

## Rutherford's Prediction and Geiger's and Marsden's Results

With the encouraging result described in Example 4-2, Geiger and Marsden proceeded to conduct a series of experiments in which they measured

1. The number of particles per unit area on the screen, scattered through angles between  $\theta$  and  $\theta + d\theta$ ;
2. The variation in the number scattered with foil thickness;
3. The variation in the number scattered with the atomic weight of the foil;
4. The variation in the number scattered with incident velocity  $v$ , which they varied by placing thin absorbers in the incident beam to slow down the  $\alpha$  particles.<sup>5</sup>

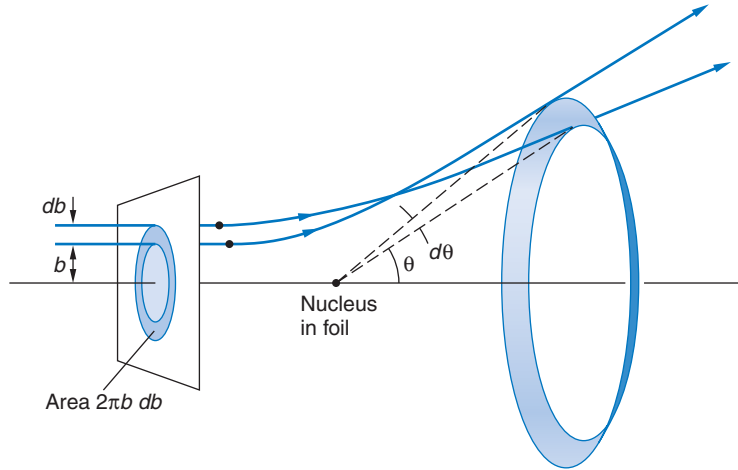
The number  $N$  scattered by one nucleus at angles between  $\theta$  and  $\theta + d\theta$  is the number incident with impact parameters between  $b$  and  $(b + db)$  (Figure 4-10). This number equals the product of the incident intensity  $I_0$  and the area ( $2\pi b db$ ) shown in Figure 4-10. The area is found by differentiating  $b$  (Equation 4-3), which yields

$$db = \frac{kq_\alpha Q}{2m_\alpha v^2} \left( -\csc^2 \frac{\theta}{2} \right) d\theta \quad 4-7$$

Ignoring the minus sign (since only the magnitude of the area is of interest here), multiplying Equation 4-7 by  $I_0 2\pi b$  and making use of a suitable trigonometric identity (see Problem 4-48), we obtain

$$N = I_0 2\pi b db = I_0 2\pi \left( \frac{kZe^2}{m_\alpha v^2} \right)^2 \frac{\sin \theta d\theta}{\sin^4(\theta/2)} \quad 4-8$$

where we have inserted  $q_\alpha = 2e$  for the alpha particle and  $Q = Ze$  for the nucleus containing  $Z$  protons. The  $N$   $\alpha$  particles scattered at angles between  $\theta$  and  $\theta + d\theta$  pass through a portion of a spherical surface of radius  $r$  centered on the scattering nucleus in the foil as shown in Figure 4-11. The area of that surface is  $A_{\text{surf}} = (2\pi r \sin \theta)(r d\theta)$ .



**FIGURE 4-10** The number of particles with impact parameters between  $b$  and  $b + db$  is proportional to the area  $2\pi b db$ . These particles are scattered into the range between  $\theta$  and  $\theta + d\theta$ .

Thus, the number of  $\alpha$  particles scattered from a single nucleus per unit area at angles between  $\theta$  and  $\theta + d\theta$  is given by<sup>6</sup>

$$\frac{N}{A_{\text{surf}}} = \frac{I_0 (kZe^2)^2}{r^2 (m_\alpha v^2)^2} \frac{1}{\sin^4(\theta/2)} \quad 4-9$$

Rutherford's nuclear model thus predicts that the number of  $\alpha$  particles  $\Delta N_1$  observed on the scintillation screen of area  $A_{\text{sc}}$  due to a single nucleus (see Figure 4-11) would be given by

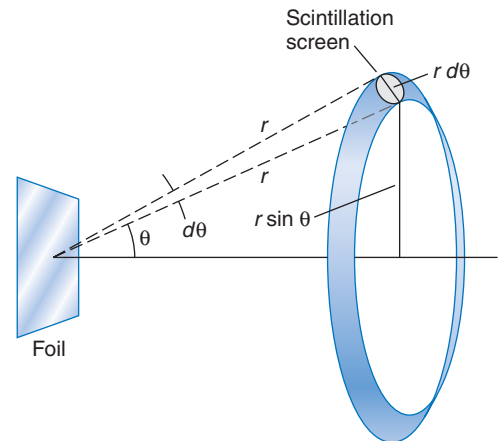
$$\Delta N_1 = \frac{N}{A_{\text{surf}}} A_{\text{sc}} = \frac{I_0 A_{\text{sc}} (kZe^2)^2}{r^2 (m_\alpha v^2)^2} \frac{1}{\sin^4(\theta/2)} \quad 4-10$$

