

3-25  $E = 300 \text{ keV}$ ,  $\theta = 30^\circ$

$$(a) \quad \Delta\lambda = \lambda' - \lambda_0 = \frac{h}{m_e c} (1 - \cos\theta) = (0.00243 \text{ nm}) [1 - \cos(30^\circ)] = 3.25 \times 10^{-13} \text{ m} \\ = 3.25 \times 10^{-4} \text{ nm}$$

$$(b) \quad E = \frac{hc}{\lambda_0} \Rightarrow \lambda_0 = \frac{hc}{E_0} = \frac{(4.14 \times 10^{-15} \text{ eVs})(3 \times 10^8 \text{ m/s})}{300 \times 10^3 \text{ eV}} = 4.14 \times 10^{-12} \text{ m}; \text{ thus,} \\ \lambda' = \lambda_0 + \Delta\lambda = 4.14 \times 10^{-12} \text{ m} + 0.325 \times 10^{-12} \text{ m} = 4.465 \times 10^{-12} \text{ m, and} \\ E' = \frac{hc}{\lambda'} \Rightarrow E' = \frac{(4.14 \times 10^{-15} \text{ eVs})(3 \times 10^8 \text{ m/s})}{4.465 \times 10^{-12} \text{ m}} = 2.78 \times 10^5 \text{ eV.}$$

$$(c) \quad \frac{hc}{\lambda_0} = \frac{hc}{\lambda'} + K_e, \text{ (conservation of energy)}$$

$$K_e = hc \left( \frac{1}{\lambda_0} - \frac{1}{\lambda'} \right) = \frac{(4.14 \times 10^{-15} \text{ eVs})(3 \times 10^8 \text{ m/s})}{\frac{1}{4.14 \times 10^{-12}} - \frac{1}{4.465 \times 10^{-12}}} = 22 \text{ keV}$$

3-28 (a) From conservation of energy we have  $E_0 = E' + K_e = 120 \text{ keV} + 40 \text{ keV} = 160 \text{ keV}$ .  
The photon energy can be written as  $E_0 = \frac{hc}{\lambda_0}$ . This gives

$$\lambda_0 = \frac{hc}{E_0} = \frac{1240 \text{ nm eV}}{160 \times 10^3 \text{ eV}} = 7.75 \times 10^{-3} \text{ nm} = 0.00775 \text{ nm.}$$

(b) Using the Compton scattering relation  $\lambda' - \lambda_0 = \lambda_c (1 - \cos\theta)$  where  
 $\lambda_c = \frac{h}{m_e c} = 0.00243 \text{ nm}$  and  $\lambda' = \frac{hc}{E'} = \frac{1240 \text{ nm eV}}{120 \times 10^3 \text{ eV}} = 10.3 \times 10^{-3} \text{ nm} = 0.0103 \text{ nm}$ .  
Solving the Compton equation for  $\cos\theta$ , we find

$$-\lambda_c \cos\theta = \lambda' - \lambda_0 - \lambda_c \\ \cos\theta = 1 - \frac{\lambda' - \lambda_0}{\lambda_c} = 1 - \frac{0.0103 \text{ nm} - 0.00775 \text{ nm}}{0.00243 \text{ nm}} = 1 - 1.049 = -0.049$$

The principle angle is  $87.2^\circ$  or  $\theta = 92.8^\circ$ .

(c) Using the conservation of momentum Equations 3.30 and 3.31 one can solve for the recoil angle of the electron.

$$p = p' \cos\theta + p_e \cos\phi$$

$p_e \sin\phi = p' \sin\theta$ ; dividing these equations one can solve for the recoil angle of the electron

$$\tan\phi = \frac{p' \sin\theta}{p - p' \cos\theta} = \left( \frac{h}{\lambda'} \right) \frac{\sin\theta}{\frac{h}{\lambda_0} - \frac{h}{\lambda' \cos\theta}} = \left( \frac{hc}{\lambda'} \right) \frac{\sin\theta}{\frac{hc}{\lambda_0} - \frac{hc}{\lambda' \cos\theta}} \\ = \frac{120 \text{ keV}(0.9988)}{160 \text{ keV} - 120 \text{ keV}(-0.049)} = 0.7232$$

and  $\phi = 35.9^\circ$ .

- 3-30 Maximum energy transfer occurs when the scattering angle is 180 degrees. Assuming the electron is initially at rest, conservation of momentum gives

$$hf + hf' = p_e c = \sqrt{(m_e c^2 + K)^2 - m^2 c^4} = \sqrt{(511 + 50)^2} = 178 \text{ keV}$$

while conservation of energy gives  $hf - hf' = K = 30 \text{ keV}$ . Solving the two equations gives  $E = hf = 104 \text{ keV}$  and  $hf' = 74 \text{ keV}$ . (The wavelength of the incoming photon is  $\lambda = \frac{hc}{E} = 0.0120 \text{ nm}$ .)

