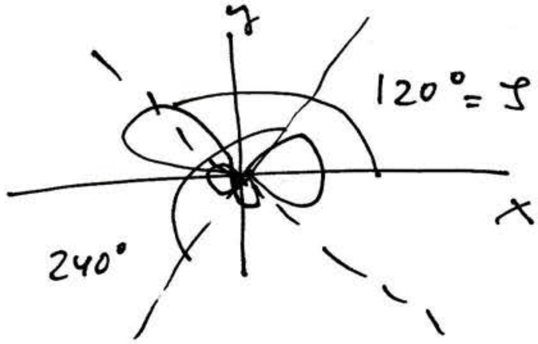
Problem 1

$\Phi_1 = \frac{1}{\sqrt{3}} s + \sqrt{\frac{2}{3}}\, p_x$

$\Phi_2 = \frac{1}{\sqrt{3}} s - \frac{1}{\sqrt{6}} p_x + \frac{1}{\sqrt{2}} p_y$

$\Phi_3 = \frac{1}{\sqrt{3}} s - \frac{1}{\sqrt{6}} p_x - \frac{1}{\sqrt{2}} p_y$

$\Phi_4 = p_z$



1) check that they are $\perp$. Obviously, $\langle \Phi_4 | \Phi_i \rangle = 0$ for $i = 1, 2, 3$

$\langle \Phi_1 | \Phi_2 \rangle = \frac{1}{3} - \sqrt{\frac{2}{18}} = 0 \qquad \langle \Phi_2 | \Phi_3 \rangle = \frac{1}{3} + \frac{1}{6} - \frac{1}{2} = 0$

$\langle \Phi_1 | \Phi_3 \rangle = \frac{1}{3} - \sqrt{\frac{2}{18}} = 0$

2) check that Φ_1, Φ_2, Φ_3 are in x-y plane and at 120°

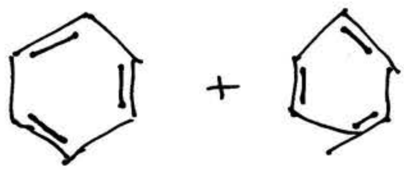
Φ_1 points along +x axis

Φ_2 points along axis with $\tan\vartheta = -\frac{1}{\sqrt{2}} \Big/ \frac{1}{\sqrt{6}} = \sqrt{3} \Rightarrow \vartheta = 120^\circ$

Φ_3 points along axis with $\tan\vartheta = +\frac{1/\sqrt{2}}{1/\sqrt{6}} = \sqrt{3} \Rightarrow \vartheta = 240^\circ$

Problem 2

superposition of single and double bonds $\Rightarrow$ length is in-between



Problem 3

$$\frac{d\vec{v}}{dt} = -\frac{e}{m}\vec{E} \Rightarrow \vec{v} = \vec{v}_0 - \frac{e}{m} t \vec{E}$$

Gain in kinetic energy:

$$\Delta \mathcal{E} = \frac{1}{2} m (v^2 - v_0^2) = \frac{1}{2} m \left(-\frac{2e}{m} t \vec{v}_0 \cdot \vec{E} + \frac{e^2 t^2 E^2}{m^2} \right)$$

On the average, $\langle \vec{v}_0 \cdot \vec{E} \rangle = 0 \Rightarrow$

(a) $\Delta E = \dfrac{(eEt)^2}{2m}$ energy lost in collision at time t

(b) Probability that collision occurs at time t, $t+dt$ interval is

$$P(t)\,dt = \frac{dt}{\tau} e^{-t/\tau}$$

The mean time between collisions is $\langle t \rangle = \int_0^\infty dt\, t\, P(t) = \tau$

The average energy lost in a collision is

$$\langle \Delta E \rangle = \frac{e^2 E^2}{2m} \langle t^2 \rangle = \frac{e^2 E^2}{2m} \int_0^\infty dt\, t^2 P(t) = \frac{e^2 E^2}{2m} \cdot 2\tau^2$$

so the average energy lost per unit time is $\dfrac{\langle \Delta E \rangle}{\tau} = \dfrac{e^2 E^2 \tau}{m}$

If there are n electrons per unit volume, energy transfered per unit volume per unit time is $n \cdot \dfrac{e^2 E^2}{m} \tau = \sigma E^2$

