PHYSICS 210A : STATISTICAL PHYSICS
HW ASSIGNMENT #5

(1) Using the argument we used in class and in §5.5.3 of the notes, predict the surface temperatures of the remaining planets in our solar system. In each case, compare your answers with the most reliable source you can find. In cases where there are discrepancies, try to come up with a convincing excuse.

(2) Read carefully the new and improved §5.6.4 of the lecture notes ("Melting and the Lindemann criterion"). Using the data in Table 5.1, and looking up the atomic mass and lattice constant of tantalum (Ta), find the temperature T_{L} where the Lindemann criterion predicts Ta should melt.

(3) Consider a two-dimensional gas of fermions which obey the dispersion relation

$$\varepsilon(\boldsymbol{k}) = \varepsilon_0 \Big((k_x^2 + k_y^2)\, a^2 + \tfrac{1}{2} (k_x^4 + k_y^4)\, a^4 \Big) .$$

Sketch, on the same plot, the Fermi surfaces for $\varepsilon_{\mathrm{F}} = 0.1\,\varepsilon_0$, $\varepsilon_{\mathrm{F}} = \varepsilon_0$, and $\varepsilon_{\mathrm{F}} = 10\,\varepsilon_0$.

(4) Show that the chemical potential of a three-dimensional ideal nonrelativistic Fermi gas is given by

$$\mu(n,T) = \varepsilon_{\mathrm{F}} \left[1 - \frac{\pi^2}{12} \left(\frac{k_{\mathrm{B}} T}{\varepsilon_{\mathrm{F}}} \right)^2 - \frac{\pi^4}{80} \left(\frac{k_{\mathrm{B}} T}{\varepsilon_{\mathrm{F}}} \right)^4 + \ldots \right]$$

and the average energy per particle is

$$\frac{E}{N} = \tfrac{3}{5}\, \varepsilon_{\mathrm{F}} \left[1 + \frac{5\pi^2}{12} \left(\frac{k_{\mathrm{B}} T}{\varepsilon_{\mathrm{F}}} \right)^2 - \frac{\pi^4}{16} \left(\frac{k_{\mathrm{B}} T}{\varepsilon_{\mathrm{F}}} \right)^4 + \ldots \right] ,$$

where $\mu_0(n)$ is the Fermi energy at $T = 0$. Compute the heat capacity $C_V(T)$ to terms of order T^3. How does the T^3 contribution to the electronic heat capacity compare with the contribution from phonons?

(5) Consider a three-dimensional Bose gas of particles which have two internal polarization states, labeled by $\sigma = \pm 1$. The single particle energies are given by

$$\varepsilon(\boldsymbol{p}, \sigma) = \frac{\boldsymbol{p}^2}{2m} + \sigma \Delta ,$$

where $\Delta > 0$.

(a) Find the density of states per unit volume $g(\varepsilon)$.

(b) Find an implicit expression for the condensation temperature $T_{\mathrm{c}}(n, \Delta)$. When $\Delta \to \infty$, your expression should reduce to the familiar one derived in class.

(c) When $\Delta = \infty$, the condensation temperature should agree with the familiar result for three-dimensional Bose condensation. Assuming $\Delta \gg k_{\mathrm{B}} T_{\mathrm{c}}(n, \Delta = \infty)$, find analytically the leading order difference $T_{\mathrm{c}}(n, \Delta) - T_{\mathrm{c}}(n, \Delta = \infty)$.