

**Show all steps in your calculations. Justify all answers. Write clearly.**

**Suggestion: do the problems you find easiest first**

**Some constants:**  $hc = 12,400 eV\text{\AA}$ ,  $k_B = 1/11,600 eV/K$ ,  $m_e c^2 = 511,000 eV$

$\hbar c = 1973 eV\text{\AA}$  ;  $ke^2 = 14.4 eV\text{\AA}$  ;  $1\text{\AA} = 10^{-10} m$  ;  $m_{neutron} c^2 = 939.6 MeV$   
 $\hbar^2/m_e = 7.62 eV\text{\AA}^2$  ;  $\mu_B = 5.79 \times 10^{-5} eV/T$

**Problem 1** (10 pts)

There are 7 electrons in a two-dimensional square box of side length L. Electrons have spin 1/2 and obey the Pauli principle, for this problem we assume that they don't interact with each other and that there is no spin-orbit coupling. When light of wavelengths in the range  $1,000\text{\AA} < \lambda < 10,000\text{\AA}$  is incident on this box, the longest wavelength photon that is absorbed has wavelength 4122\text{\AA}.

- What is the length L, in \text{\AA}?
- What is the second longest wavelength photon that can be absorbed by this system?

**Problem 2** (10 pts)

An electron in a hydrogen-like ion is described by the wavefunction

$$\psi(r, \theta, \phi) = Cr^2 e^{-r/a_0} \sin\theta \cos\theta e^{-i\phi}$$

- Give the values of the quantum numbers  $n, \ell, m_\ell$  and of the ionic charge Z. Justify your answers.
- Find the most probable value of r for this electron and compare with the Bohr atom result.
- Taking spin-orbit coupling into account, what are the possible values of the total angular quantum number  $j$  for this electron? What are the possible values of  $m_j$ ?

**Problem 3** (10 pts)

The ground state energy of a one-dimensional harmonic oscillator is 0.01 eV.

- At what temperature is the average energy of this system 0.02 eV?
- At the temperature found in (a), what is the average energy of this system predicted by the equipartition theorem? Is it higher or lower than what was found in (a)? Why?
- For a 3-dimensional solid where the atomic vibrations are modeled by such harmonic oscillators, by how much does the molar heat capacity differ from the Dulong-Petit value  $C_V = 3R$  at the temperature found in (a)? Give your answer in percent. Is it higher or lower than the Dulong-Petit value?