Since the number of scattering nuclei per unit area of the foil is  $nt$ , then the scintillation detector should observe a total  $\Delta N = nt \Delta N_1$  particles with scattering angles between  $\theta$  and  $\theta + d\theta$ , where

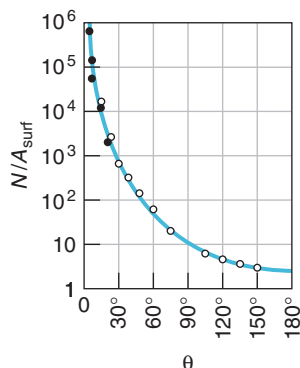
$$\Delta N = \left( \frac{I_0 A_{\text{sc}} nt}{r^2} \right) \left( \frac{kZe^2}{2E_k} \right)^2 \frac{1}{\sin^4(\theta/2)} \quad 4-6$$

where we have written  $E_k = \frac{1}{2} m_\alpha v^2$  for the kinetic energy of the incident  $\alpha$  particles.

Equation 4-6 is the prediction of Rutherford's nuclear model for the number  $\Delta N$  of  $\alpha$  particles that would reach the scintillation detector. Note that Equation 4-6 predicts that  $\Delta N$  will be proportional to  $1/\sin^4(\theta/2)$ , to  $Z^2$ , to  $t$ , and to  $E_k^{-2}$ . Figure 4-12 illustrates the data obtained by Geiger and Marsden for the number of scattered  $\alpha$  particles/unit area versus  $\theta$  (Equation 4-9). The kinetic energy  $E_k$  of the incident  $\alpha$  particles was 7.7 MeV. In addition, their experiments included measurement of the predicted dependence of  $\Delta N$  on the other parameters. To describe what they found, we quote from Geiger and Marsden's paper "Deflection of  $\alpha$  Particles Through Large Angles"<sup>7</sup>:



**FIGURE 4-11** Particles scattered at angles between  $\theta$  and  $\theta + d\theta$  fall onto the portion of a spherical surface of radius  $r$  centered on the foil. The area of that surface is  $(2\pi r \sin \theta) (rd\theta) = A_{\text{surf}}$ . The scintillation screen is located at the distance  $r$  from the foil and occupies a fraction  $f_{\text{sc}}$  of that area equal to the (area of screen)/(area of surface)  $= A_{\text{sc}}/A_{\text{surf}} = f_{\text{sc}}$ , and thus sees a number of particles  $\Delta N_1 = (N/A_{\text{surf}}) (A_{\text{sc}}) = N f_{\text{sc}}$  due to scattering from a single nucleus.



**FIGURE 4-12** The number of scattered  $\alpha$  particles per unit area as a function of  $\theta$ . The solid curve is  $\sin^{-4}(\theta/2)$ . The data are from Geiger and Marsden for 7.7 MeV  $\alpha$  particles. [From R. D. Evans, *The Atomic Nucleus*, McGraw-Hill, New York, 1955.]

The experiments described in the foregoing paper were carried out to test a theory of the atom proposed by Prof. Rutherford, the main feature of which is that there exists at the center of the atoms an intense, highly concentrated electrical charge. The verification is based on the laws of scattering which were deduced from this theory. The following relations have been verified experimentally [bold added]:

- 1. The number of  $\alpha$  particles emerging from a scattering foil at an angle  $\theta$  with the original beam varies as  $1/\sin^4(\theta/2)$** , when the  $\alpha$  particles were counted on a definite area at a constant distance from the foil. This relation has been tested for angles varying from  $5^\circ$  to  $150^\circ$ , and over this range the number of  $\alpha$  particles varied from 1 to 250,000 in good agreement with the theory.
- 2. The number of  $\alpha$  particles scattered in a definite direction is directly proportional to the thickness of the scattering foil** for small thicknesses. For large thickness the decrease of velocity of the  $\alpha$  particles in the foil causes a somewhat more rapid increase in the amount of scattering.
- 3. The scattering per atom of foils of different materials varies approximately as the square of the atomic weight.** This relation was tested for foils of atomic weight from that of carbon to that of gold.<sup>8</sup>
- 4. The amount of scattering by a given foil is approximately proportional to the inverse fourth power of the velocity of the incident  $\alpha$  particles** [i.e., to  $E_k^{-2}$ ]. This relation was tested over a range of velocities such that the number of scattered particles varied as 1:10.

The excellent agreement over four orders of magnitude between the measurements of  $\Delta N$  (see Equation 4-6) by Geiger and Marsden and its predicted linear dependence on  $1/\sin^4(\theta/2)$  is illustrated clearly in Figure 4-9a, where data for both gold and silver scattering foils are plotted. In Figure 4-9b the predicted linear dependence on the foil thickness  $t$  is also seen to be very good for four different scattering foils. The predicted dependence of  $\Delta N$  on  $E_k$  was also found. These agreements all serve to indicate that the force law  $F = kq_\alpha Q/r^2$  used to derive Equations 4-9 and 4-6 is correct and that Rutherford's nuclear model of the atom is in accord with experimental observations.