PHYSICS 220 : GROUP THEORY
PROBLEM SET #3

[**1**] Construct homogeneous polynomial basis functions of the lowest order for all C_{4v} IRREPs.

[**2**] Is C_{4v} simply reducible? Why or why not? If yes, construct all its CGCs.

[**3**] Find all the $\mathsf{SU}(2)$ CGCs for $\frac{1}{2} \otimes 1 = \frac{1}{2} \oplus \frac{3}{2}$.

[**4**] By applying the raising and lowering operators, show that the $\mathsf{SU}(2)$ CGCs satisfy a recursion relation relating $\begin{pmatrix} j_1 & j_2 \\ m_1 & m_2 \end{pmatrix} \begin{vmatrix} j \\ m\pm1 \end{vmatrix}$, $\begin{pmatrix} j_1 & j_2 \\ m_1\mp1 & m_2 \end{pmatrix} \begin{vmatrix} j \\ m \end{vmatrix}$, and $\begin{pmatrix} j_1 & j_2 \\ m_1 & m_2\mp1 \end{pmatrix} \begin{vmatrix} j \\ m \end{vmatrix}$.