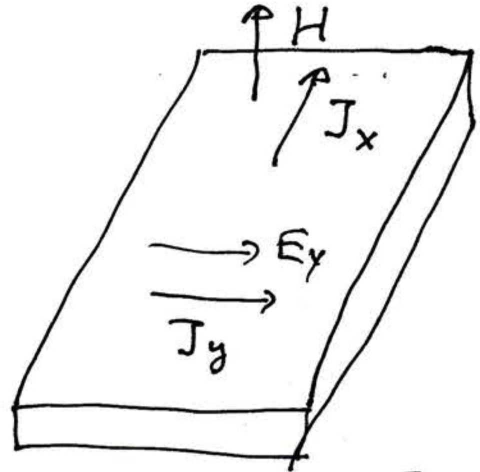
In a wire of cross section A, length L, energy lost per unit time = power =

$$P = \sigma E^2 \cdot A \cdot L = \frac{\sigma A}{L} (EL)^2 = \frac{V^2}{R} \quad \text{with } R = \frac{\rho L}{A}, \; \rho = \frac{1}{\sigma}$$

$V = E \cdot L$. Now $J = \sigma E \Rightarrow I = JA = \dfrac{\sigma A}{L}(EL) = \dfrac{V}{R} \Rightarrow$

$$\Rightarrow \boxed{P = I^2 R}$$

Problem 4



$$J_x = n_1 e_1 v_{1x} + n_2 e_2 v_{2x} = \sum_i \frac{n_i e_i^2 \tau_i}{m_i} E_x = \sum_i n_i |e_i| \mu_i E_x$$

$$v_{ix} = \frac{e_i}{m_i} \tau_i E_x \qquad \mu_i \equiv \frac{|e_i| \tau_i}{m_i}$$

$$J_y = n_1 e_1 v_{1y} + n_2 e_2 v_{2y} = \sum_i n_i e_i v_{iy}$$

Force in y direction: $F_{iy} = e_i E_y + e_i \frac{v_{ix}}{c} H$

$$v_{iy} = \frac{F_{iy}}{m_i} \tau_i = \frac{e_i \tau_i}{m_i} E_y + \frac{e_i}{m_i} \tau_i \frac{v_{ix}}{c} H$$

$$J_y = \sum_i \left(n_i \frac{e_i^2 \tau_i}{m_i} E_y + \frac{n_i e_i^2}{m_i} \tau_i \frac{e_i}{m_i} \frac{\tau_i}{c} E_x H \right) \Rightarrow$$

$$J_y = \sum_i \left(n_i |e_i| \mu_i E_y + n_i e_i \mu_i^2 \frac{E_x H}{c} \right)$$

$$J_y = 0 \Rightarrow E_y = -\frac{E_x H}{c} \frac{n_1 e_1 \mu_1^2 + n_2 e_2 \mu_2^2}{n_1 |e_1| \mu_1 + n_2 |e_2| \mu_2}$$

and $J_x = (n_1 |e_1| \mu_1 + n_2 |e_2| \mu_2) E_x \Rightarrow$

$$\Rightarrow \boxed{R_H = \frac{E_y}{J_x H} = -\frac{1}{c} \frac{n_1 e_1 \mu_1^2 + n_2 e_2 \mu_2^2}{(n_1 |e_1| \mu_1 + n_2 |e_2| \mu_2)^2}}$$

(b) $$R_H = -\frac{1}{c}\frac{n_1 e_1 \mu_1^2 + n_2 e_2 \mu_2^2}{(n_1 |e_1| \mu_1 + n_2 |e_2| \mu_2)^2}$$

If $n_2 \sim 0$, or $\mu_2 \sim 0$, or $e_2 \sim 0$

$$R_H = -\frac{1}{c}\frac{n_1 e_1 \mu_1^2}{(n_1 |e_1| \mu_1)^2} = -\frac{1}{n_1 e_1 c}$$

as in 1 band case

(c) $R_H = 0 \Rightarrow \quad n_1 e_1 \mu_1^2 + n_2 e_2 \mu_2^2 = 0$

so the sign of e_1 and e_2 have to be opposite

Assume $\quad e_1 = -e$ (electrons), $e_2 = e$ (holes)

$R_H = 0 \Rightarrow \boxed{n_1 \mu_1^2 = n_2 \mu_2^2}$

this is called a "compensated metal"

(d) The paper says that the density of carriers "experiences a fourfold enhancement".

This is deduced from the formula

$$R_H = -\frac{1}{n_1 e c}$$

so they measure a decrease in the magnitude of R_H by a factor of 4.

Assume instead a situation with 2 carriers with charge $-e$ and $+e$, and for simplicity $\mu_1 = \mu_2$. Then,

$$R_H = -\frac{1}{ec} \frac{n_1 - n_2}{(n_1 + n_2)^2}$$

If the carrier concentration is changing so that n_1 and n_2 become close, i.e. $R_H \to 0$, R_H can decrease by a factor 4 with a much smaller change in carrier concentration than a factor of 4, which is more plausible.

For example, $n_2 = 1$, n_1 changing from 1.05 to 1.01 will give a change in R_H of a factor 4.8, with a change in carrier concentration of 2% instead of 400%.