PHYSICS 210A : STATISTICAL PHYSICS
HW ASSIGNMENT #2

(1) Consider a d-dimensional ideal gas with dispersion $\varepsilon(\boldsymbol{p}) = A|\boldsymbol{p}|^\alpha$, with $\alpha > 0$. Find the density of states $D(E)$, the statistical entropy $S(E)$, the equation of state $p = p(N, V, T)$, the heat capacity at constant volume $C_V(N, V, T)$, and the heat capacity at constant pressure $C_p(N, V, T)$.

(2) Find the velocity distribution $f(\boldsymbol{v})$ for the particles in problem (1). Compute the most probable speed, mean speed, and root-mean-square velocity.

(3) A spin-1 Ising magnet is described by the noninteracting Hamiltonian

$$H = -\mu_0 \mathsf{H} \sum_{i=1}^{\mathsf{N}} \sigma_i \ ,$$

where $\sigma_i = -1, 0, +1$.

(a) Find the entropy $S(\mathsf{H}, T, \mathsf{N})$.

(b) Suppose the system starts off at a temperature $T = 10\,\text{mK}$ and a field $\mathsf{H} = 20\,\text{T}$. The field is then lowered adiabatically to $\mathsf{H} = 1\,\text{T}$. What is the final temperature of the system?

(4) Consider an adsorption model where each of N sites on a surface can accommodate either one or two adsorbate molecules. When one molecule is present the energy is $\varepsilon = -\Delta$, but when two are present the energy is $\varepsilon = -2\Delta + U$, where U models the local interaction of two adsorbate molecules at the same site. You should think of there being two possible binding locations within each adsorption site, so there are four possible states per site: unoccupied (1 possibility), singly occupied (2 possibilities), and doubly occupied (1 possibility). The surface is in equilibrium with a gas at temperature T and number density n.

(a) Find the surface partition function.

(b) Find the fraction f_j which contain j adsorbate molecules, where $j = 0, 1, 2$.

(5) Consider a system of dipoles with the Hamiltonian

$$H = \sum_{i<j} J_{ij}^{\alpha\beta}\, n_i^\alpha\, n_j^\beta - \mu_0 \sum_i \mathsf{H}_i^\alpha\, n_i^\alpha \ ,$$

where

$$J_{ij}^{\alpha\beta} = \frac{J}{R_{ij}^3} \left(\delta^{\alpha\beta} - 3\, \hat{R}_{ij}^\alpha\, \hat{R}_{ij}^\beta \right) .$$

Here $\boldsymbol{R}_i$ is the spatial position of the dipole $\hat{\boldsymbol{n}}_i$, and $\boldsymbol{R}_{ij} = \boldsymbol{R}_i - \boldsymbol{R}_j$ with $\hat{R}_{ij}^\alpha \equiv R_{ij}^\alpha / R_{ij}$ the unit direction vector from j to i. The dipole vectors m_i^α are three-dimensional unit vectors. H_i^α is the local magnetic field.

(a) Find an expression for the free energy $F\big(T, \{\vec{\mathsf{H}}_i\}\big)$ valid to order β^2, where $\beta = 1/k_{\rm B}T$.

(b) Obtain an expression for the uniform field magnetic susceptibility tensor $\chi_{\alpha\beta}$.

(c) An experimentalist plots the quantity $T\chi_{\alpha\beta}$ *versus* T^{-1} for large temperatures. What should the data resemble if the dipoles are arranged in a cubic lattice structure? How about if they are arranged in a square lattice in the (x, y) plane? (You'll need to separately consider the various cases for the indices α and β. You will also need to numerically evaluate certain lattice sums.)

(6) The general form of the kinetic energy for a rotating body is

$$T = \tfrac{1}{2}I_1\big(\dot\phi\,\sin\theta\sin\psi + \dot\theta\cos\psi\big)^2 + \tfrac{1}{2}I_2\,\big(\dot\phi\,\sin\theta\cos\psi - \dot\theta\,\sin\psi\big)^2 + \tfrac{1}{2}I_3\,\big(\dot\phi\cos\theta + \dot\psi\big)^2\ ,$$

where (ϕ, θ, ψ) are the Euler angles.

(a) Find the Hamiltonian $H(p_\phi, p_\theta, p_\psi)$ for a free asymmetric rigid body.

(b) Compute the rotational partition function,

$$\xi_{\rm rot}(T) = \frac{1}{h^3}\int\limits_{-\infty}^{\infty}\!dp_\phi\int\limits_{-\infty}^{\infty}\!dp_\theta\int\limits_{-\infty}^{\infty}\!dp_\psi\int\limits_0^{2\pi}\!d\phi\int\limits_0^{\pi}\!d\theta\int\limits_0^{2\pi}\!d\psi\; e^{-H(p_\phi,p_\theta,p_\psi)/k_{\rm B}T}$$

and show that you recover eqn. 4.271 in the notes.