3-31 (a)  $E' = \frac{hc}{\lambda'}, \lambda' = \lambda_0 + \Delta\lambda$

$$\lambda_0 = \frac{hc}{E_0} = \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(3 \times 10^8 \text{ m/s})}{0.1 \text{ MeV}} = 1.243 \times 10^{-11} \text{ m}$$

$$\Delta\lambda = \left(\frac{h}{m_e c}\right)(1 - \cos\theta) = \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(1 - \cos 60^\circ)}{(9.11 \times 10^{-31} \text{ kg})(3 \times 10^8 \text{ m/s})} = 1.215 \times 10^{-12} \text{ m}$$

$$\lambda' = \lambda_0 + \Delta\lambda = 1.364 \times 10^{-11} \text{ m}$$

$$E' = \frac{hc}{\lambda'} = \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(3 \times 10^8 \text{ m/s})}{1.364 \times 10^{-11} \text{ m}} = 9.11 \times 10^4 \text{ eV}$$

- 4-1 F corresponds to the charge passed to deposit one mole of monovalent element at a cathode. As one mole contains Avogadro's number of atoms,

$$e = \frac{96\,500 \text{ C}}{6.02 \times 10^{23}} = 1.60 \times 10^{-19} \text{ C}.$$

- 4-3 Thomson's device will work for positive and negative particles, so we may apply

$$\frac{q}{m} \approx \frac{V\theta}{B^2 ld}.$$

(a)  $\frac{q}{m} \approx \frac{V\theta}{B^2 ld} = (2\,000 \text{ V}) \frac{0.20 \text{ radians}}{(4.57 \times 10^{-2} \text{ T})^2} (0.10 \text{ m})(0.02 \text{ m}) = 9.58 \times 10^7 \text{ C/kg}$

- (b) As the particle is attracted by the negative plate, it carries a positive charge and is

$$\text{a proton. } \left( \frac{q}{m_p} = \frac{1.60 \times 10^{-19} \text{ C}}{1.67 \times 10^{-27} \text{ kg}} = 9.58 \times 10^7 \text{ C/kg} \right)$$

(c)  $v_x = \frac{E}{B} = \frac{V}{dB} = \frac{2\,000 \text{ V}}{0.02 \text{ m}} (4.57 \times 10^{-2} \text{ T}) = 2.19 \times 10^6 \text{ m/s}$

- (d) As  $v_x \sim 0.01c$  there is no need for relativistic mechanics.

- 4-8 (a) From Equation 4.16 we have  $\Delta n_\infty \left(\frac{\sin\phi}{2}\right)^{-4}$  or  $\Delta n_2 = \Delta n_1 \frac{\left(\frac{\sin\phi_1}{2}\right)^4}{\left(\frac{\sin\phi_2}{2}\right)^4}$ . Thus the

number of  $\alpha$ 's scattered at 40 degrees is given by

$$\Delta n_2 = (100 \text{ cpm}) \frac{\left(\frac{\sin 20}{2}\right)^4}{\left(\frac{\sin 40}{2}\right)^4} = (100 \text{ cpm}) \left(\frac{\sin 10}{\sin 20}\right)^4 = 6.64 \text{ cpm}.$$

Similarly

$$\Delta n \text{ at } 60 \text{ degrees} = 1.45 \text{ cpm}$$

$$\Delta n \text{ at } 80 \text{ degrees} = 0.533 \text{ cpm}$$

$$\Delta n \text{ at } 100 \text{ degrees} = 0.264 \text{ cpm}$$

(b) From 4.16 doubling  $\left(\frac{1}{2}\right)m_\alpha v_\alpha^2$  reduces  $\Delta n$  by a factor of 4. Thus  $\Delta n$  at 20 degrees =  $\left(\frac{1}{4}\right)(100 \text{ cpm}) = 25 \text{ cpm}$ .

(c) From 4.16 we find  $\frac{\Delta n_{\text{Cu}}}{\Delta n_{\text{Au}}} = \frac{Z_{\text{Cu}}^2 N_{\text{Cu}}}{Z_{\text{Au}}^2 N_{\text{Au}}}$ ,  $Z_{\text{Cu}} = 29$ ,  $Z_{\text{Au}} = 79$ .

$N_{\text{Cu}}$  = number of Cu nuclei per unit area

= number of Cu nuclei per unit volume \* foil thickness

$$= \left[ (8.9 \text{ g/cm}^3) \left( \frac{6.02 \times 10^{23} \text{ nuclei}}{63.54 \text{ g}} \right) \right] t = 8.43 \times 10^{22} t$$

$$N_{\text{Au}} = \left[ (19.3 \text{ g/cm}^3) \left( \frac{6.02 \times 10^{23} \text{ nuclei}}{197.0 \text{ g}} \right) \right] t = 5.90 \times 10^{22} t$$

$$\text{So } \Delta n_{\text{Cu}} = \Delta n_{\text{Au}} (29)^2 \frac{8.43 \times 10^{22}}{(79)^2} (5.90 \times 10^2) = (100) \left(\frac{29}{79}\right)^2 \left(\frac{8.43}{5.90}\right) = 19.3 \text{ cpm}.$$

4-9 The initial energy of the system of  $\alpha$  plus copper nucleus is 13.9 MeV and is just the kinetic energy of the  $\alpha$  when the  $\alpha$  is far from the nucleus. The final energy of the system may be evaluated at the point of closest approach when the kinetic energy is zero and the potential energy is  $k(2e) \frac{Ze}{r}$  where  $r$  is approximately equal to the nuclear radius of copper. Invoking conservation of energy  $E_i = E_f$ ,  $K_\alpha = (k) \frac{2Ze^2}{r}$  or

$$r = (k) \frac{2Ze^2}{K_\alpha} = \frac{(2)(29)(1.60 \times 10^{-19})^2 (8.99 \times 10^9)}{(13.9 \times 10^6 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})} = 6.00 \times 10^{-15} \text{ m}.$$