In Figure 4–7, $\mathbf{P}_1$ is the initial momentum of the α and $\mathbf{P}_2$ the final momentum. It is evident from the vector diagram that the total change in momentum $\Delta\mathbf{P} = \mathbf{P}_2 - \mathbf{P}_1$ is along the z' axis. The magnitude of $\mathbf{P}_1$ and $\mathbf{P}_2$ is MV. From the isosceles triangle

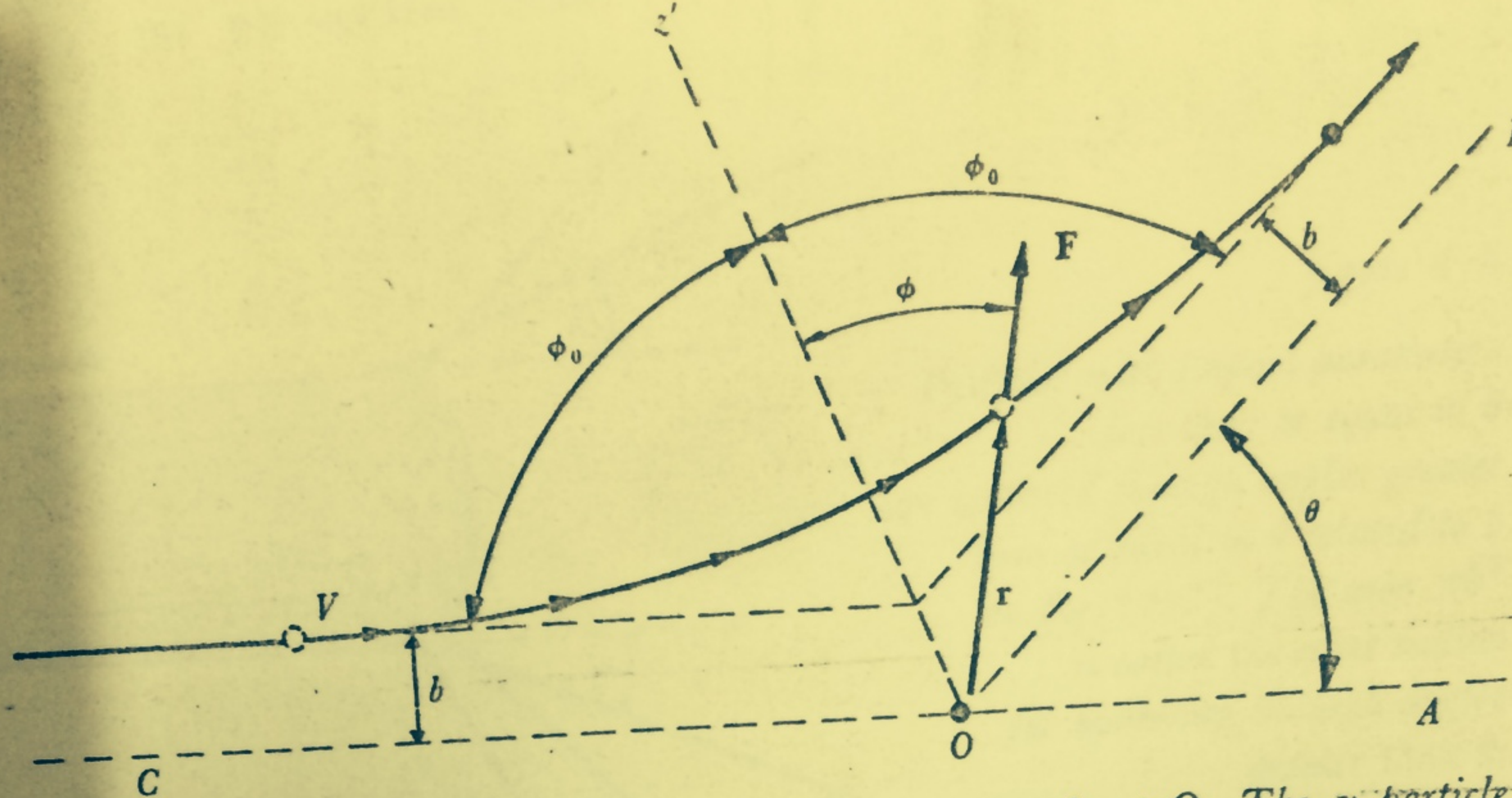

figure 4-6 · *Rutherford scattering geometry. The nucleus is at O. The α particle has initial momentum MV parallel to line COA and final momentum of the same magnitude (by conservation of energy) parallel to line OB. The distance b is called the* impact parameter. *The change in momentum is along the symmetry axis z′. The scattering angle θ can be related to the impact parameter by setting this change in momentum equal to the component of the impulse in the z′ direction* $\Delta P = \int F \cos\phi \, dt$.

formed by $\mathbf{P}_1$, $\mathbf{P}_2$, and $\Delta\mathbf{P}$, we find the magnitude of $\Delta\mathbf{P}$ to be $\frac{1}{2}\Delta P/MV = \sin\frac{1}{2}\theta$, or $\Delta P = 2MV\sin\frac{1}{2}\theta$. We now write Newton's law for the α particle:

$$\mathbf{F} = d\mathbf{P}/dt$$

or

$$d\mathbf{P} = \mathbf{F}\,dt$$

The force F is given by Coulomb's law, $Kq_\alpha Q/r^2$, and is in the radial direction. Taking components along the z' axis and integrating, we have

$$\int (dP)_{z'} = \Delta P = \int F\cos\phi\,dt = \int F\cos\phi\,\frac{dt}{d\phi}\,d\phi \qquad (4\text{–}5)$$

