average pressure of 105 mm-Hg. Calculate the power output of the heart, in watts, assuming 70 beats per minute.

93. Four lawn sprinkler heads are fed by a 1.9-cm-diameter pipe. The water comes out of the heads at an angle of 35° to the horizontal and covers a radius of 7.0 m. (*a*) What is the velocity of the water coming out of each sprinkler head? (Assume zero air resistance.) (*b*) If the output diameter of each head is 3.0 mm, how many liters of water do the four heads deliver per second? (*c*) How fast is the water flowing inside the 1.9-cm-diameter pipe?

94. A bucket of water is accelerated upward at $1.8\,g$. What is the buoyant force on a 3.0-kg granite rock (SG = 2.7) submerged in the water? Will the rock float? Why or why not?

95. The stream of water from a faucet decreases in diameter as it falls (Fig. 13–58). Derive an equation for the diameter of the stream as a function of the distance y below the faucet, given that the water has speed v_0 when it leaves the faucet, whose diameter is d.

FIGURE 13–58 Problem 95. Water coming from a faucet.

96. You need to siphon water from a clogged sink. The sink has an area of $0.38\,\text{m}^2$ and is filled to a height of 4.0 cm. Your siphon tube rises 45 cm above the bottom of the sink and then descends 85 cm to a pail as shown in Fig. 13–59. The siphon tube has a diameter of 2.0 cm. (*a*) Assuming that the water level in the sink has almost zero velocity, estimate the water velocity when it enters the pail. (*b*) Estimate how long it will take to empty the sink.

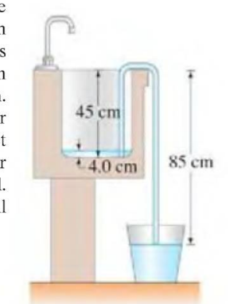


FIGURE 13–59 Problem 96.

97. An airplane has a mass of 1.7×10^6 kg, and the air flows past the lower surface of the wings at 95 m/s. If the wings have a surface area of $1200\,\text{m}^2$, how fast must the air flow over the upper surface of the wing if the plane is to stay in the air?

98. A drinking fountain shoots water about 14 cm up in the air from a nozzle of diameter 0.60 cm. The pump at the base of the unit (1.1 m below the nozzle) pushes water into a 1.2-cm-diameter supply pipe that goes up to the nozzle. What gauge pressure does the pump have to provide? Ignore the viscosity; your answer will therefore be an underestimate.

99. A hurricane-force wind of 200 km/h blows across the face of a storefront window. Estimate the force on the $2.0\,\text{m} \times 3.0\,\text{m}$ window due to the difference in air pressure inside and outside the window. Assume the store is airtight so the inside pressure remains at 1.0 atm. (This is why you should not tightly seal a building in preparation for a hurricane).

100. Blood from an animal is placed in a bottle 1.30 m above a 3.8-cm-long needle, of inside diameter 0.40 mm, from which it flows at a rate of $4.1\,\text{cm}^3/\text{min}$. What is the viscosity of this blood?

101. Three forces act significantly on a freely floating helium-filled balloon: gravity, air resistance (or drag force), and a buoyant force. Consider a spherical helium-filled balloon of radius $r = 15$ cm rising upward through 0°C air, and $m = 2.8$ g is the mass of the (deflated) balloon itself. For all speeds v, except the very slowest ones, the flow of air past a rising balloon is turbulent, and the drag force F_D is given by the relation

$$F_D = \tfrac{1}{2} C_D \rho_{air} \pi r^2 v^2$$

where the constant $C_D = 0.47$ is the "drag coefficient" for a smooth sphere of radius r. If this balloon is released from rest, it will accelerate very quickly (in a few tenths of a second) to its terminal velocity v_T, where the buoyant force is cancelled by the drag force and the balloon's total weight. Assuming the balloon's acceleration takes place over a negligible time and distance, how long does it take the released balloon to rise a distance $h = 12$ m?

* 102. If cholesterol buildup reduces the diameter of an artery by 15%, by what % will the blood flow rate be reduced, assuming the same pressure difference?

103. A two-component model used to determine percent body fat in a human body assumes that a fraction $f(<1)$ of the body's total mass m is composed of fat with a density of $0.90\,\text{g/cm}^3$, and that the remaining mass of the body is composed of fat-free tissue with a density of $1.10\,\text{g/cm}^3$. If the specific gravity of the entire body's density is X, show that the percent body fat ($= f \times 100$) is given by

$$\%\ \text{Body fat} = \frac{495}{X} - 450.$$

* Numerical/Computer

* 104. (III) Air pressure decreases with altitude. The following data show the air pressure at different altitudes.

Altitude (m)	Pressure (kPa)
0	101.3
1000	89.88
2000	79.50
3000	70.12
4000	61.66
5000	54.05
6000	47.22
7000	41.11
8000	35.65
9000	30.80
10,000	26.50

(*a*) Determine the best-fit quadratic equation that shows how the air pressure changes with altitude. (*b*) Determine the best-fit exponential equation that describes the change of air pressure with altitude. (*c*) Use each fit to find the air pressure at the summit of the mountain K2 at 8611 m, and give the % difference.

Answers to Exercises

A: (*d*).

B: The same. Pressure depends on depth, not on length.

C: Lower.

D: (*a*).

E: (*e*).

F: Increases.

G: (*b*).

An object attached to a coil spring can exhibit oscillatory motion. Many kinds of oscillatory motion are sinusoidal in time, or nearly so, and are referred to as being simple harmonic motion. Real systems generally have at least some friction, causing the motion to be damped. The automobile spring shown here has a shock absorber (yellow) that purposefully dampens the oscillation to make for a smooth ride. When an external sinusoidal force is exerted on a system able to oscillate, resonance occurs if the driving force is at or near the natural frequency of oscillation.

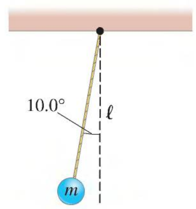
CHAPTER 14

Oscillations

CHAPTER-OPENING QUESTION—Guess now!

A simple pendulum consists of a mass m (the "bob") hanging on the end of a thin string of length ℓ and negligible mass. The bob is pulled sideways so the string makes a 5.0° angle to the vertical; when released, it oscillates back and forth at a frequency f. If the pendulum was raised to a 10.0° angle instead, its frequency would be

(a) twice as great.
(b) half as great.
(c) the same, or very close to it.
(d) not quite twice as great.
(e) a bit more than half as great.



Many objects vibrate or oscillate—an object on the end of a spring, a tuning fork, the balance wheel of an old watch, a pendulum, a plastic ruler held firmly over the edge of a table and gently struck, the strings of a guitar or piano. Spiders detect prey by the vibrations of their webs; cars oscillate up and down when they hit a bump; buildings and bridges vibrate when heavy trucks pass or the wind is fierce. Indeed, because most solids are elastic (see Chapter 12), they vibrate (at least briefly) when given an impulse. Electrical oscillations are necessary in radio and television sets. At the atomic level, atoms vibrate within a molecule, and the atoms of a solid vibrate about their relatively fixed positions. Because it is so common in everyday life and occurs in so many areas of physics, oscillatory motion is of great importance. Mechanical oscillations are fully described on the basis of Newtonian mechanics.

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