

Homework Set #1.

Consider the Harmonic Oscillator

$$L(x, \dot{x}) = \frac{1}{2} \dot{x}^2 - \frac{1}{2} x^2$$

under the following discretization procedure of the Feynman Path integral (Lecture 2):

$$T_0 = 2\pi \quad \text{classical period of oscillator}$$

$$\Delta t = \frac{T_0}{128}$$

$$N_D = 600 \quad x_0 = -4, \quad x_D = +4$$

$$x_{\text{start}} = 0.75$$

$$\psi_0 = \left(\frac{\alpha}{\pi}\right)^{\frac{1}{4}} e^{-\frac{\alpha}{2} (x - x_{\text{start}})^2} \quad \text{initial wavefunction}$$

$$\alpha = 2$$

1. Calculate the propagator K from the elementary K_ϵ matrix $(N_D+1) \times (N_D+1)$ dimensional,
 $\epsilon = \frac{T_0}{128} = \Delta t$ for time period $\frac{T_0}{16}$

$$K = (\Delta x)^{N-1} \cdot K_\epsilon^N(\Delta t)$$

2. Evolve the wavefunction in time with $\frac{T_0}{16}$ stepsize and measure $\langle x \rangle$ as a function of time. Make a plot
3. Calculate $\langle E \rangle$, $\langle K \rangle$, $\langle V \rangle$ as a function of time. Make a plot.
4. Calculate the evolution of the wavefunction as a function of time. Make plot.
5. Compare your plots with page 4 and 5 plots of Lecture 4-5
6. Bonus (142) Animation of the wavefunction