where we have changed the variable of integration from t to ϕ. We can write $dt/d\phi$ in terms of the angular momentum of the α about the origin. Since the force is central (i.e., it acts along the line joining the α and the origin), there is no torque about the origin, and the angular momentum is conserved. Initially, the angular momentum is MVb. At a later time, it is $Mr^2\,d\phi/dt$. Thus conservation of angular momentum implies

$$Mr^2\frac{d\phi}{dt} = MVb \qquad (4\text{–}6)$$

Using Eq. (4–6) for $d\phi/dt$ in Eq. (4–5) and $Kq_\alpha Q/r^2$ for F, we have

$$\Delta P = \int \frac{Kq_\alpha Q}{r^2} \cos\phi \frac{r^2}{Vb}\, d\phi = \frac{Kq_\alpha Q}{Vb} \int \cos\phi\, d\phi$$

or

$$\Delta P = \frac{Kq_\alpha Q}{Vb} (\sin\phi_2 - \sin\phi_1)$$

where ϕ_1 and ϕ_2 are the initial and final values of ϕ. From Figure 4–6 we see that $\phi_1 = -\phi_0$, $\phi_2 = +\phi_0$, where $2\phi_0 + \theta = 180°$. Thus $\sin\phi_2 - \sin\phi_1 = 2\sin(90 - \frac{1}{2}\theta) = 2\cos\frac{1}{2}\theta$. Writing ϕ in terms of θ and using our previous result for the net momentum change, $\Delta P = 2MV\sin\frac{1}{2}\theta$, we have, finally,

$$2MV\sin\tfrac{1}{2}\theta = \frac{Kq_\alpha Q}{Vb} 2\cos\tfrac{1}{2}\theta$$

or

$$b = \frac{Kq_\alpha Q}{MV^2} \cot\tfrac{1}{2}\theta \qquad (4\text{–}7)$$

Of course, it is not possible to choose or to know the impact parameter for any α particle; however, all such particles with impact parameters less than, or equal to, a particular b will be scattered through an angle θ greater than or equal to that given by Eq. (4–7). Let the intensity of the incident α-particle beam be I_0 particles per

figure 4–7 *Momentum diagram for Rutherford scattering. The magnitude of the momentum change ΔP is related to the scattering angle θ by $\Delta P = 2MV\sin\frac{1}{2}\theta$.*