PHYSICS 210A : STATISTICAL PHYSICS
HW ASSIGNMENT #4

(1) $\nu = 8$ moles of a diatomic ideal gas are subjected to a cyclic quasistatic process, the thermodynamic path for which is an ellipse in the (V, p) plane. The center of the ellipse lies at $(V_0, p_0) = (0.25\,\mathrm{m}^3, 1.0\,\mathrm{bar})$. The semimajor and semiminor axes of the ellipse are $\Delta V = 0.10\,\mathrm{m}^3$ and $\Delta p = 0.20\,\mathrm{bar}$.

(a) What is the temperature at $(V, p) = (V_0 + \Delta V, p_0)$?

(b) Compute the net work per cycle done by the gas.

(c) Compute the internal energy difference $E(V_0 - \Delta V, p_0) - E(V_0, p_0 - \Delta p)$.

(d) Compute the heat Q absorbed by the gas along the upper half of the cycle.

(**2**) Consider a thermodynamic system for which $E(S, V, N) = aS^4/NV^2$.

(a) Find the equation of state $p = p(T, V, N)$.

(b) Find the equation of state $\mu = \mu(T, p)$.

(c) ν moles of this substance are taken through a Joule-Brayton cycle. The upper isobar lies at $p = p_2$ and extends from volume V_A to V_B. The lower isobar lies at $p = p_1$. Find the volumes V_C and V_D.

(d) Find the work done per cycle W_cyc, the heat Q_AB, and the cycle efficiency.

(**3**) A diatomic gas obeys the equation of state

$$p = \frac{RT}{v - b} - \frac{a}{v^2} + \frac{cRT}{v^3} \ ,$$

where a, b, and c are constants.

(a) Find the adiabatic equation of state relating temperature T and molar volume v.

(b) What is the internal energy per mole, $\varepsilon(T, v)$?

(c) What is the Helmholtz free energy per mole, $f(T, v)$?

(**4**) Consider the thermodynamics of a solid in equilibrium with a vapor at temperature T and pressure p, but separated by a quasi-liquid layer of thickness d. Let the number density of the liquid be n_ℓ. The Gibbs free energy per unit area of the quasi-liquid layer is taken as

$$g_{qll}(T, p) = n_\ell\, \mu_\ell(T, p)\, d + \gamma(d) \ ,$$

where $\gamma(d)$ is an effective surface tension which interpolates between $\gamma(0) = \gamma_{sv}$ and $\gamma(\infty) = \gamma_{s\ell} + \gamma_{\ell v}$. The phenomenon of premelting requires $\gamma(0) > \gamma(\infty)$.

(a) Show that $\mu_{qll}(T,p) = \mu_\ell(T,p) + n_\ell^{-1}\gamma'(d) = \mu_s(T,p)$.

(b) Expand T relative to some point $(T_{\rm m}, p)$ along the melting curve to lowest order in $T - T_{\rm m}$. Show $\Delta\mu(T,p) \equiv \mu_s(T,p) - \mu_\ell(T,p) = \ell_{\rm m}(T - T_{\rm m})/T_{\rm m}$, where $\ell_{\rm m}$ is the latent heat of melting.

(c) Assume

$$\gamma(d) = \gamma_{sv} + (\gamma_{s\ell} + \gamma_{\ell v} - \gamma_{sv}) \cdot \frac{d^2}{d^2 + \sigma^2} \ ,$$

where σ is a molecular length scale. Assuming $d \gg \sigma$, find the dependence of the thickness d of the quasi-liquid layer on the reduced temperature $t \equiv (T_{\rm m} - T)/T_{\rm m}$.