HW set 2

Problem 1

(a) Derive the conductivity sum rule

$$\int_0^\infty d\omega \sigma_1(\omega) = C\frac{ne^2}{m} \qquad (1)$$

using the Drude form for $\sigma_1(\omega)$. Find the value of C, which is independent of τ.

(b) To derive Eq. (1) classically in general (without using the Drude form for $\sigma_1(\omega)$):

(i) Assume an impulsive electric field $E(t) = \delta(t)$ is applied to a system of particles of mass m, charge e and number density n. Fourier-analyze the current density and electric field as

$$J(t) = \int_{-\infty}^{\infty} d\omega J(\omega) e^{-i\omega t} \qquad E(t) = \delta(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega e^{-i\omega t}$$

and using $J(\omega) = \sigma(\omega)E(\omega)$ show that

$$J(t>0) = \frac{2}{\pi}\int_0^\infty d\omega \sigma_1(\omega)\cos(\omega t) \qquad (2)$$

(ii) Find $J(t = 0^+)$ by direct calculation of the change in momentum of the particles under the electric field $E(t) = \delta(t)$.

(iii) Using the result of (ii) and Eq. (2), derive Eq. (1).

Problem 2

AM, 2.1

Problem 3

AM, 2.3

Problem 4

Assume the electronic density of state of a certain metal is of the form

$$g(\varepsilon) = \ln\frac{\varepsilon_0}{|\varepsilon|} \qquad \text{for } -\varepsilon_0 < \varepsilon < \varepsilon_0, \text{ 0 otherwise}$$

so it has a logarithmic singularity at $\varepsilon = 0$.

Find the behavior of the specific heat and Pauli paramagnetic susceptibility at very low T.