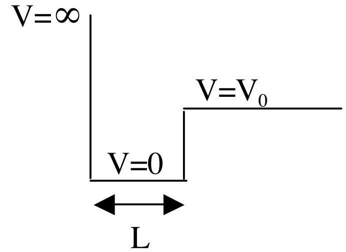
Show all steps in your calculations. Justify all answers. Write clearly.
Suggestion: do the problems you find easiest first

Some constants: $hc = 12,400\, eVA,\ \ k_B = 1/11,600\, eV/K,\ \ m_e c^2 = 511,000\, eV$

$\hbar c = 1973\, eVA$; $ke^2 = 14.4\, eVA$; $1A = 10^{-10}\, m$; $m_{neutron} c^2 = 939.6\, MeV$

Problem 1 (10 pts)



An electron is in the lowest state of the potential well shown in the figure. The width of the well is L=2.3562 A=(3/4)π A.

(a) What would be the energy of this electron if $V_0=\infty$? ($\hbar^2/m_e = 7.62\ eVA^2$)

(b) If the energy of the electron is 3.81 eV, what is the value of V_0, in eV?

(c) Make a plot of the wavefunction for case (b) that is qualitatively correct.

(d) For extra credit (3 pts) Find the minimum value of V_0 (in eV) that will bind an electron in this well, and make a plot of the wavefunction for that case.

Problem 2 (10 pts)

An electron is described by the wavefunction $\psi(x) = Ce^{-\alpha x^2/2}$.

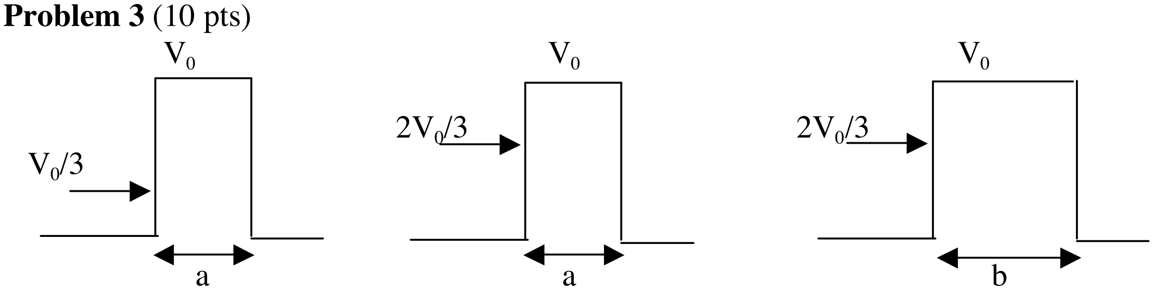
(a) Find C in terms of α.

(b) Find its average momentum, <p> in terms of α. Justify your answer.

(c) Find $\langle p^2 \rangle$ and Δp in terms of α.

$$\int_{-\infty}^{\infty} dx\, e^{-\lambda x^2} = \sqrt{\frac{\pi}{\lambda}} \quad ; \quad \int_{-\infty}^{\infty} dx\, x^2 e^{-\lambda x^2} = \frac{1}{2}\sqrt{\frac{\pi}{\lambda^3}} \quad ; \quad \int_{-\infty}^{\infty} dx\, x^4 e^{-\lambda x^2} = \frac{3}{4}\sqrt{\frac{\pi}{\lambda^5}}$$

Problem 3 (10 pts)



For the barrier on the left, for every 10,000 particles of mass m incident, 100 particles tunnel through. The incident particle energy is $V_0/3$. V_0=barrier height.

(a) For the case in the middle (same barrier), for every 10,000 particles of mass m incident with energy $2V_0/3$, how many tunnel through?

(b) For the barrier on the right and particles of mass m, for what width b do 100 particles tunnel through for every 10,000 incident with energy $2V_0/3$? Give b in terms of a.

(c) For the case in the middle and particles of mass M, 100 tunnel through for every 10,000 incident. Give M in terms of m.