

PHYSICS 200B : CLASSICAL MECHANICS
PROBLEM SET #1

[1] Evaluate all cases of $\{A_i, A_j\}$, where

$$\begin{aligned} A_1 &= \frac{1}{4}(x^2 + p_x^2 - y^2 - p_y^2) & A_3 &= \frac{1}{2}(x p_y - y p_x) \\ A_2 &= \frac{1}{2}(x y + p_x p_y) & A_4 &= x^2 + y^2 + p_x^2 + p_y^2 . \end{aligned}$$

[2] Determine the generating function $F_3(p, Q)$ which produces the same canonical transformation as the generating function $F_2(q, P) = q^2 \exp(P)$.

[3] Show explicitly that the canonical transformation generated by an arbitrary $F_1(q, Q, t)$ preserves the symplectic structure of Hamilton's equations. That is, show that

$$M_{aj} \equiv \frac{\partial \Xi_a}{\partial \xi_j}$$

is symplectic. *Hint : Start by writing $p_\sigma = \frac{\partial F_1}{\partial q_\sigma}$ and $P_\sigma = -\frac{\partial F_1}{\partial Q_\sigma}$, and then evaluate the differentials dp_σ and dP_σ .*

[4] Consider the small oscillations of an anharmonic oscillator with Hamiltonian

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 q^2 + \alpha q^3 + \beta q p^2$$

under the assumptions $\alpha q \ll m\omega^2$ and $\beta q \ll \frac{1}{m}$.

(a) Working with the generating function

$$F_2(q, P) = qP + a q^2 P + b P^3 ,$$

find the parameters a and b such that the new Hamiltonian $\tilde{H}(Q, P)$ does not contain any anharmonic terms up to third order (*i.e.* no terms of order Q^3 nor of order QP^2).

(b) Determine $q(t)$.

[5] A particle of mass m moves in one dimension subject to the potential

$$U(x) = \frac{k}{\sin^2(x/a)} .$$

(a) Obtain an integral expression for Hamilton's characteristic function.

(b) Under what conditions may action-angle variables be used?

- (c) Assuming that action-angle variables are permissible, determine the frequency of oscillation by the action-angle method.
- (d) Check your result for the oscillation frequency in the limit of small oscillations.