PHYSICS 210A : STATISTICAL PHYSICS
HW ASSIGNMENT #3

(1) Consider an ultrarelativistic ideal gas in three space dimensions. The dispersion is $\varepsilon(\boldsymbol{p}) = pc$.

(a) Find T, p, and μ within the microcanonical ensemble (variables S, V, N).

(b) Find F, S, p, and μ within the ordinary canonical ensemble (variables T, V, N).

(c) Find Ω, S, p, and N within the grand canonical ensemble (variables T, V, μ).

(d) Find G, S, V, and μ within the Gibbs ensemble (variables T, p, N).

(e) Find H, T, V, and μ within the S-p-N ensemble. Here $\mathsf{H} = E + pV$ is the enthalpy.

(2) Consider a system composed of spin tetramers, each of which is described by the Hamiltonian

$$\hat{H} = -J(\sigma_1\sigma_2 + \sigma_1\sigma_3 + \sigma_1\sigma_4 + \sigma_2\sigma_3 + \sigma_2\sigma_4 + \sigma_3\sigma_4) - \mu_0 H(\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4) \ .$$

The individual tetramers are otherwise noninteracting.

(a) Find the single tetramer partition function ζ.

(b) Find the magnetization per tetramer $m = \mu_0 \langle \sigma_1 + \sigma_2 + \sigma_3 + \sigma_4 \rangle$.

(c) Suppose the tetramer number density is $n_{\rm t}$. The magnetization density is $M = n_{\rm t} m$. Find the zero field susceptibility $\chi(T) = (\partial M / \partial H)_{H=0}$.

(3) For an ideal gas, find the difference $C_\varphi - C_V$ for the following functions φ. You are to assume N is fixed in each case.

(a) $\varphi(p, V) = p^3 V^2$

(b) $\varphi(p, T) = p\, e^{T/T_0}$

(c) $\varphi(T, V) = V T^{-1}$

(4) Find an expression for the energy density $\varepsilon = E/V$ for a system obeying the Dieterici equation of state,

$$p(V - Nb) = N k_{\rm B} T \, e^{-Na/V k_{\rm B} T} \ ,$$

where a and b are constants. Your expression for $\varepsilon(v, T)$ should involve an integral which can be expressed in terms of the exponential integral,

$$\mathsf{Ei}(x) = \int\limits_{-\infty}^{x} \! dt \, \frac{e^t}{t} \ .$$