HW problem week 4

Your turned in assignment should be clearly written and easy to follow! Learning how to explain your work in a way that is as easy as possible to follow is an important part of your training as a physicist. An incoherent mess of equations with a correct final answer could receive less points than a solution which is clearly explained at every step but has an algebra mistake somewhere. Once you've solved the problem, you can rewrite it on a new piece of paper for clarity if you need to.

In class, we derived the Bohr model of the atom following the assumption that the angular momentum $\vec{L}$ is quantized in units of $\hbar$. In this problem, you will repeat the derivation using the relativistic expressions for energy and momentum. In this problem (and always in life), you should try to combine physical constants into α, the fine structure constant, wherever possible.

1. Assume that the electron travels in a circular orbit with constant angular speed ω. Using Newton's law $\vec{F} = d\vec{p}/dt$ with the relativistic expression for the momentum, obtain an expression relating the radius r and the velocity v. Hint: Since the speed is constant, the relativistic answer for $d\vec{p}/dt$ is related in a simple way to the nonrelativistic answer.

2. By quantizing the relativistic angular momentum (still $\vec{r} \times \vec{p}$) to integer multiples of $\hbar$, and using the result of part 1, show that the speed of the n'th Bohr orbit is the same as in the nonrelativistic case:
$$v_n = \frac{Z\alpha c}{n}.$$

3. Using the result of parts 1 and 2, calculate the radius of the n'th Bohr orbit.

4. The formula $E = \sqrt{(mc^2)^2 + p^2c^2} = \gamma mc^2$ is for a free particle; in the presence of a potential we add the potential energy U. Using the result of the previous parts, calculate the relativistic answer for the total energy.

5. By expanding in powers of α using the Taylor expansion
$$\sqrt{1+\epsilon} \approx 1 + \epsilon/2 - \epsilon^2/8 + O(\epsilon^3)$$
for small ϵ, show that the energy levels are of the form
$$E_n = (\text{rest energy}) + (\text{bohr result}) - \frac{mc^2}{8}\left(\frac{\alpha Z}{n}\right)^4 + O(\alpha^6).$$

6. The relativistic Bohr model of the atom actually makes a prediction for size of the largest stable element. By looking at the results derived in this problem and imposing some physical assumptions on the radius, velocity, or energy, find a condition on the atomic number Z of a hydrogen like atom.