# Quantum Mechanics PHYS 212B

Problem Set 3

Due Tuesday, February 2, 2016

**Exercise 3.1** Consider a particle in a definite initial momentum state in a large box of volume $l^2$. Now turn on a potential $V(r)$ inside the box. What is the rate at which the particle makes transitions to other momentum states? What is the differential scattering cross section corresponding to this transition rate?

**Solution 3.1** See lecture notes or Baym's Page 252.

**Exercise 3.2** Calculate the electron energy spectrum for a generic beta decay process. The mass difference between the initial and final nuclei involved is $\Delta M$ - this is the energy shared by the final state electron and antineutrino. Assume a contact weak interaction potential, energy-independent matrix element $|\langle \Psi_f \hat{H} |\Psi_i\rangle|^2$, and that the electron and antineutrino can be described as being in plane wave states.

**Solution 3.2** See lecture notes.

**Exercise 3.3** Consider a particle bound in a one-dimensional simple harmonic oscillator potential. Initially it is in its ground state. At $t = 0$ a perturbation $H(x,t) = Ax^2 e^{-t/\tau}$ is turned on. Here A is the normalization constant. What is the probability that after a sufficiently long time $t \gg \tau$ the system will have made a transition to a given excited state. Consider all final state.

**Solution 3.3** The first order perturbation is

$$\begin{aligned}
\langle n|\psi_t\rangle =& \frac{1}{i\hbar}\int_0^t dt' e^{-i\epsilon_n t'/\hbar}\langle n|Ax^2 e^{-t'/\tau}|0\rangle \\
=& \frac{\langle n|Ax^2|0\rangle}{i\hbar}\int_0^t dt' e^{\left(-\frac{1}{\tau}-\frac{i\epsilon_n}{\hbar}\right)t'} \\
=& \frac{\langle n|Ax^2|0\rangle}{i\hbar}\frac{e^{\left(-\frac{1}{\tau}-\frac{i\epsilon_n}{\hbar}\right)t}-1}{-\frac{1}{\tau}-\frac{i\epsilon_n}{\hbar}} \\
\approx& \frac{\langle n|Ax^2|0\rangle}{i\hbar}\frac{1}{\frac{1}{\tau}+\frac{i\epsilon_n}{\hbar}}
\end{aligned}$$

Since $t \gg \tau$, $e^{-t/\tau} \approx 0$.

We have $Ax^2 = A(a + a^\dagger)^2$, the nonzero element for $\langle n|Ax^2|0\rangle$ is $\langle 2|Ax^2|0\rangle$,

$$\langle 2|Ax^2|0\rangle = \frac{\sqrt{2}A\hbar}{2m\omega}$$

Finally,

$$\langle 2|\psi_t\rangle = \frac{\sqrt{2}A}{2m\omega}\frac{1}{\frac{i}{\tau}-\frac{5\omega}{2}}$$

For given exited state $|n > 0\rangle$, the probability

$$P(|n=2\rangle) = |\langle 2|\psi_t\rangle|^2 = \frac{A^2}{2m\omega^2}\frac{1}{1/\tau^2 + 25\omega^2/4}$$

$$P(|n \neq 2\rangle) = 0$$